Density and momentum dependence of nuclear symmetry energy in the relativistic Hartree-Fock approximation

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Hartree-Fock approach

The theoretical advantages of a Hartree-Fock (HF) approach

- HF theory is a method of the simplest approximation for solving the many-body system.
- The N-body wave function of the system is approximated by **a single Slater determinant**.
- **Antisymmetry** of the wave function can be satisfied.

  "Pauli exclusion principle"

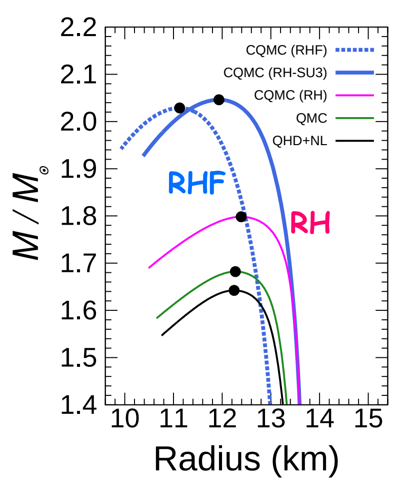
- **Nuclear tensor force** can be included automatically via **exchange (Fock) diagrams**.

$$S_{12} = 3\,(\sigma_1 \cdot q)\,(\sigma_2 \cdot q) - \sigma_1 \cdot \sigma_2 q^2\,,$$

- In a relativistic framework, **the nuclear tensor force** is also seen in **the exchange (Fock) diagrams**.

L. J. Jiang, S. Yang, J. M. Dong, and W. H. Long, Phys. Rev. C **91**, 025802 (2015).
L. J. Jiang, S. Yang, B. Y. Sun, W. H. Long, and H. Q. Gu, Phys. Rev.=C **91**, 034326 (2015).

Neutron-star matter



The advantage of **the relativisic Hartree-Fock (RHF) calculation** is also seen in dense nuclear matter.

⇒ **Equation of state for neutron stars**

- Fock contribution suppresses **the hyperon appearance** in the core of neutron stars.
- The maximum mass of a neutron star can reach the $2M_\odot$, even if **the hyperons** are taken into account.
- **The radius** of a neutron star in **the RHF model becomes smaller** than that in **the RH models**.

√ Fock terms play a important role in supporting a massive neutron star.

TM, T. Katayama, and K. Saito, Phys. Lett. B **709**, 242 (2012).
TM, T. Katayama, and K. Saito, Astrophys. J. Suppl. **203**, 22 (2012).
TM, M. K. Cheoun, and K. Saito, Phys. Rev. C **88**, 015802 (2013).

Motivation

There are many theoretical studies which are focused on the nuclear properties around the normal nuclear matter density.

- Non-relativistic framework:

  If we consider the properties of dense nuclear matter or neutron-star matter, **"relativity"** may affect the equation of state.

- Relativistic mean-field theory:

  Only **the direct diagram** is included and **the exchange contribution is not considered**.
  Called **relativistic Hartree (RH)** models

Our aim:

- Using the **relativistic Hartree-Fock** approximation, we study the nuclear properties not only around the saturation density but also at higher densities.
- **Nuclear symmetry energy** and **Fock contribution**.

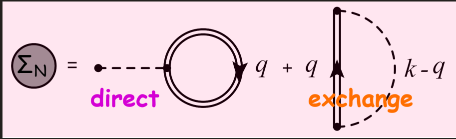
Nucleon self-energy

The nucleon self-energy is given by the Lorentz covariant form with **scalar (s)**, **time (0)**, and **space (v)** components.

$$\Sigma_N(k) = \Sigma_N^s(k) - \gamma_0 \Sigma_N^0(k) + \left(\vec{\gamma}\cdot\hat{k}\right)\Sigma_N^v, \quad N = n, p.$$

Within the relativistic Hartree-Fock approximation, the Σ_N is composed of the **direct** and **exchange** diagrams.

$$\Sigma_N^i(k) = \Sigma_N^{i,dir} + \Sigma_N^{i,ex}, \quad i = s, 0, v.$$



Inserting this form into the Dirac equation, we get the effective nucleon mass and momentum in nuclear matter.

$$M_N^*(k) = M_N + \Sigma_N^s(k), \quad k_N^*(k) = \left(k^0 + \Sigma_N^0(k), \vec{k} + \hat{k}\Sigma_N^v(k)\right).$$

RHF calculation for nuclear matter

Lagrangian density for uniform hadronic matter:

