Nuclear Matter Fourth-Order Symmetry Energy

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- The symmetry energy (Esym)
- The fourth-order symmetry energy (Esym,4)
- Isospin-quartic term in nuclear mass (asym,4)
- Esym,4 vs asym,4
- Summary

Refs: [1] Rui Wang/LWC, PLB773, 62 (2017);
[2] Jie Pu/Z. Zhang/LWC, arXiv:1708.02132

“7th International Symposium on Nuclear Symmetry Energy – NuSYM17”,
September 4 – 7, 2017, GANIL, Caen, France

Outline

- **The symmetry energy (Esym)**
- **The fourth-order symmetry energy (Esym,4)**
- **Isospin-quartic term in nuclear mass (asym,4)**
- **Esym,4 vs asym,4**
- **Summary**

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The Nuclear Matter Symmetry Energy

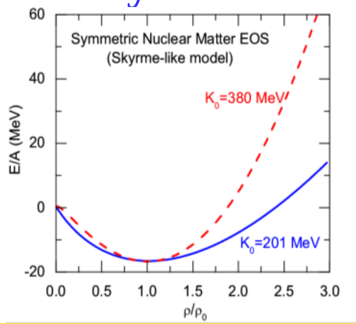
EOS of Isospin Asymmetric Nuclear Matter (Parabolic law)

$$E(\rho,\delta) = E(\rho,0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \quad \delta = (\rho_n - \rho_p)/\rho$$

Symmetric Nuclear Matter
(relatively well-determined)

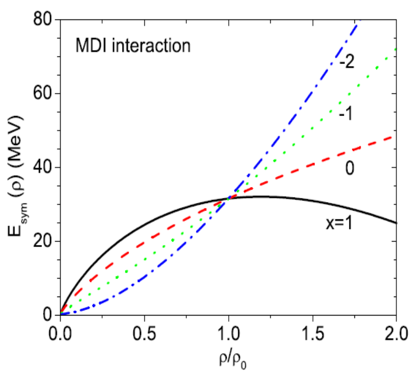
Isospin asymmetry

Symmetry energy term
(largely uncertain)



Nuclear Matter Symmetry Energy

$$E_{\text{sym}}(\rho) \equiv \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial \delta^2}$$



$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{\text{sym}}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots\cdots, \quad (\rho \sim \rho_0)$$

$E_{\text{sym}}(\rho_0) \approx 30$ MeV (LD mass formula: *Myers & Swiatecki, NPA81; Pomorski & Dudek, PRC67*)

$$L \equiv 3\rho_0 \left.\frac{\partial E_{\text{sym}}(\rho)}{\partial \rho}\right|_{\rho=\rho_0}$$ (Many-Body Theory: L : –50 ~ 200 MeV; Exp: ???)

$$K_{\text{sym}} \equiv 9\rho_0^2 \left.\frac{\partial^2 E_{\text{sym}}(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0}$$ (Many-Body Theory: K_{sym} : –700 ~ 466 MeV; Exp: ???)

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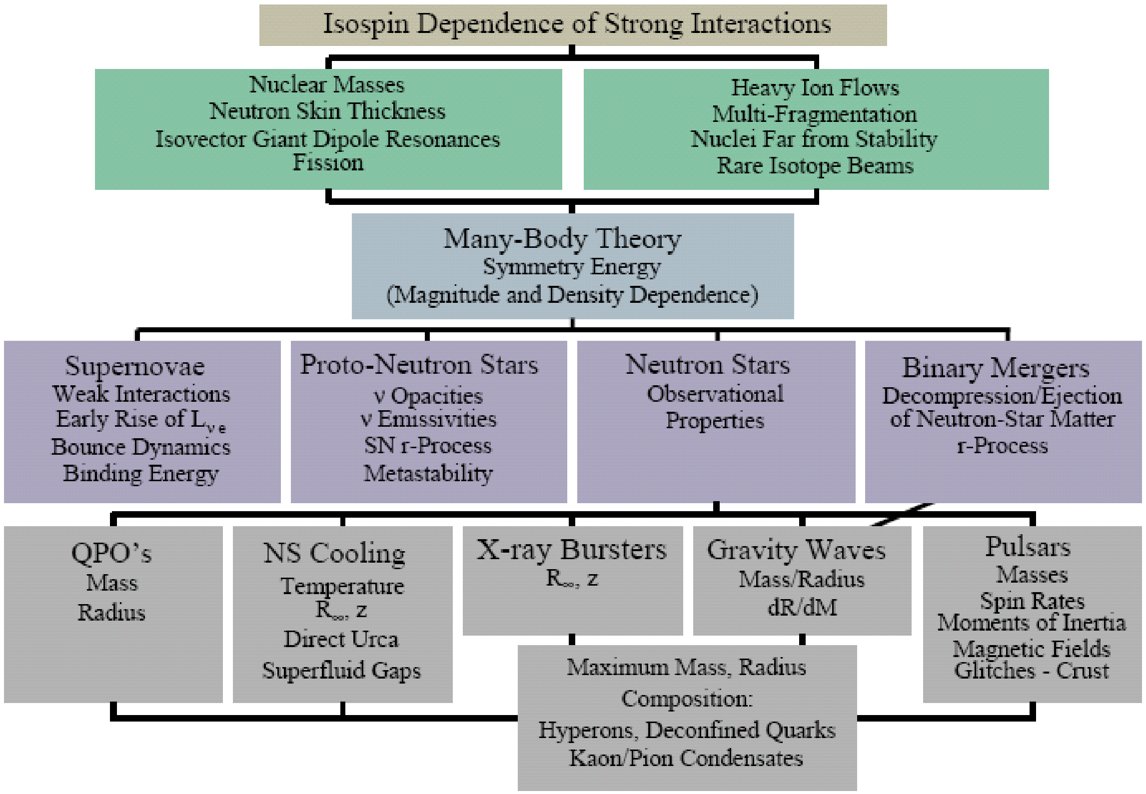
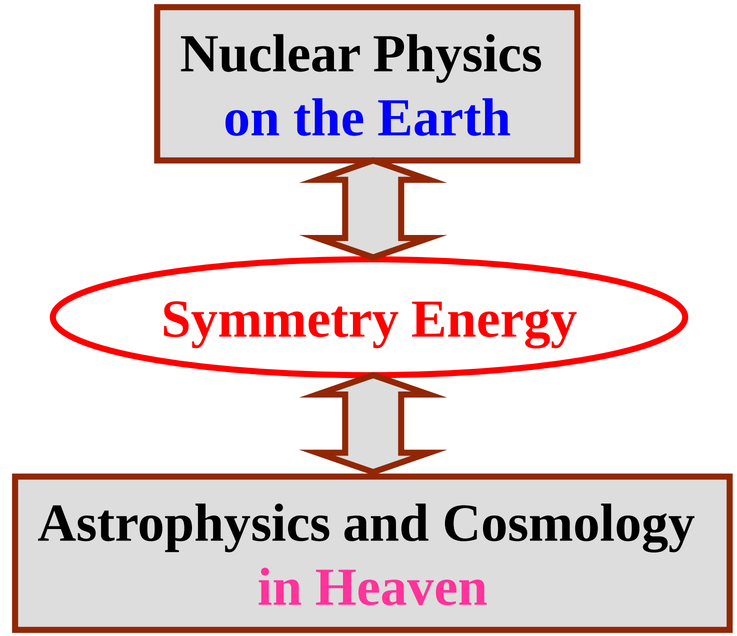
Why Symmetry Energy?

21.65.Ef

The multifaceted influence of the nuclear symmetry energy

A.W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, ***Phys. Rep.* 411, 325 (2005).**





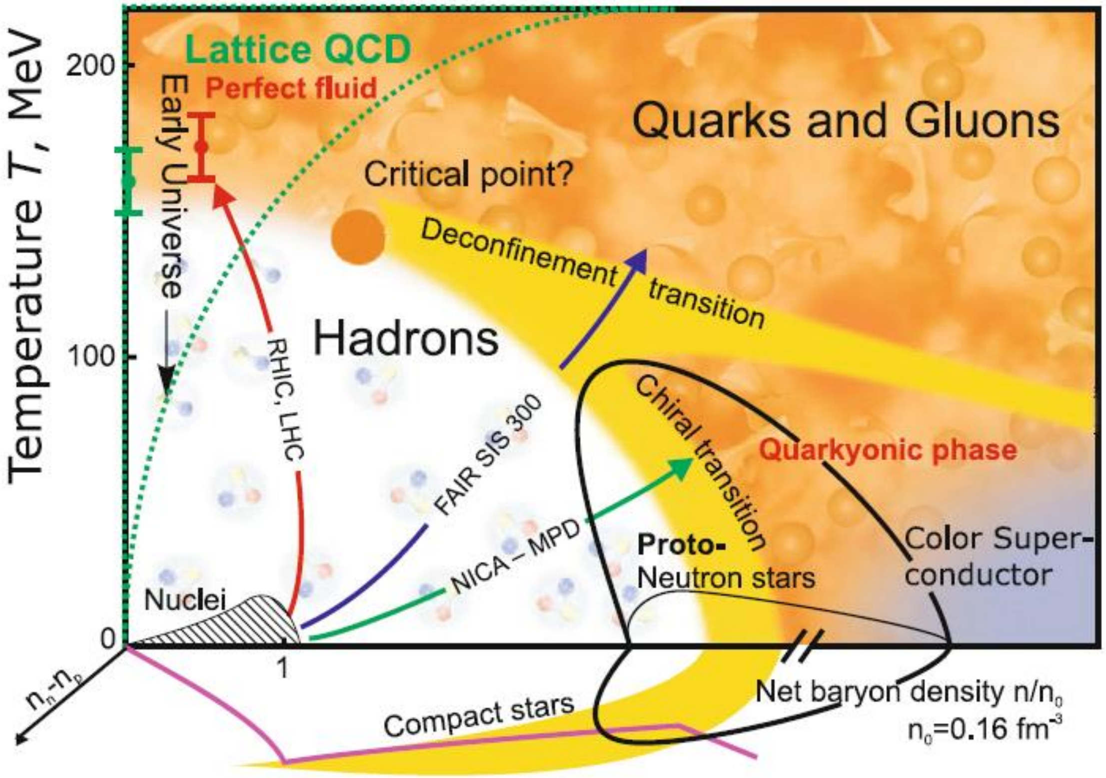
The symmetry energy is also related to some issues of fundamental physics:

1. **The precision tests of the SM through atomic parity violation observables (Sil et al., PRC2005)**
2. **Possible time variation of the gravitational constant (Jofre et al. PRL2006; Krastev/Li, PRC2007)**
3. **Non-Newtonian gravity proposed in the grand unified theories (Wen/Li/Chen, PRL2009)**
4. **Dark Matter (Zheng/Zhang/Chen, JCAP2014; Zheng/Sun/Chen, ApJ2015)**

Phase Diagram of Strong Interaction Matter

QCD Phase Diagram in 3D: density, temperature, and isospin

V.E. Fortov, Extreme States of Matter – on Earth and in the Cosmos, Springer-Verlag Berlin Heidelberg 2011



Esym: Important for understanding the EOS of strong interaction matter and QCD phase transitions at extreme isospin conditions

1. Heavy Ion Collisions (Terrestrial Lab);
2. Compact Stars(In Heaven); …

Quark Matter
Symmetry Energy ?

M. Di Toro et al. NPA775, 102(2006); Pagliara/Schaffner-Bielich, PRD81, 094024(2010); Shao et al., PRD85, 114017(2012);Chu/Chen, ApJ780, 135 (2014); LWC, arXiv:1708.04433

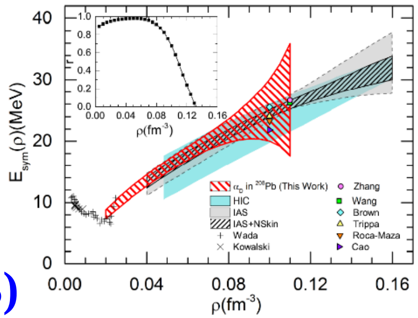
At extremely high baryon density, the main degree of freedom could be the deconfined quark matter rather than confined baryon matter, and there we should consider quark matter symmetry energy (isospin symmetry is still satisfied). The isopsin asymmetric quark matter could be produced/exist in HIC/Compact Stars

E_{sym}: Current Status

● Cannot be that all the constraints obtained so far on E_{sym} (ρ_0) and L are equivalently reliable since some of them don't have any overlap. However, essentially all the constraints seem to agree with:

$$E_{sym}(\rho_0) = 32.5 \pm 2.5 \text{ MeV}$$
$$L = 55 \pm 25 \text{ MeV}$$

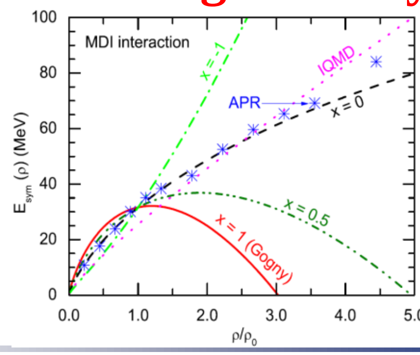
Z. Zhang/LWC, PLB726, 234 (2013)



● The symmetry energy at subsaturation densities have been relatively well-constrained

● All the constraints on the high density Esym come from HIC's (FOPI), and all of them are based on transport models. The constraints on the high density Esym are still elusive and controversial for the moment !!!

Xiao/Li/Chen/Yong/Zhang, PRL102, 062502 (2009)



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Fourth-Order Symmetry Energy

EOS of Isospin Asymmetric Nuclear Matter

$$E(\rho,\delta) = E(\rho,0) + E_{\mathrm{sym}}(\rho)\delta^2 + E_{\mathrm{sym},4}(\rho)\delta^4 + O(\delta^6), \quad \delta = (\rho_n - \rho_p)/\rho$$

Symmetric Nuclear Matter (relatively well-determined)

Symmetry energy term (largely uncertain)

4th-Order Symmetry energy term (poorly known)

Nuclear Matter Fourth-Order Symmetry Energy

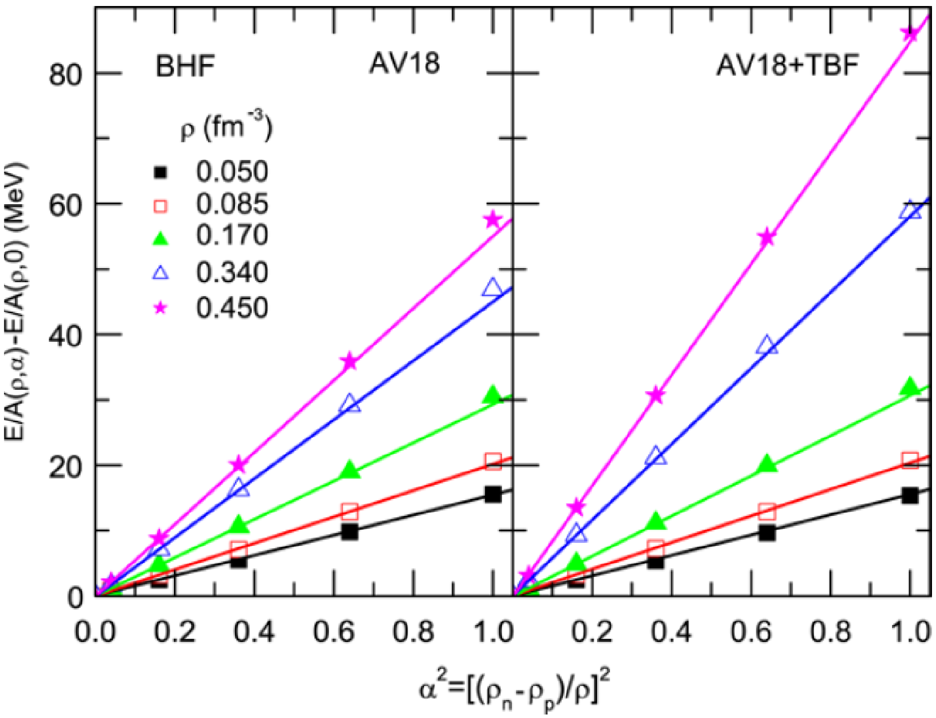
$$E_{\mathrm{sym},4}(\rho) \equiv \frac{1}{4!}\frac{\partial^4 E(\rho,\delta)}{\partial\delta^4}$$

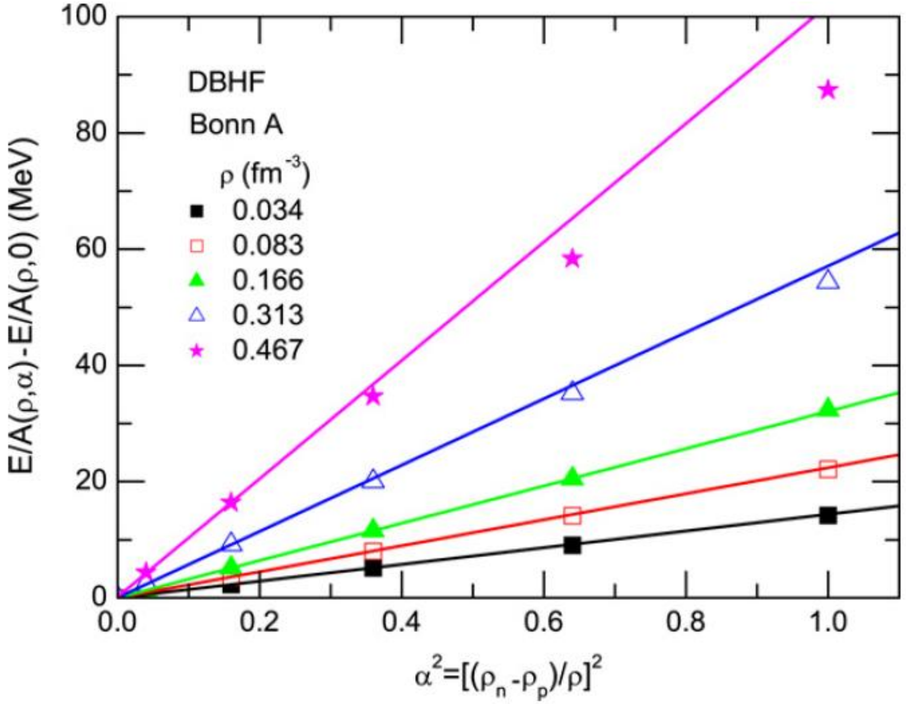
- **Esym,4 is strongly model dependent, although most models predict quite small values compared to Esym – Parabolic Law**
- **Essentially no experimental or empirical information on Esym,4**
- **No any fundamental theory/principle require Esym,4 must be small**

Parabolic Law from BHF/DBHF

W. Zuo, A. Lejeune, U. Lombardo, J.F. Mathiot, Eur. Phys. J. A 14 (2002) 469.

E.N.E. van Dalena, C. Fuchs, A. Faessler, Eur. Phys. J. A 31 (2007) 29.