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{int}.$$

Interaction Lagrangian density: mesons (σ, ω, $\vec{\pi}$, and $\vec{\rho}$)

$$\mathcal{L}_{int} = \sum_{N=n,p} \left(\mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\pi + \mathcal{L}_\rho\right),$$

scalar $$\mathcal{L}_\sigma = g_{\sigma N}\, \bar{\psi}_N \sigma \psi_N,$$

vector $$\mathcal{L}_\omega = -\, g_{\omega N}\, \bar{\psi}_N \gamma_\mu \omega^\mu \psi_N,$$

pseudovector $$\mathcal{L}_\pi = -\frac{f_{\pi N}}{m_\pi}\, \bar{\psi}_N \gamma_5 \gamma_\mu \partial^\mu \vec{\pi} \psi_N \cdot \vec{\tau}_N,$$

$$\mathcal{L}_\rho = -\, g_{\rho N}\, \bar{\psi}_N \gamma_\mu \vec{\rho}^{\,\mu} \psi_N \cdot \vec{\tau}_N + \frac{f_{\rho N}}{2\mathcal{M}}\, \bar{\psi}_N \sigma_{\mu\nu} \partial^\nu \vec{\rho}^{\,\mu} \psi_N \cdot \vec{\tau}_N.$$

vector **tensor**

We fit the π-N and ρ-N coupling constants to their well-known physical values: ${f_{\pi N}}^2/4\pi = 0.08$ and ${g_{\rho N}}^2/4\pi = 0.55$.

$f_{\rho N}/g_{\rho N} = 6.0$. ($\pi$N-scattering data)

The following nonlinear term is also introduced in order to reproduce the saturation properties of nuclear matter at the mean-field level.

$$U_{NL} = \frac{1}{3} g_2 \bar{\sigma}^3 + \frac{1}{4} g_3 \bar{\sigma}^4.$$

Coupling constants and U_N^{SEP}

Relativistic Hartree-Fock calculations with **enhanced Fock contribution** **(RHF-EFC)**

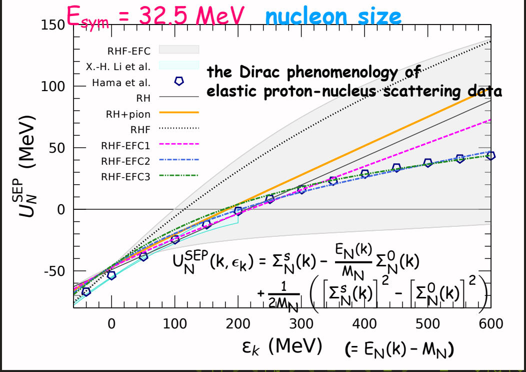
Coupling constants, $g_{\sigma N}$, $g_{\omega N}$, g_2, and g_3 are determined in order to reproduce the saturation properties at the normal nuclear matter density, $\rho_0 = 0.16\ \mathrm{fm}^{-3}$.

- $E = -0.16$ MeV,
- $p = 0$,
- $M_N^*/M_N = 0.70$,
- $K_0 = 250$ MeV.

We introduce **a form factor** and **a weight parameter** at each exchange vertex:

$$g_{MN} \to g_{MN} w_M \frac{1}{\left(1 - p^2/\Lambda_i^2\right)^2}, \quad M = \sigma, \omega, \rho.$$

Model	RH	RH+pion	RHF	RHF-EFC1	RHF-EFC2	RHF-EFC3
M_L^*/M_N	0.754	0.762	0.733	0.762	0.763	0.762
U_N^{SEP} (MeV)	−52.7	−51.8	−55.3	−51.8	−51.7	−51.9
J_0 (MeV)	−362	−319	−368	−338	−378	−417
E_{sym} (MeV)	23.5	26.7	46.3	32.5*	32.5*	32.5*
L (MeV)	67.0	73.7	123.6	88.1	105.2	113.5
K_{sym} (MeV)	32.0	21.9	−41.0	11.1	83.8	116.7
K_{asy} (MeV)	−370	−420	−783	−517	−547	−564
$K_{sat,2}$ (MeV)	−273	−326	−601	−398	−388	−375
w_σ	–	–	1.00	0.75	1.50	2.00
w_ω	–	–	1.00	0.80	2.12	2.88
w_ρ	–	–	1.00	0.25	0.50	0.60
w_π	–	1.00	1.00	1.00	1.00	1.00



Numerical results

Nuclear matter properties at the saturation density
and its momentum dependence

Nuclear symmetry energy

$$E_{sym}(\rho_B) = E_{sym}^{kin}(\rho_B) + E_{sym}^{pot,dir}(\rho_B) + E_{sym}^{pot,ex}(\rho_B)$$