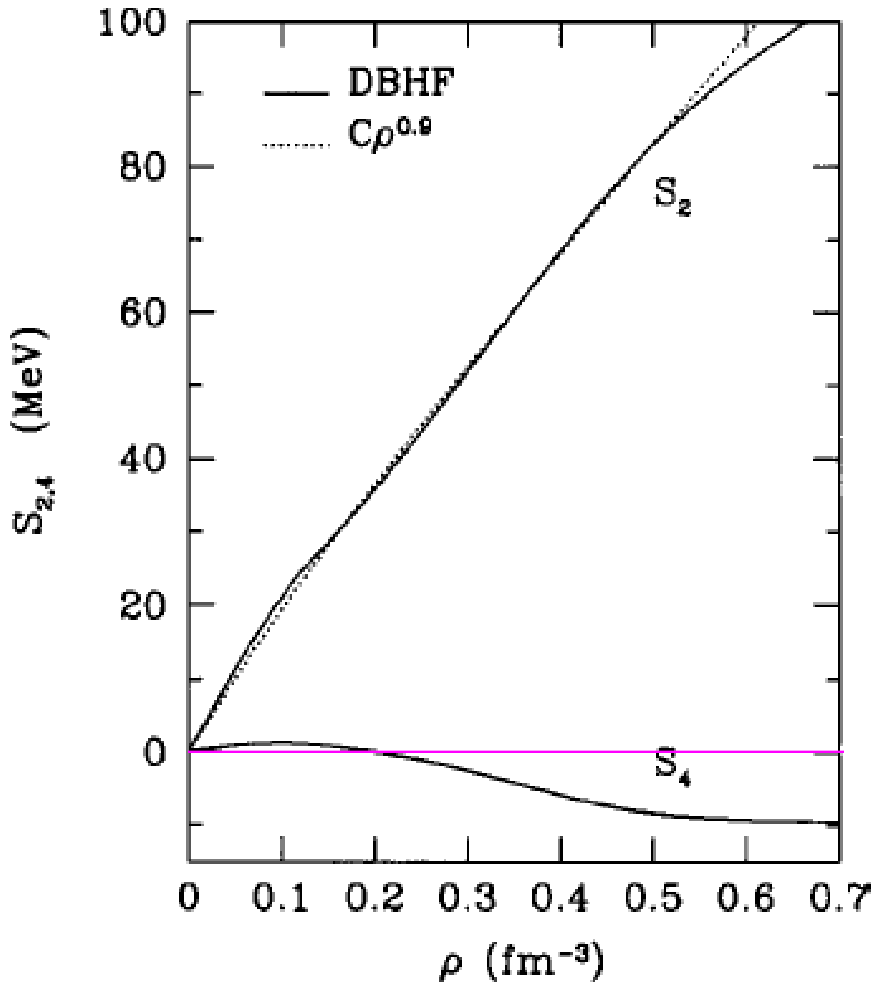




The parabolic approximation seems good at lower densities but failed at higher densities. More careful calculations and quantitative results are needed to confirm

Esym,4 in DBHF

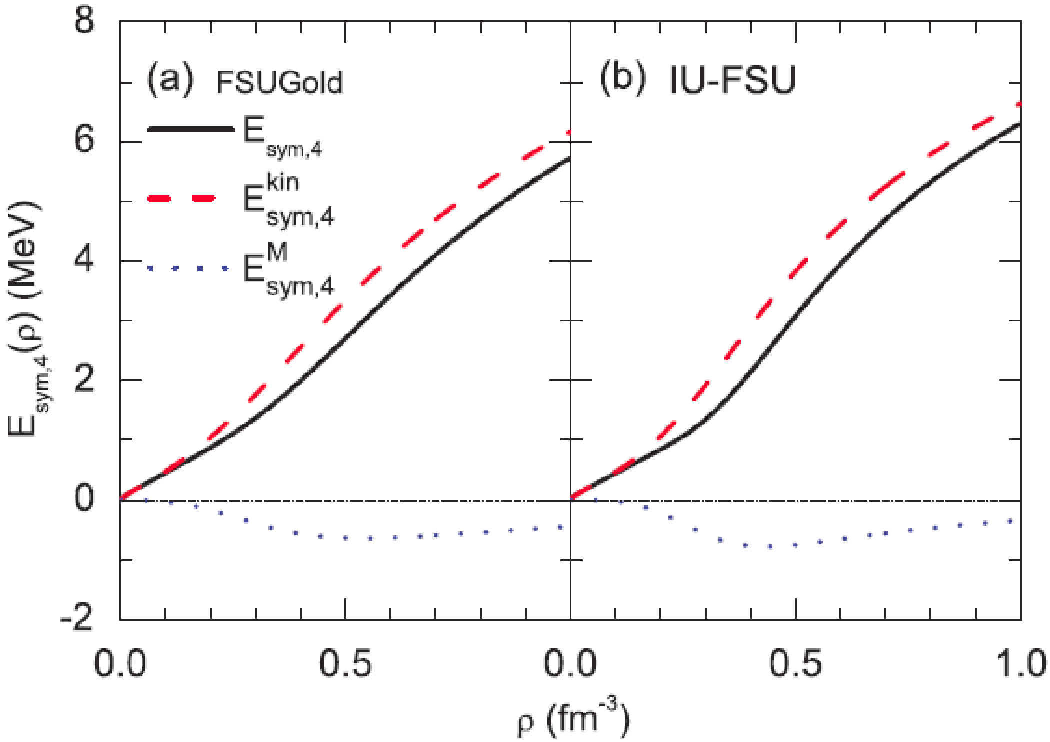
C.H. Lee, T.T.S Kuo, G.Q. Li, and G.E. Brown, PRC57, 3488 (1998)



Esym,4 is small, at least at lower densities in DBHF

Esym,4 in the RMF models

B.J. Cai/LWC, PRC 85, 024302 (2012)

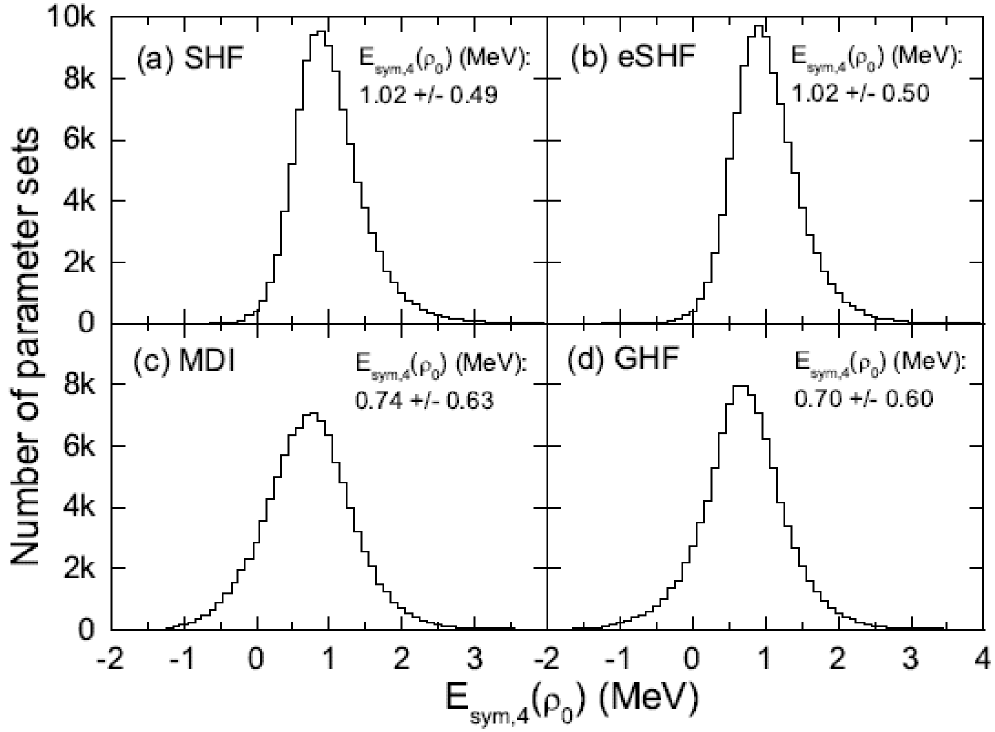


	FSUGold	IU-FSU	FSU-I	FSU-II	FSU-III	FSU-IV	FSU-V
ρ_0	0.148	0.155	0.148	0.148	0.148	0.148	0.148
$E_0(\rho_0)$	−16.3	−16.4	−16.3	−16.3	−16.3	−16.3	−16.3
$E_{sym}(\rho_0)$	32.5	31.3	37.4	35.5	33.9	31.4	30.9
$E_{sym,4}(\rho_0)$	0.66	0.67	0.66	0.66	0.66	0.66	0.78

Esym,4 is small in the RMF models, at least at lower densities

Esym,4 in non-relativistic mean field models

J. Pu/Z. Zhang/LWC, arXiv:1708.02132



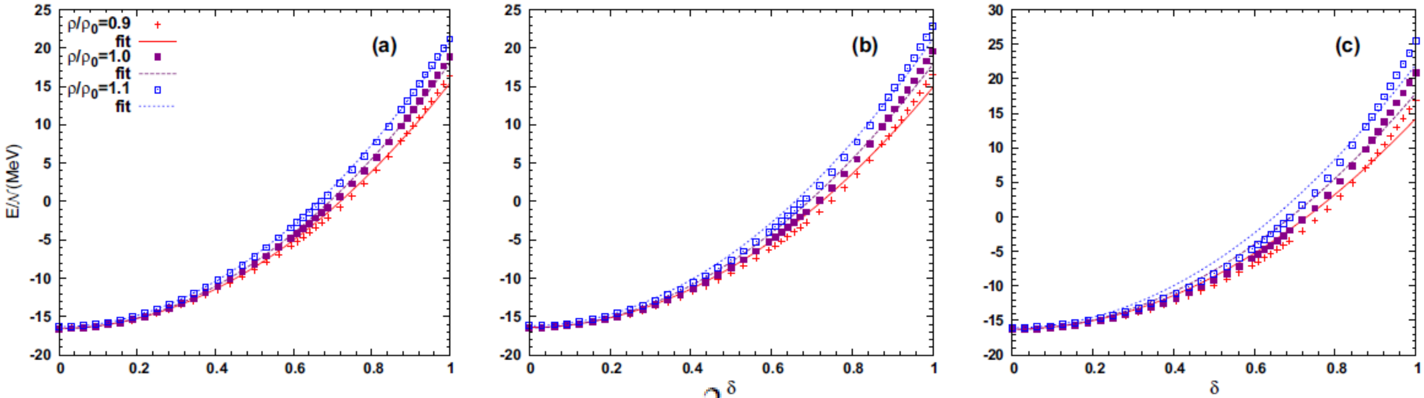
Systematic analysis by using 0.1 million samples of parameter sets for each energy density functionals

$$E_{\mathrm{sym},4}(\rho) = \frac{\hbar^2}{162m}\left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}}\left[\frac{3m}{m_v^*(\rho)} - \frac{2m}{m_s^*(\rho)}\right]$$

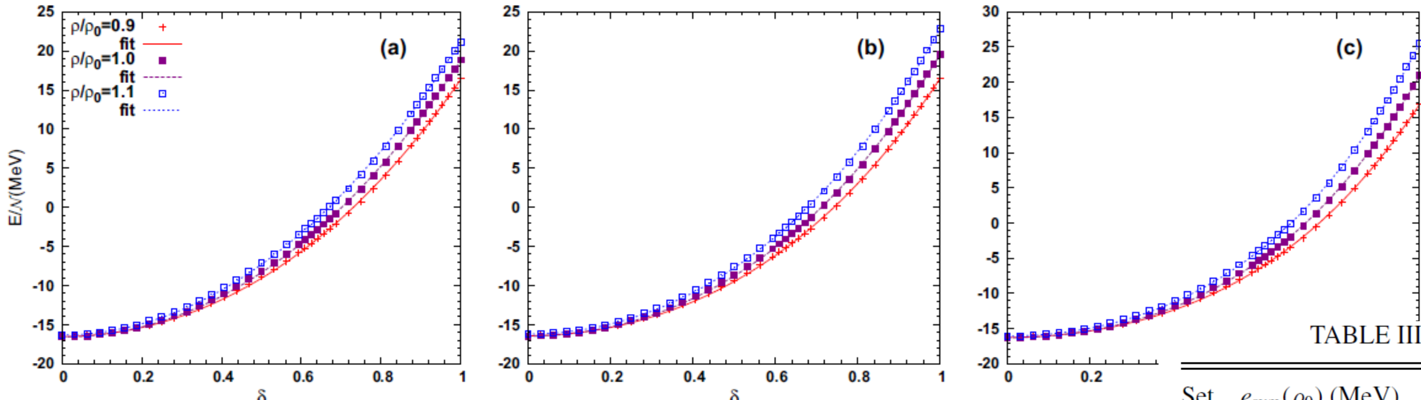
- **Esym,4 is essentially smaller than about 2 MeV**
- **Esym,4 is strong correlated with the nucleon isoscaclar and isovector masses**

Esym,4 in the QMD models

R. Nandi & S. Schramm, PRC94, 025806 (2016)



$$e(\rho,\delta) = e_0(\rho) + e_{\mathrm{sym}}(\rho)\delta^2$$



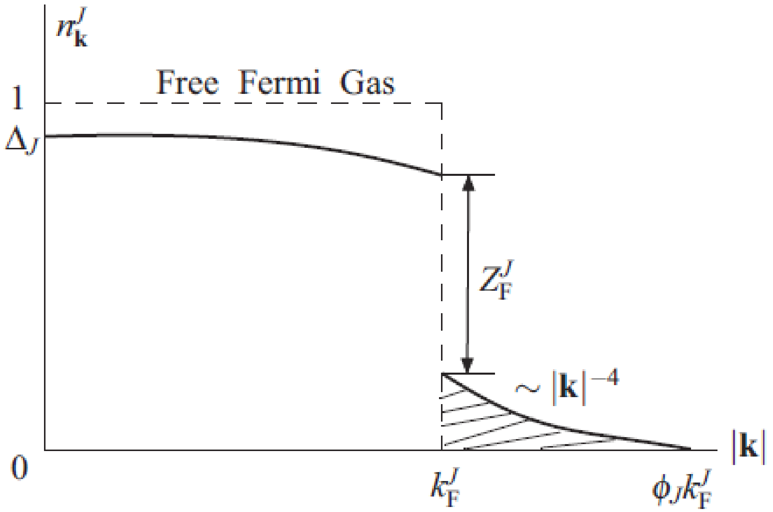
$$e(\rho,\delta) = e_0(\rho) + e_{\mathrm{sym}}(\rho)\delta^2 + e_{\mathrm{sym},4}(\rho)\delta^4$$

TABLE III. Symmetry energy coefficients.

Set	$e_{\mathrm{sym}}(\rho_0)$ (MeV)	$e_{\mathrm{sym},4}(\rho_0)$ (MeV)	L (MeV)	$L_{\mathrm{sym},4}$ (MeV)
I	32.1	3.27	102.2	– 33.2
II	28.9	7.07	91.7	0.0
III	24.5	12.7	76.1	50.0

Esym4 could be large in the QMD model, depending on interactions

Kinetic part of Esym,4 with Short Range Correlation



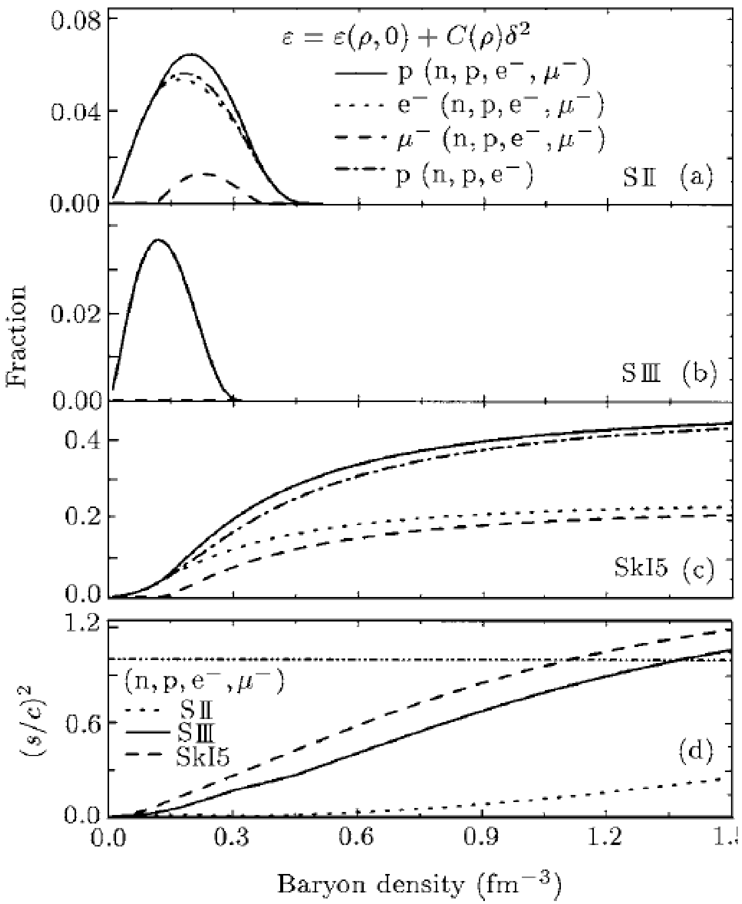
B.J. Cai & B.A. Li
PRC92, 011601(R) (2015)

The energy of a free gas of neutrons and protons is well known to be approximately isospin parabolic with a negligibly small quartic term of only 0.45 MeV at the saturation density of nuclear matter $\rho_0 = 0.16\ \text{fm}^{-3}$. Using an isospin-dependent single-nucleon momentum distribution including a high (low) momentum tail (depletion) with its shape parameters constrained by recent high-energy electron scattering and medium-energy nuclear photodisintegration experiments as well as the state-of-the-art calculations of the deuteron wave function and the equation of state of pure neutron matter near the unitary limit within several modern microscopic many-body theories, we show for the first time that the kinetic energy of interacting nucleons in neutron-rich nucleonic matter has a significant quartic term of 7.18 ± 2.52 MeV. Such a large quartic term has broad ramifications in determining the equation of state of neutron-rich nucleonic matter using observables of nuclear reactions and neutron stars.