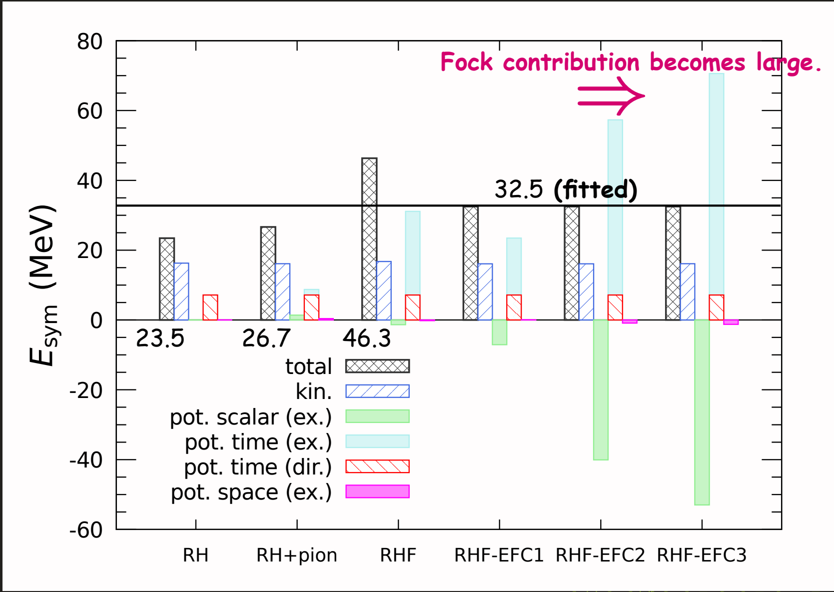
$$= \frac{1}{6}\frac{k_F^*}{E_F^*}k_F + \frac{1}{2}\frac{g_{\rho N}^2}{m_\rho^2}\rho_B$$

$$+ \frac{1}{8}\rho_B\left(\frac{M_F^*}{E_F^*}\left[\partial\Sigma_F^{ex,s}(\rho_B)\right] - \left[\partial\Sigma_F^{ex,0}(\rho_B)\right] + \frac{k_F^*}{E_F^*}\left[\partial\Sigma_F^{ex,v}(\rho_B)\right]\right),$$

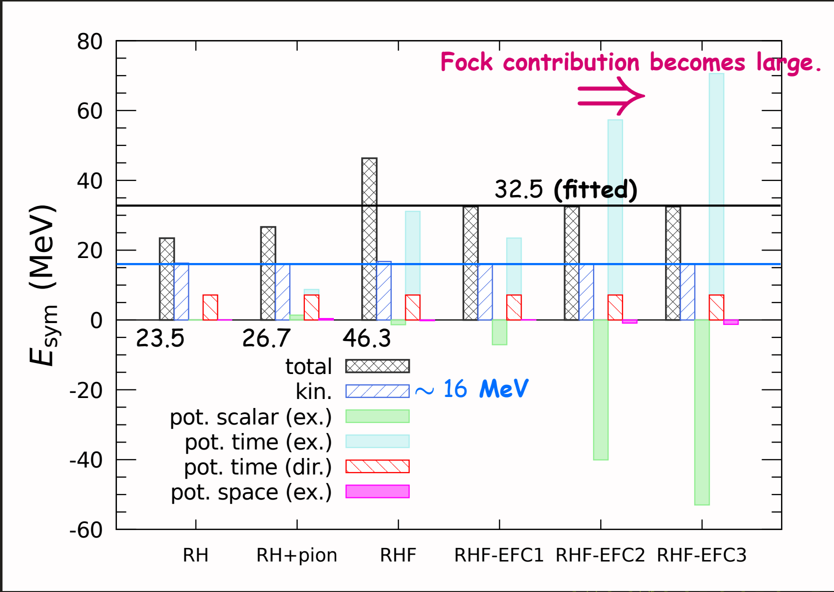
with $\rho_B = \rho_n + \rho_p$ and

$$\left[\partial\Sigma_F^{ex,i}(\rho_B)\right] = \left(\frac{\partial}{\partial\rho_p} - \frac{\partial}{\partial\rho_n}\right)\left[\Sigma_p^{ex,i}(k_{F_p}) - \Sigma_n^{ex,i}(k_{F_n})\right]_{\rho_3=0}, \quad i = s, 0, v.$$

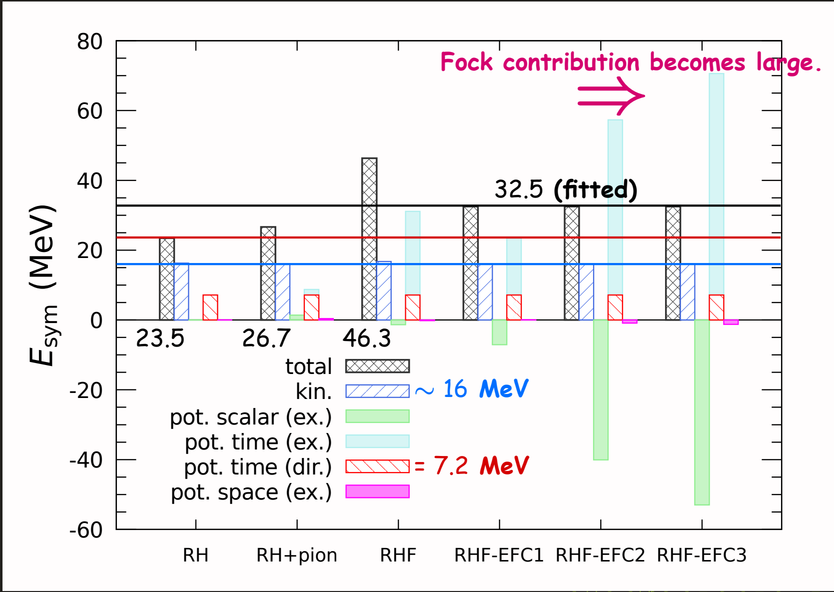
Nuclear symmetry energy



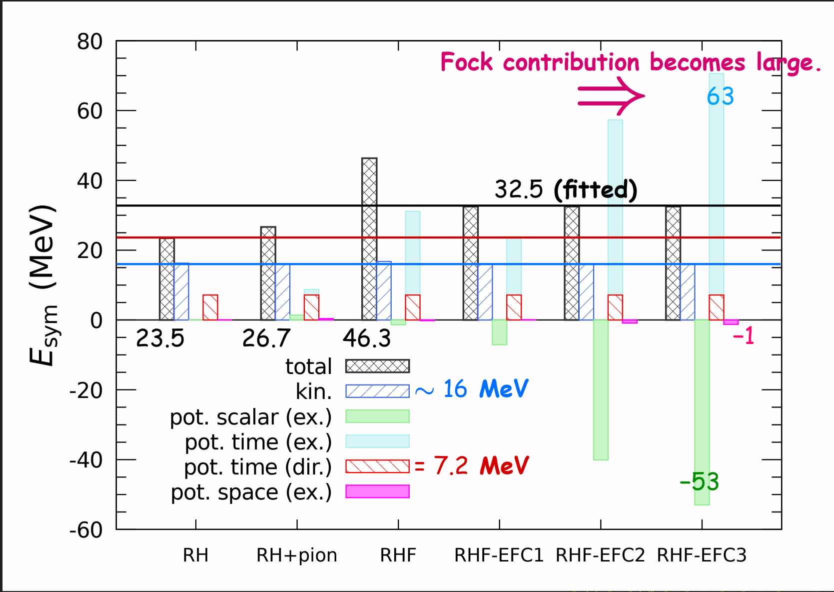
Nuclear symmetry energy



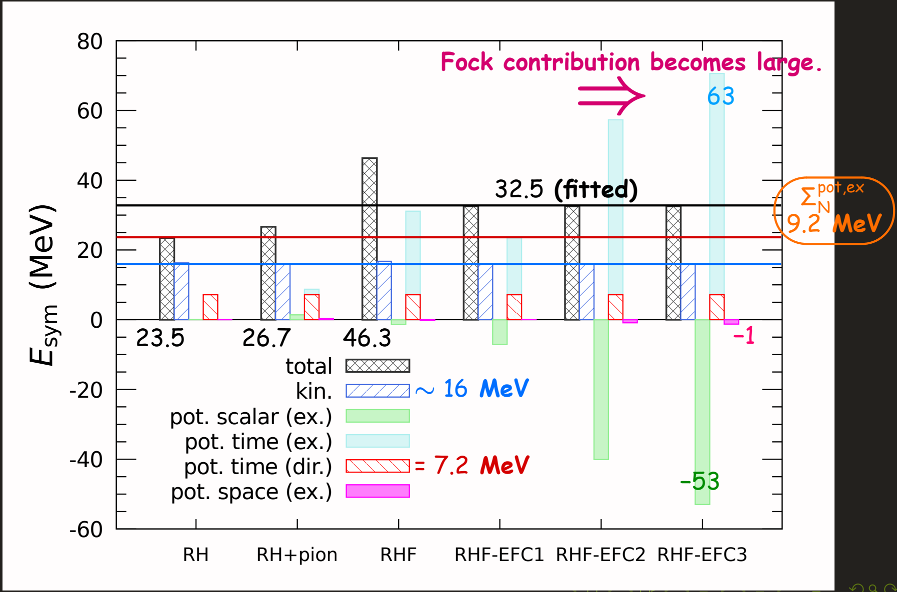
Nuclear symmetry energy



Nuclear symmetry energy



Nuclear symmetry energy

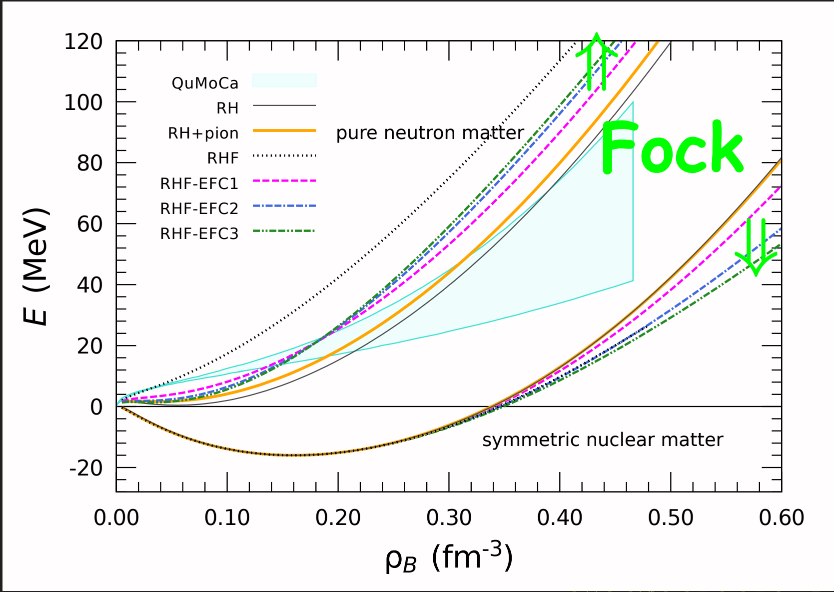


Numerical results

Density dependence of nuclear matter properties

- Binding energy for symmetric nuclear and pure neutron matter
- Nuclear symmetry energy

Nuclear binding energy