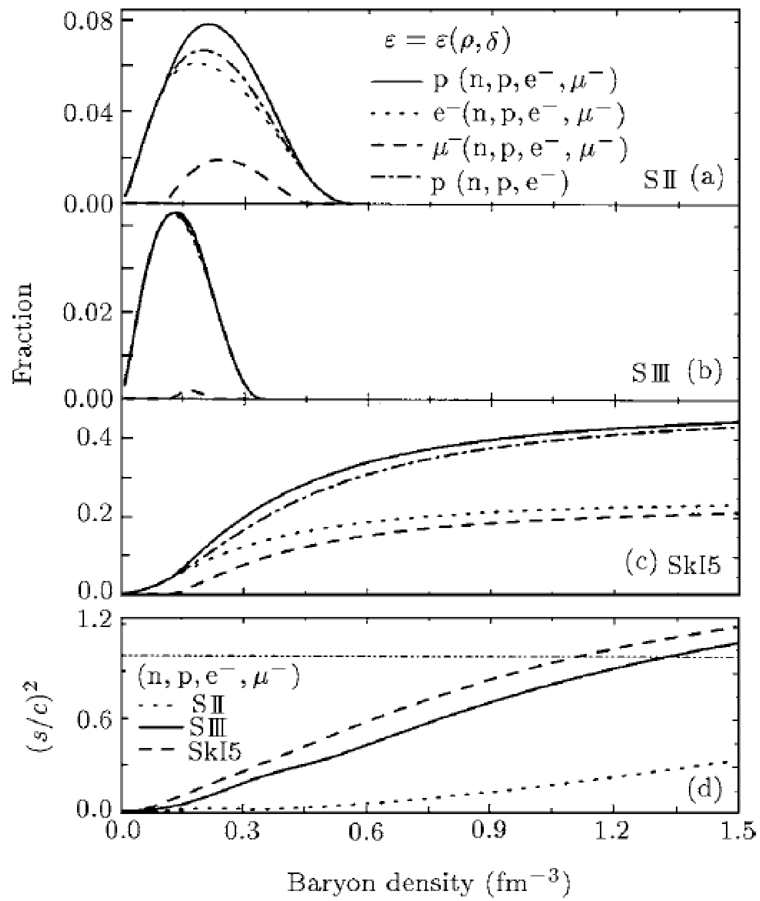
Effects beyond mean field approximation could be important for Esym,4

Esym,4 and proton fraction in NStar

F.S Zhang/LWC, Chin. Phys. Lett. 18, 142 (2001)



Parabolic law

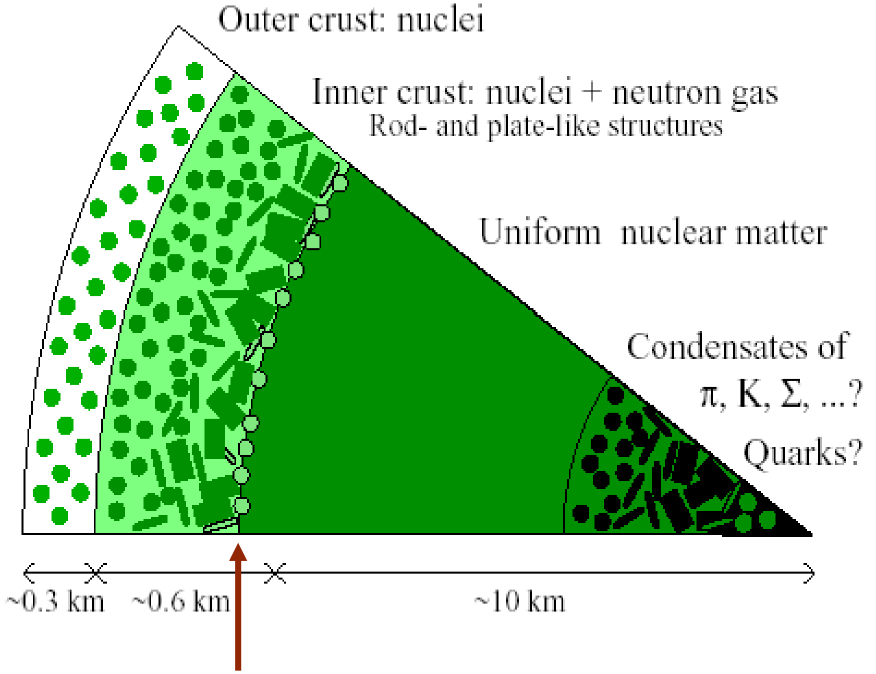


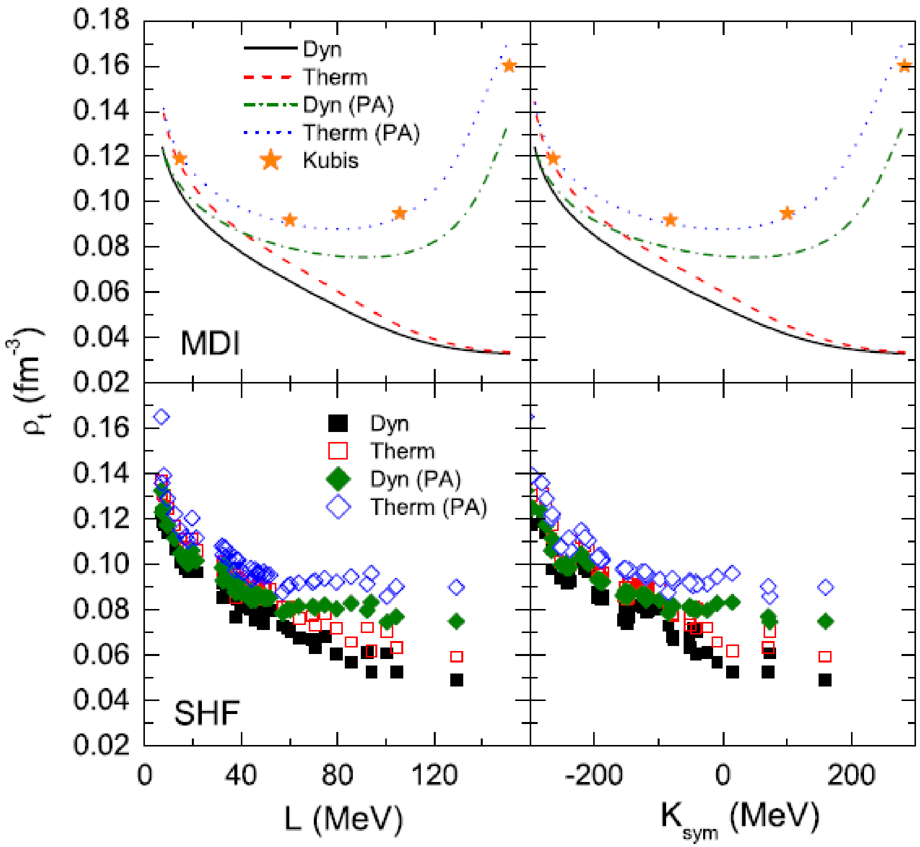
Full calcualtion

The higher-order terms of the symmetry energy have obvious effects on the proton fraction and the parabolic law of the symmetry energy is not enough to determine the proton fraction

Esym,4 and core-crust edge in NStar

J Xu/LWC/B.A. Li/H.R. Ma, PRC 79, 035802 (2009); ApJ697, 1549 (2009)





Core-Crust edge

(important for Glitch of NStar)

The core-crust transition density and pressure of Nstar are sensitive to the high-order terms of the symmetry energy

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Liquid-Drop Model (Bethe-Weizsäcker mass formula)

Order in I^2 ↓ / Order in $A^{-1/3}$ →			
	$I^2 A$	$I^2 A^{2/3}$	$I^2 A^{1/3}$
	$I^4 A$	$I^4 A^{2/3}$	$I^4 A^{1/3}$
	$I^6 A$	$I^6 A^{2/3}$	$I^6 A^{1/3}$

$$B(A,Z) = A \times \left[c_{00}(A,Z) + c_{01}(A,Z)A^{-\frac{1}{3}} + a_{\text{sym}}(A,Z)I^2 + a_{\text{sym},4}(A)I^4 \right.$$
$$\left. + a_c(1+\chi_0)Z^2 A^{-\frac{4}{3}} \right]$$

The isospin-quartic term is very small for known finite nuclei due to the very small value of I^4. Therefore, it is very hard to determine asym,4 from nuclear mass – need special treatments !

Isospin-Quartic Energy of Finite Nuclei

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H. Jiang/M. Bao/LWC/Y.M. Zhao/A. Arima, PRC 90, 064303 (2014)

"Experimental" symmetry energy of finite nuclei

$$e(Z,A) = B(Z,A) - \left\{ a_v A + a_s A^{2/3} + E_W + a_{\rm Coul}\frac{Z^2}{A^{1/3}}[1 - Z^{-2/3}] + a_{\rm pair} A^{-1/3}\delta_{np} \right\}$$

In order to remove as much as possible the uncertainty from the theoretical treatment of the volume and surface energy, the shell correction, Coulomb, and pairing interaction, etc., in nuclear masses, we take the double difference of $e(Z,A)$ and define

$$R_{ip-jn}(Z,A) = e(Z,A) + e(Z-i, A-i-j) - e(Z, A-j) - e(Z-i, A-i). \quad (2)$$

$$e(Z,A) = \left(c_2^{(\rm V)} I^2 + c_4^{(\rm V)} I^4 + c_6^{(\rm V)} I^6\right) A - \left(c_2^{(\rm S)} I^2 + c_4^{(\rm S)} I^4 + c_6^{(\rm S)} I^6\right) A^{2/3} + \left(c_2^{(\rm C)} I^2 + c_4^{(\rm C)} I^4 + c_6^{(\rm C)} I^6\right) A^{1/3}$$

$$R_{ip-jn}(Z,A) = c_2^{(\rm V)} X_2^{(\rm V)} + c_4^{(\rm V)} X_4^{(\rm V)} + c_6^{(\rm V)} X_6^{(\rm V)} + c_2^{(\rm S)} X_2^{(\rm S)} + c_4^{(\rm S)} X_4^{(\rm S)} + c_6^{(\rm S)} X_6^{(\rm S)} + c_2^{(\rm C)} X_2^{(\rm C)} + c_4^{(\rm C)} X_4^{(\rm C)} + c_6^{(\rm C)} X_6^{(\rm C)}$$

$$R_{ip-jn}(Z,A) = c_2^{(\rm V)} X_2^{(\rm V)} + c_4^{(\rm V)} X_4^{(\rm V)} + c_2^{(\rm S)} X_2^{(\rm S)} + c_4^{(\rm S)} X_4^{(\rm S)}$$

Isospin-Quartic Energy of Finite Nuclei

H. Jiang/M. Bao/LWC/Y.M. Zhao/A. Arima, PRC 90, 064303 (2014)

Csym,4 from the double difference of "experimental" symmetry energy of finite nuclei

TABLE IV. The resultant $c_2^{(V)}$, $c_2^{(S)}$, $c_4^{(V)}$, and $c_4^{(S)}$ (in MeV) obtained by using different cases of symmetry energy based on the total database. Cases A, B, and C are the same as in Fig. 2.

E_W	Case	$c_2^{(V)}$	$c_2^{(S)}$	$c_4^{(V)}$	$c_4^{(S)}$
Form (1)	A	30.97 ± 0.33	50.81 ± 1.12		
	B	31.22 ± 0.31	57.44 ± 1.12	8.47 ± 0.49	
	C	25.25 ± 0.54	38.33 ± 1.81	42.52 ± 2.63	100.56 ± 7.64
Form (2)	A	31.97 ± 0.31	58.32 ± 1.06		
	B	32.07 ± 0.31	60.89 ± 1.12	3.28 ± 0.50	
	C	25.39 ± 0.54	39.54 ± 1.80	41.31 ± 2.62	112.30 ± 7.61

Analyzing the binding energy of 2348 measured nuclei with 20<A<270 [AME2012]

The above results are precise and robust (the effects of odd-even staggering, a_v, a_s, a_{Coul}, a_{pair} are very small)

Isospin-Quartic Energy of Finite Nuclei

J.L Tian/H.T. Cui/T. Gao/N. Wang, Chin. Phys. C 40, 094101 (2016)

Chinese Physics C Vol. 40, No. 9 (2016) 094101

Effect of Wigner energy on the symmetry energy coefficient in nuclei *

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[1] School of Physics and Electrical Engineering, Anyang Normal University, Anyang 455000, China
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Abstract: The nuclear symmetry energy coefficient (including the coefficient $a_{\rm sym}^{(4)}$ of the I^4 term) of finite nuclei is extracted by using the differences of available experimental binding energies of isobaric nuclei. It is found that the extracted symmetry energy coefficient $a_{\rm sym}^{*}(A,I)$ decreases with increasing isospin asymmetry I, which is mainly caused by Wigner correction, since $e_{\rm sym}^{*}$ is the summation of the traditional symmetry energy $e_{\rm sym}$ and the Wigner energy $e_{\rm W}$. We obtain the optimal values $J=30.25\pm0.10$ MeV, $a_{\rm ss}=56.18\pm1.25$ MeV, $a_{\rm sym}^{(4)}=8.33\pm1.21$ MeV and the Wigner parameter $x=2.38\pm0.12$ through a polynomial fit to 2240 measured binding energies for nuclei with $20\leqslant A\leqslant 261$ with an rms deviation of 23.42 keV. We also find that the volume symmetry coefficient $J\simeq 30$ MeV is insensitive to the value x, whereas the surface symmetry coefficient $a_{\rm ss}$ and the coefficient $a_{\rm sym}^{(4)}$ are very sensitive to the value of x in the range $1\leqslant x\leqslant 4$. The contribution of the $a_{\rm sym}^{(4)}$ term increases rapidly with increasing isospin asymmetry I. For very neutron-rich nuclei, the contribution of the $a_{\rm sym}^{(4)}$ term will play an important role.

A precise value of asym,4 is obtained by analyzing the I dependence of the binding energy of isobaric nuclei considering the Effects from Wigner energy.

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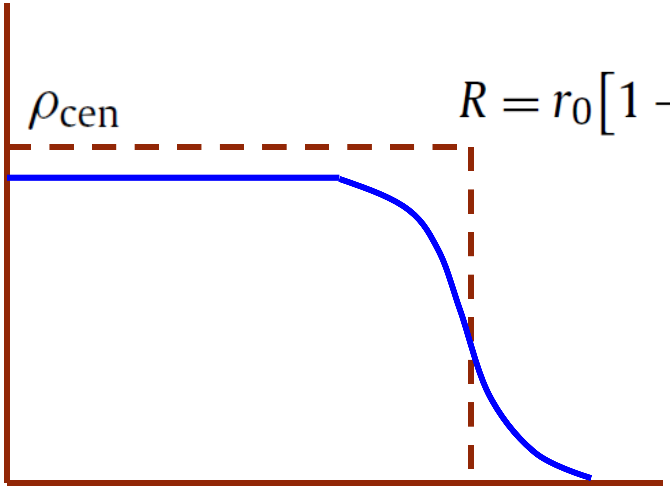
$a_{sym,4}(A)$



$$R = r_0\left[1+3\chi_{cen}(N,Z)\right]^{-1/3}A^{1/3} \qquad \frac{4}{3}\pi\rho_0 r_0^3 = 1$$

$$\chi_{cen} = (\rho_{cen}-\rho_0)/3\rho_0$$

$$\Delta_v = N_v - Z_v \qquad \Delta_s = N_s - Z_s$$

$$N_v + N_s = N \qquad Z_v + Z_s = Z$$

$$B_v \approx A\left[E_0(\rho_0) + \frac{1}{2}K_0\chi_{cen}^2 + E_{sym}(\rho_0)\left(\frac{\Delta_v}{A}\right)^2 + L\chi_{cen}\left(\frac{\Delta_v}{A}\right)^2 + E_{sym,4}(\rho_0)\left(\frac{\Delta_v}{A}\right)^4\right].$$

$$B_s = \left[\sigma_0 - \sigma_I\left(\frac{\Delta_s}{S}\right)^2\right]4\pi R^2 + \frac{2\sigma_I}{4\pi R^2}\Delta_s^2 \approx E_{s0}(1-2\chi_{cen})A^{\frac{2}{3}} + \beta(1+2\chi_{cen})A^{\frac{4}{3}}\left(\frac{\Delta_s}{A}\right)^2$$

$$B_c = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0}\frac{1}{R}Z^2 \approx a_c A^{-1/3}Z^2(1+\chi_{cen}) \qquad a_c = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 r_0}$$

$$\frac{\partial B(N,Z)}{\partial \chi_{cen}} = 0 \qquad \frac{\partial B(N,Z)}{\partial \Delta_v} = 0$$

$a_{sym,4}(A)$

$$\frac{\partial B(N,Z)}{\partial \chi_{cen}} = 0 \qquad \frac{\partial B(N,Z)}{\partial \Delta_v} = 0$$

$$\frac{\partial B}{\partial \chi_{cen}} = A\left[K_0\chi_{cen} + E_{sym}(\rho_0)\left(\frac{\Delta_v}{A}\right)^2\right] - 2E_{s0}A^{\frac{2}{3}} + 2\beta A^{\frac{4}{3}}\left(\frac{\Delta_s}{A}\right)^2 + a_c Z^2 A^{-\frac{1}{3}} = 0$$

$$\frac{\partial B}{\partial \Delta_v} = 2\left(E_{sym}(\rho_0) + L\chi_{cen}\right)\frac{\Delta_v}{A} + 4E_{sym,4}(\rho_0)\left(\frac{\Delta_v}{A}\right)^3 - 2\beta A^{\frac{1}{3}}\frac{\Delta_s}{A}(1 + 2\chi_{cen}) = 0$$

$$\chi_{cen} = \chi_0 + \chi_2\left(\frac{\Delta_v}{A}\right)^2 + \mathcal{O}\left[\left(\frac{\Delta_v}{A}\right)^4\right] \qquad \left(\frac{\Delta_v}{A}\right)^2 = D_2 I^2 + \mathcal{O}(I^4)$$

$$\chi_0(A,Z) = \frac{2E_{s0}A^{2/3} - a_c Z^2 A^{-1/3}}{AK_0} \qquad \chi_2 = -\frac{L}{K_0}$$

$$I^2 = \left(\frac{\Delta_v}{A} + \frac{\Delta_s}{A}\right)^2 \approx \left[\frac{\left(E_{sym}(\rho_0) + \beta A^{\frac{1}{3}}\right)^2}{\beta^2 A^{2/3}} + O(A^{\frac{1}{3}})\right]\left(\frac{\Delta_v}{A}\right)^2 \qquad D_2(A) = \frac{1}{\left(1 + \frac{E_{sym}(\rho_0)}{\beta}A^{-1/3}\right)^2}$$

$a_{sym,4}(A)$

$$B(A,Z) = A \times \left[c_{00}(A,Z) + c_{01}(A,Z)A^{-\frac{1}{3}} + a_{sym}(A,Z)I^2 + a_{sym,4}(A)I^4 + a_c(1+\chi_0)Z^2A^{-\frac{4}{3}}\right]$$

$$c_{00}(A,Z) = E_0(\rho_0) + \frac{1}{2}K_0\chi_0^2(A,Z)$$

$$c_{01}(A,Z) = E_{s0}[1 - 2\chi_0(A,Z)]$$

$$a_{sym}(A,Z) = D_2(A)\left[E_{sym}(\rho_0) + a_c Z^2 \chi_2 A^{-\frac{4}{3}} + \left(\frac{E_{sym}^2(\rho_0)}{\beta} - 2E_{s0}\chi_2\right)A^{-\frac{1}{3}}\right]$$

$$a_{sym,4}(A) = D_2^2(A)\left(E_{sym,4}(\rho_0) - \frac{L^2}{2K_0}\right)$$

asym depends on the Coulomb and Surface coefficients ac and Es0 !!!