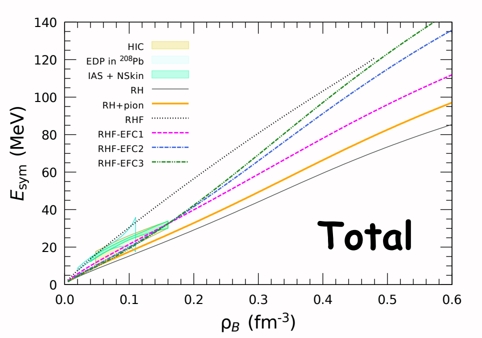
Symmetry energy

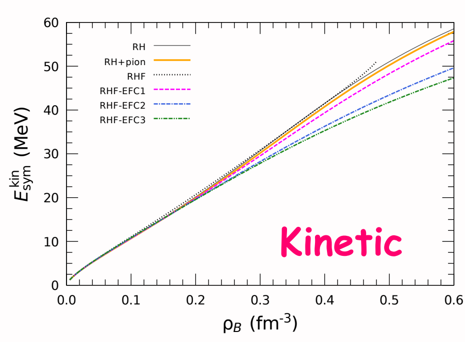
Nuclear symmetry energy can be divided into the **kinetic** and **potential** parts.

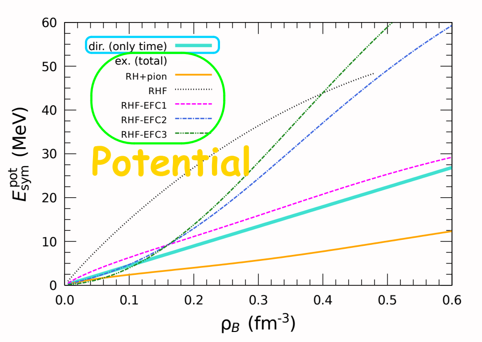
$$E_{sym}(\rho_B) = E_{sym}^{kin}(\rho_B) + E_{sym}^{pot,dir}(\rho_B) + E_{sym}^{pot,ex}(\rho_B).$$

$$\frac{1}{6}\frac{k_F^*}{E_F^*}k_F \qquad \frac{1}{2}\frac{g_{\rho N}^2}{m_\rho^2}\rho_B \qquad \text{Fock}$$

Fock contribution is composed of the scalar (s), time(0), and space(v) components.

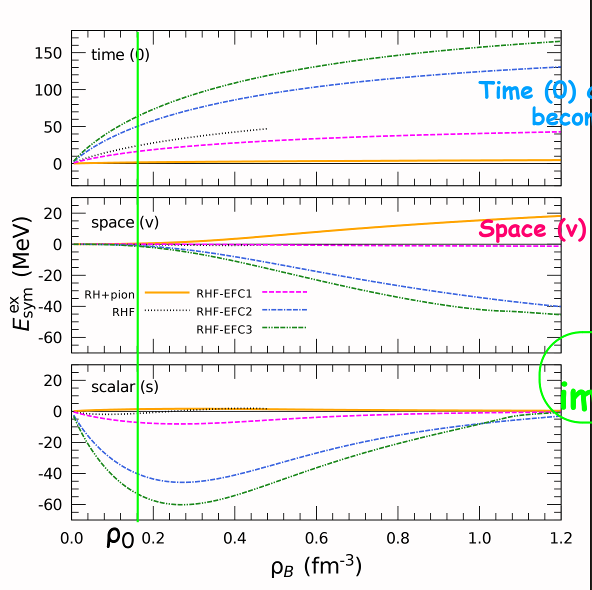






Fock contribution of symmetry energy

$E_{sym}^{pot,ex}(\rho_B)$ { scalar (s), time (0), space (v) }



Time (0) component becomes dominant.

Space (v) component is not small.

⇓

Σ_N^v is important.

Summary

- Using the **relativistic Hartree-Fock** approximation, we study how the **Fock contribution** affects **nuclear symmetry energy** at higher densities as well as at the saturation density.

Saturation density, ρ_0:

- We estimate the strength of nuclear symmetry energy.

$$E_{sym}(\rho_0) = E_{sym}^{kin}(\rho_0) + E_{sym}^{pot,dir}(\rho_0) + E_{sym}^{pot,ex}(\rho_0),$$
$$32.5\ \text{MeV} \sim 16.1\ \text{MeV},\ 7.2\ \text{MeV},\ 9.2\ \text{MeV}$$

Higher densities:

- **Fock contribution** increases **nuclear symmetry energy**.
- Time (0) component of $E_{sym}^{pot,ex}(\rho_B)$ becomes dominant as the density increases.
- We have to consider the space (v) component as well as scalar (s) and time (0) components self-consistently.

Thank You for Your Attention.

This work was supported by JSPS KAKENHI Grant Number JP17K14298.

Lorentz covariant decomposition of ϵ and E_{sym}

Energy density for nuclear matter:

$$\begin{aligned}\epsilon &= \epsilon_{nucl} + \epsilon_{NL} \\ &= \sum_{N=n,p} \frac{1}{\pi^2} \int_0^{k_{F_N}} dk\, k^2 \left[e_N^{kin}(k) + e_N^{pot}(k) \right] - \frac{1}{2} \left(\frac{1}{3} g_2 \bar{\sigma}^3 + \frac{1}{2} \bar{\sigma}^4 \right),\end{aligned}$$

with

$$e_N^{kin}(k) = E_N^*(k), \quad e_N^{pot}(k) = -\frac{1}{2} \left[\frac{\Sigma_N^s(k) M_N^*(k)}{E_N^*(k)} + \Sigma_N^0(k) + \frac{\Sigma_N^v(K) k_N^*(k)}{E_N^*(k)} \right].$$

Nuclear symmetry energy: $\rho_B = \rho_n + \rho_p$ and $\rho_3 = \rho_p - \rho_n$.

$$\begin{aligned}E_{sym}(\rho_B) &= \frac{1}{2} \rho_B \left[\frac{\partial^2 E(\rho_B, \rho_3)}{\partial \rho_3^2} \right]_{\rho_3=0,\, \rho_B:\mathrm{fixed}} = E_{sym}^{kin}(\rho_B) + E_{sym}^{pot,dir}(\rho_B) + E_{sym}^{pot,ex}(\rho_B) \\ &= \frac{1}{6} \frac{k_F^*}{E_F^*} k_F + \frac{1}{2} \frac{g_{\rho N}^2}{m_\rho^2} \rho_B + \frac{1}{8} \rho_B \left(\frac{M_F^*}{E_F^*} \left[\partial \Sigma_F^{ex,s}(\rho_B) \right] - \left[\partial \Sigma_F^{ex,0}(\rho_B) \right] + \frac{k_F^*}{E_F^*} \left[\partial \Sigma_F^{ex,v}(\rho_B) \right] \right),\end{aligned}$$