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$a_{sym}(A)$

Neglecting the corrections due to the central density variation of finite nuclei (i.e., setting $\chi_0=\chi_2=0$) and the higher-order I^4 term, our mass formula is reduced to Danielewicz's mass formula **P. Danielewicz, Nucl. Phys. A 727, 233 (2003)**

$$a_{sym}(A,Z) = D_2(A)\left[E_{sym}(\rho_0) + a_c Z^2 \chi_2 A^{-\frac{4}{3}} + \left(\frac{E_{sym}^2(\rho_0)}{\beta} - 2E_{s0}\chi_2\right)A^{-\frac{1}{3}}\right]$$

$$a_{sym}(A) = E_{sym}(\rho_0) \Big/ \left(1 + \frac{E_{sym}(\rho_0)}{\beta} A^{-\frac{1}{3}}\right)$$

In addition, in the limit of $A \to \infty$

$$E_{sat}(\delta) = B(A,Z)/A = E_0(\rho_0) + E_{sym}(\rho_0)\delta^2 + \left(E_{sym,4}(\rho_0) - \frac{L^2}{2K_0}\right)\delta^4 + \mathcal{O}(\delta^6)$$

is reduced to the well known binding energy formula of ANM at saturation

L.W. Chen et al., PRC80, 014322 (2009)

$a_{sym}(A)$, c_{surf}, c_{Cou}

$$B(N,Z)=c_{vol}A+c_{sur}A^{2/3}+c_{cou}\frac{Z^2(1-Z^{-2/3})}{A^{1/3}}+c_{sym}\frac{(N-Z)^2}{A}+c_p\frac{(-1)^N+(-1)^Z}{A^{2/3}}$$

Analyzing the binding energy of 2348 measured nuclei with 20<A<270, we obtain

$$c_{sym}=22.2\ \text{MeV}\qquad c_{sur}=17.33\ \text{MeV}\qquad c_{cou}=0.709\ \text{MeV}$$

$$c_{sym}\approx\langle a_{sym}\rangle=\sum\frac{1}{N_{MN}}a_{sym}(A,Z)\qquad \langle X(A,Z)\rangle \text{ is defined as } \sum X(A,Z)/N_{MN}$$

$$c_{sym}=E_{sym}(\rho_0)\sum\frac{1}{N_{MN}}D_2(A)-\frac{a_cL}{K_0}\sum\frac{1}{N_{MN}}Z^2D_2(A)A^{-\frac{4}{3}}+\left(\frac{E^2_{sym}(\rho_0)}{\beta}+\frac{2E_{s0}L}{K_0}\right)\sum\frac{1}{N_{MN}}D_2(A)A^{-\frac{1}{3}}$$

$$=E_{sym}(\rho_0)\langle D_2(A)\rangle-\frac{a_cL}{K_0}\langle Z^2D_2(A)A^{-\frac{4}{3}}\rangle+\left(\frac{E^2_{sym}(\rho_0)}{\beta}+\frac{2E_{s0}L}{K_0}\right)\langle D_2(A)A^{-\frac{1}{3}}\rangle$$

$$c_{sur}\approx E_{s0}(1-2\langle\chi_0(A,Z)\rangle)=E_{s0}\left[1-2\left(\frac{E_{s0}\langle 2A^{-\frac{1}{3}}\rangle-a_c\langle Z^2A^{-\frac{4}{3}}\rangle}{K_0}\right)\right]$$

$$c_{cou}\approx a_c(1+\langle\chi_0(A,Z)\rangle)=a_c\left[1+\left(\frac{E_{s0}\langle 2A^{-\frac{1}{3}}\rangle-a_c\langle Z^2A^{-\frac{4}{3}}\rangle}{K_0}\right)\right]$$

$a_{sym}(A)$, c_{surf}, c_{Cou}

$E_{sym}(\rho_0)/\beta = 2.4$ $\langle D_2(A)\rangle = 0.45$

$\langle D_2(A)A^{-\frac{1}{3}}\rangle = 0.09$ $\langle Z^2 D_2(A)A^{-\frac{4}{3}}\rangle = 2.14$

$K_0 = 240$ MeV $L = 45.2$ MeV

$c_{sym} \approx 21.8$ MeV $E_{s0} = 17.96$ MeV $a_c = 0.70$ MeV

(Prediction from our mass formula)

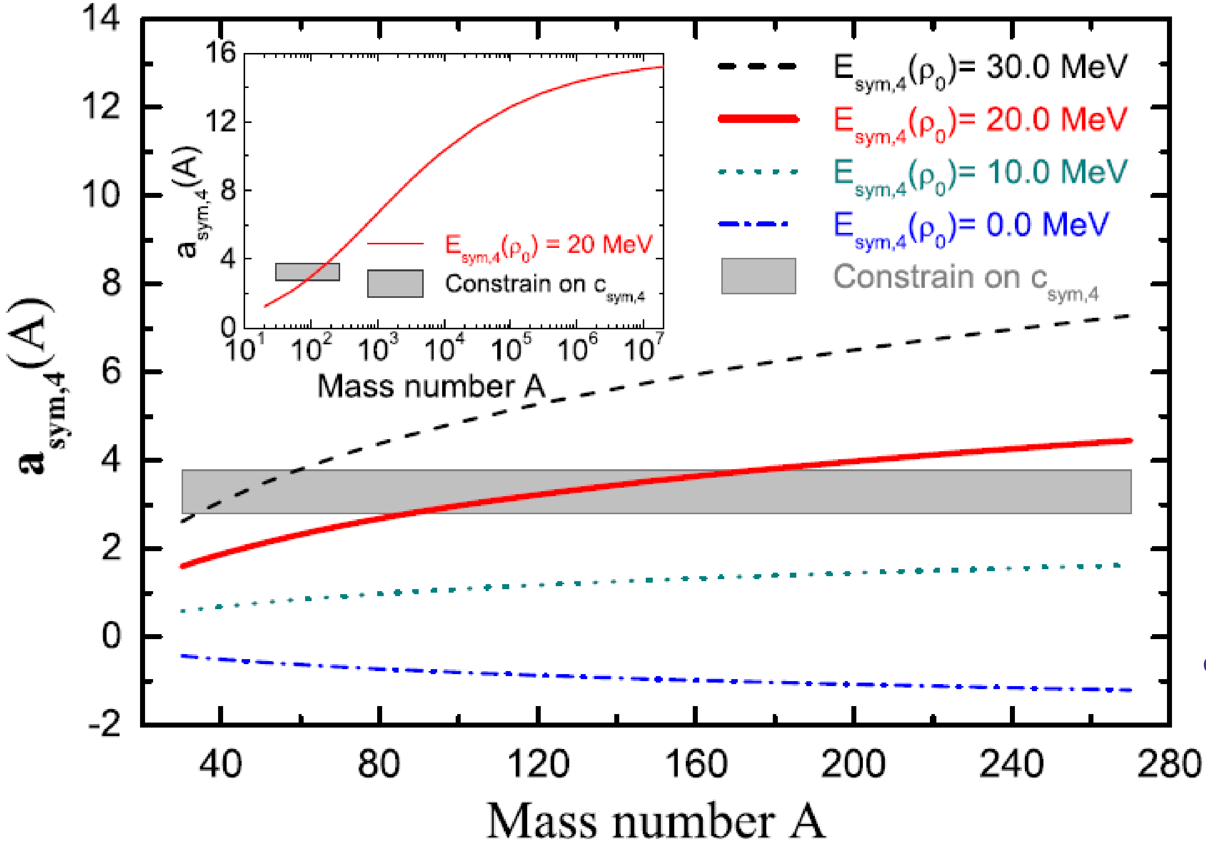
$c_{sym} = 22.2$ MeV $c_{sur} = 17.33$ MeV $c_{cou} = 0.709$ MeV

(From fitting nuclear masses by using the simple Bethe-Weizsäcker formula)

$a_{sym,4}(A)$ vs $E_{sym,4}$

$$a_{\mathrm{sym},4}(A) = D_2^2(A)\left(E_{\mathrm{sym},4}(\rho_0) - \frac{L^2}{2K_0}\right)$$

R. Wang/LWC, PLB773, 62 (2017)



$$D_2(A) = \frac{1}{(1 + \frac{E_{\mathrm{sym}}(\rho_0)}{\beta} A^{-1/3})^2}$$

$$E_{\mathrm{sym}}(\rho_0)/\beta = 2.4$$

P. Danielewicz, NPA 727, 233 (2003); Danielewicz/Singh/Lee NPA 958, 147 (2017)

$$L = 45.2 \text{ MeV}$$

Z. Zhang/LWC, PLB726, 234 (2013)

$$K_0 = 240 \text{ MeV}$$

asym,4(A) vs Esym,4(rho0)

$E_{sym,4}$ from c_{sym}

$$a_{sym,4}(A) = D_2^2(A)\left(E_{sym,4}(\rho_0) - \frac{L^2}{2K_0}\right)$$

$$D_2(A) = \frac{1}{(1+\frac{E_{sym}(\rho_0)}{\beta}A^{-1/3})^2}$$

$$c_{sym,4} \approx \langle a_{sym,4}\rangle = \sum \frac{1}{N_{MN}} a_{sym,4}(A)$$

$$= \left(E_{sym,4}(\rho_0) - \frac{L^2}{2K_0}\right)\langle D_2^2(A)\rangle$$

$$E_{sym}(\rho_0)/\beta = 2.4$$

$$K_0 = 240 \text{ MeV}$$

$$E_{sym,4}(\rho_0) = \frac{\langle a_{sym,4}(A)\rangle}{\langle D_2^2(A)\rangle} + \frac{L^2}{2K_0}$$

$$L = 45.2 \text{ MeV}$$

$\langle D_2^2(A)\rangle = 0.2$ $L = 45.2 \pm 10.0$ MeV $K_0 = 240 \pm 40$ MeV

$E_{sym}(\rho_0)/\beta = 2.4 \pm 0.4$

Csym = 3.28+/-0.50 MeV

H. Jiang/M. Bao/LWC/Y.M. Zhao/A. Arima, PRC 90, 064303 (2014)

$$E_{sym,4}(\rho_0) = 20.0 \pm 4.6 \text{ MeV}$$

$E_{sym,4}$ from c_{sym}

$$E_{sym,4}(\rho_0) = 20.0 \pm 4.6 \text{ MeV}$$

- **Significantly larger than the predictions of mean field models**
- **Such a significant Esym,4 definitely needs further investigation:**

Effects of beyond mean field approximation, Short-range correlations, tensor force, ….

Note:

$$E_{sym,4}(\rho_0) = 2 \text{ MeV} \Longleftrightarrow c_{sym,4} \approx \langle a_{sym,4} \rangle = -0.45 \text{ MeV}$$

Decomposition of the Esym,4 according to the Hugenholtz-Van Hove (HVH) theorem

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)
C. Xu, B.A. Li, and L.W. Chen, PRC82, 054607 (2010)
R. Chen, B.J. Cai. L.W. Chen, B.A. Li, X.H. Li, and C. Xu, PRC85, 024305 (2012).

$$t(k_{F_n}) + U_n(\rho, \delta, k_{F_n}) = \frac{\partial \varepsilon(\rho, \delta)}{\partial \rho_n}$$

Hugenholtz-Van Hove theorem

N. M. Hugenholtz, L. Van Hove, Physica **24**, 363 (1958)

$$t(k_{F_p}) + U_p(\rho, \delta, k_{F_p}) = \frac{\partial \varepsilon(\rho, \delta)}{\partial \rho_p}$$

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho, k_F),$$

Brueckner/Dabrowski, Phys. Rev. 134 (1964) B722

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho, k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F} \cdot k_F + 3U_{sym,2}(\rho, k_F)$$

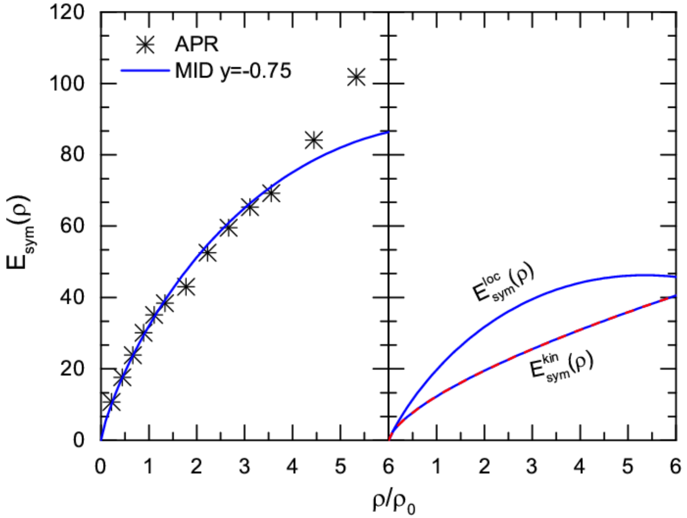
$$E_{\text{sym},4}(\rho) = \frac{\hbar^2}{162m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3} + \left[\frac{5}{324}\frac{\partial U_0(\rho,k)}{\partial k}k - \frac{1}{108}\frac{\partial^2 U_0(\rho,k)}{\partial k^2}k^2 + \frac{1}{648}\frac{\partial^3 U_0(\rho,k)}{\partial k^3}k^3 - \frac{1}{36}\frac{\partial U_{\text{sym},1}(\rho,k)}{\partial k}k + \frac{1}{72}\frac{\partial^2 U_{\text{sym},1}(\rho,k)}{\partial k^2}k^2 + \frac{1}{12}\frac{\partial U_{\text{sym},2}(\rho,k)}{\partial k}k + \frac{1}{4}U_{\text{sym},3}(\rho,k)\right]_{k_F}$$

$E_{sym,4}(\rho)$ **has very complicated structure in potential decomposition!**

Esym,4 vs Neutron Stars

$$E_{\mathrm{sym},4}(\rho_0) = 20.0 \pm 4.6 \ \mathrm{MeV}$$

How to affect the properties of neutron stars? (preliminary)



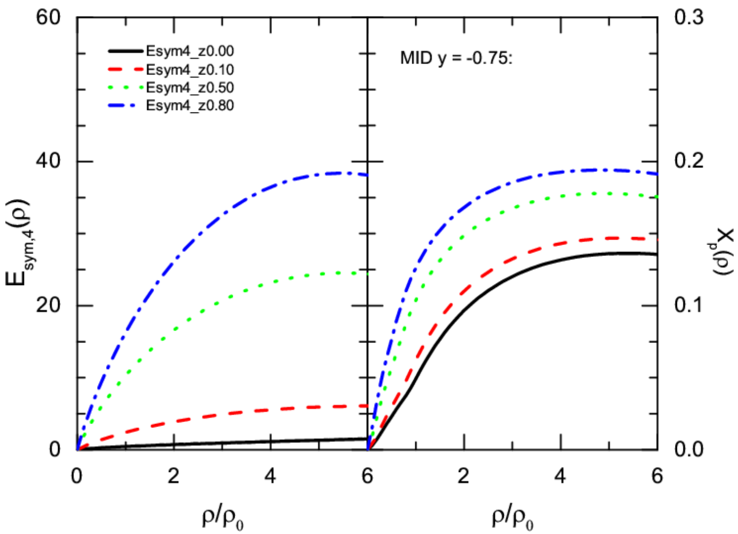
The MID Model

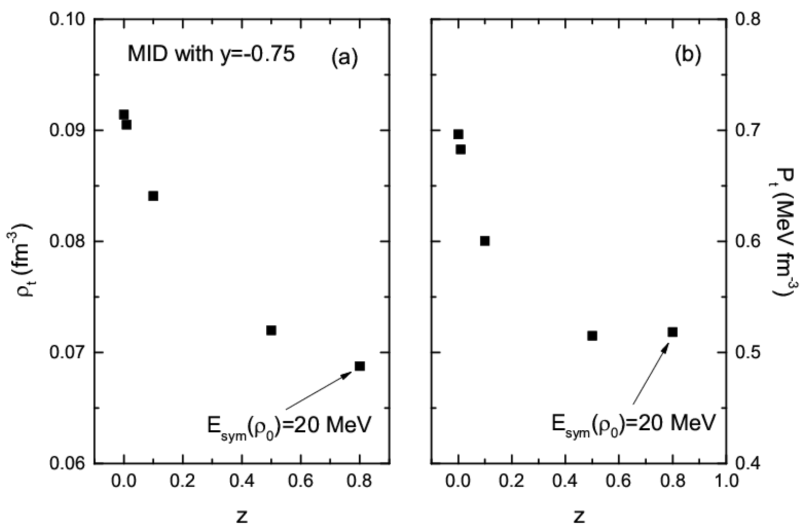
L.W. Chen, Sci. China Ser. G52, 1494 (2009)

$$V_{\mathrm{MID}}(\rho,\delta) = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0{}^{\sigma}} + \rho E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta^2$$

$$E_{\mathrm{sym}}^{\mathrm{pot}}(\rho) = E_{\mathrm{sym}}^{\mathrm{pot}}(\rho_0)(1-y)\frac{\rho}{\rho_0} + yE_{\mathrm{sym}}^{\mathrm{pot}}(\rho_0)\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\mathrm{sym}}}$$

Esym,4 (pot) ~ z Esym (pot)





Outline

- **The symmetry energy (Esym)**
- **The fourth-order symmetry energy (Esym,4)**
- **Isospin-quartic term in nuclear mass (asym,4)**
- **Esym,4 vs asym,4**
- **Summary**

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Summary

- **A relation between asym,4 and EOS of ANM is established, which provides the possibility to extract Esym,4 from asym,4**
- **A significant value of $E_{sym,4}$ = 20.0 +/- 4.5 MeV is obtained based on the $a_{sym,4}$=3.28+/-0.50 MeV extracted from nuclear mass by double difference method**
- **Such a significant Esym,4 needs further investigation, e.g., about effects beyond mean field approximation with short range correlations, tensor forces, …?**
- **Esym,4 has important impacts on proton fraction, core-crust edge of neutron stars. Such as a significant Esym,4 wil significantly enhance the proton fraction and reduce ρt and Pt.**

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Acknowledgements

Collaborators:

Peng-Cheng Chu (QTU, Qingdao)
Wei-Zhou Jiang (SEU. Nanjing)
Che Ming Ko,Zhen Zhang (TAMU. Texas)
Bao-An Li, Bao-Jun Cai (TAMU-Commerce, Texas)
Xiao-Hua Li (USC, Hengyang)
De-Hua Wen (SCUT, Guanzhou)
Zhi-Gang Xiao (Tsinghua, Beijing)
Chang Xu (NJU, Nanjing)
Jun Xu (SINAP, CAS, Shanghai)
Gao-Chan Yong (IMP, Lanzhou)
Xin Wang, Zhao-Wen Zhang, Kai-Jia Sun, Rui Wang, Jie Pu, Ying Zhou, Si-Pei Wang (SJTU, Shanghai)

Funding:

National Natural Science Foundation of China
Major State Basic Research Development Program (973 Program) in China
Shanghai Rising-Stars Program
Shanghai “Shu-Guang” Project
Shanghai “Eastern Scholar”
Science and Technology Commission of Shanghai Municipality

谢 谢！
Thanks！