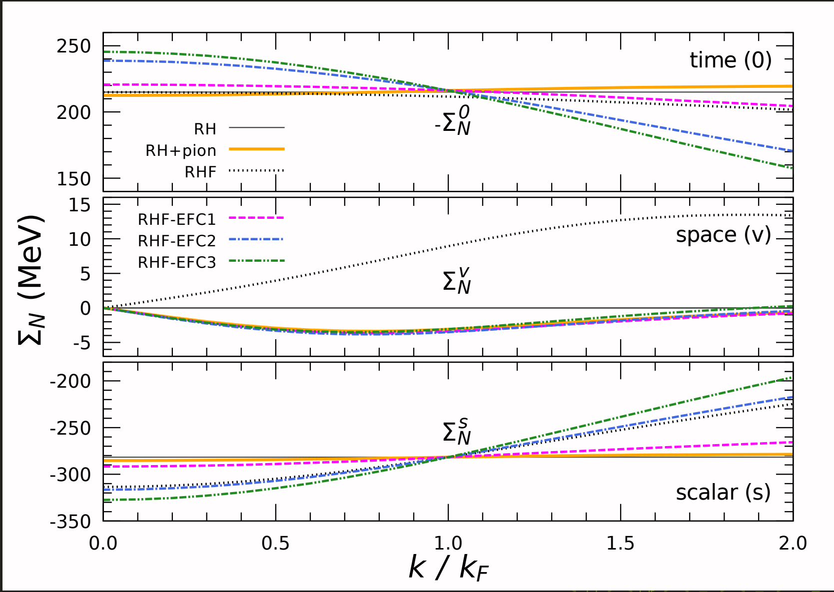
with

$$\left[\partial \Sigma_F^{ex,i}(\rho_B) \right] = \left(\frac{\partial}{\partial \rho_p} - \frac{\partial}{\partial \rho_n} \right) \left[\Sigma_p^{ex,i}(k_{F_p}) - \Sigma_n^{ex,i}(k_{F_n}) \right]_{\rho_3=0}, \quad i = s, 0, v.$$

		RHF			RHF-EFC1		
		Σ_N^s	Σ_N^0	Σ_N^v	Σ_N^s	Σ_N^0	Σ_N^v
Direct	σ	-156	0	0	-247	0	0
	ω	0	-183	0	0	-185	0
Exchange	σ	14	-15	-1	15	-15	-1
	ω	-62	-33	-1	-41	-21	-1
	π	-4	4	-3	-4	4	-3
	ρ	-73	15	14	-5	1	1
		$(-14, -63, 4)$	$(-7, 22, 0)$	$(0, -2, 16)$	$(-1, -4, 0)$	$(0, 1, 0)$	$(0, 0, 1)$
Total		-282	-212	9	-282	-216	-3

		RHF-EFC2			RHF-EFC3		
		Σ_N^s	Σ_N^0	Σ_N^v	Σ_N^s	Σ_N^0	Σ_N^v
Direct	σ	-133	0	0	-83	0	0
	ω	0	-104	0	0	-74	0
Exchange	σ	34	-36	-1	41	-43	-2
	ω	-160	-84	-3	-208	-109	-3
	π	-4	4	-3	-4	4	-3
	ρ	-18	4	3	-26	5	5
		$(-3, -16, 1)$	$(-2, 6, 0)$	$(0, 0, 4)$	$(-5, -23, 1)$	$(-3, 8, 0)$	$(0, -1, 6)$
Total		-282	-216	-4	-282	-216	-3

Nucleon self-energy at ρ_0



Matter properties

Model	RH	RH+pion	RHF	RHF-EFC1	RHF-EFC2	RHF-EFC3
M^*_L/M_N	0.754	0.762	0.733	0.762	0.763	0.762
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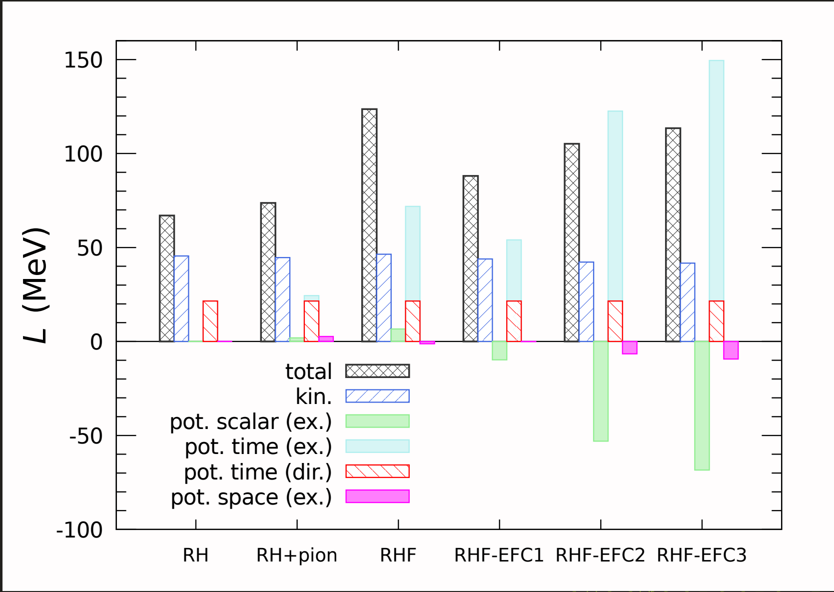
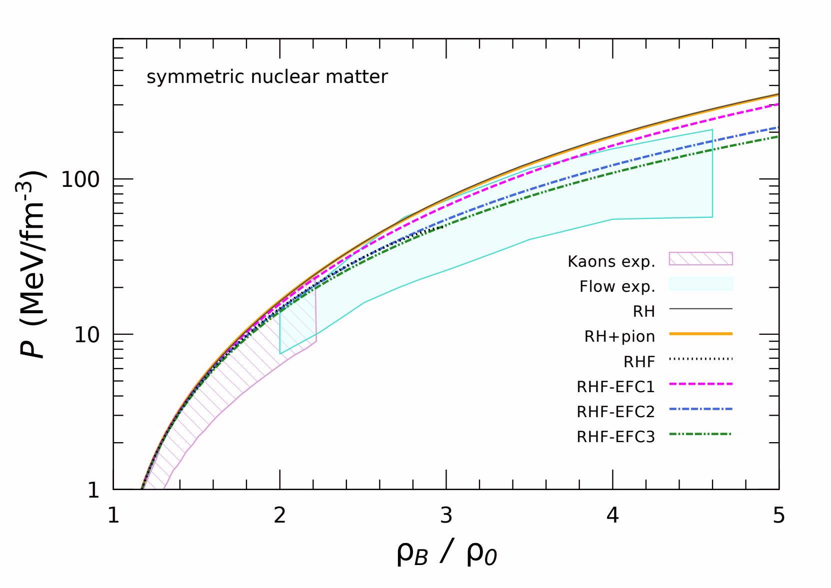
w_ω	–	–	1.00	0.80	2.12	2.88
w_ρ	–	–	1.00	0.25	0.50	0.60
w_π	–	1.00	1.00	1.00	1.00	1.00

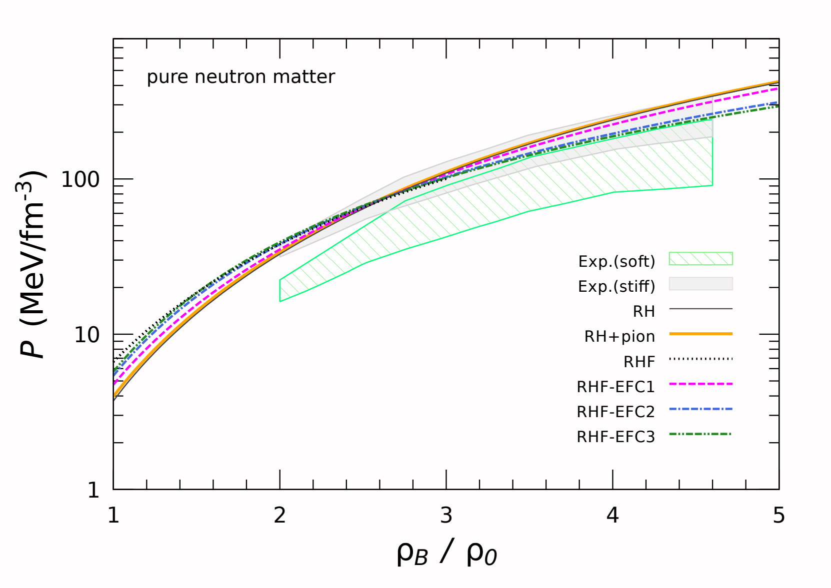
Slope parameter



Pressure: SNM



Pressure: PNM



Nucleon effective mass