Dynamical Clusters in Transport

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7th International Symposium
on Nuclear Symmetry Energy

September 4-7, 2017, GANIL, Caen, France
Central Energetic Collisions

Systems break up!

Description in terms of nucleons as elementary degrees of freedom

\[
\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial p} \frac{\partial f}{\partial r} - \frac{\partial \epsilon}{\partial r} \frac{\partial f}{\partial p} = I
\]

\( f \): distribution of nucleons in position \( r \) and momentum \( p \)

Boltzmann eq. for \( f \)

\( I \) - collision integral
Cluster Production

Even in central collisions much of mass/charge emitted inside clusters, not as individual nucleons!

<table>
<thead>
<tr>
<th>Partitioning of protons</th>
<th>Xe + Sn</th>
<th>Au + Au</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 MeV/u</td>
<td>250 MeV/u</td>
</tr>
<tr>
<td>p</td>
<td>≈ 10%</td>
<td>21%</td>
</tr>
<tr>
<td>α</td>
<td>≈ 20%</td>
<td>20%</td>
</tr>
<tr>
<td>d, t, ³He</td>
<td>≈ 10%</td>
<td>40%</td>
</tr>
<tr>
<td>A &gt; 4</td>
<td>≈ 60%</td>
<td>18%</td>
</tr>
</tbody>
</table>

INDRA data, Hudan et al., PRC 67 (2003) 064613.

Akira Ono

$^{40}\text{Ca} + ^{40}\text{Ca}$ at 35 MeV/u

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One Solution: Coalescence Model

Nucleons coalesce into a fragment if their final velocities are close enough.

\[
\frac{d\sigma_A}{d^3p} \propto \left( \frac{d\sigma_N}{d^3p/A} \right)^A
\]

Intrinsic problems:
- No energy conservation
- Perturbative

FIG. 29. Double differential cross sections for hydrogen and helium isotopes from $^{26}$Ne on U compared with...
Boltzmann Eq Models

Degrees of freedom (X):
typically nucleons, Δ, N*, pions, but clusters?

Fundamentals:
- Relativistic Landau theory (Chin/Baym)
  *Energy functional* ($\epsilon$)
- Real-time Green’s function theory
  *Production/absorption rates* ($K^<, K^>$)

\[
\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial p} \frac{\partial f}{\partial r} - \frac{\partial \epsilon}{\partial r} \frac{\partial f}{\partial p} = K^< (1 \mp f) - K^> f
\]

production absorption rate
Single-Particle Energies & Functional

\[
\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial p} \frac{\partial f}{\partial r} - \frac{\partial \epsilon}{\partial r} \frac{\partial f}{\partial p} = \mathcal{K}^< (1 \mp f) - \mathcal{K}^> f
\]

The single-particle energies $\epsilon$ are given in terms of the net energy functional $E\{f\}$ by,

\[
\epsilon(p) = \frac{\delta E}{\delta f(p)}
\]

In the local cm, the mean potential is

\[
U_{opt} = \epsilon - \epsilon_{kin}
\]

and $\epsilon_{kin} = \sqrt{p^2 + m^2}$
Many-Body Theory

Transport eq. for nucleons follows from the eq. of motion for the 1-ptcle Green’s function (KB eq.). Transport eq. for deuterons ($A = 2$) from the eq. for 2-ptcle Green’s function??

Wigner function in second quantization

$$f(p; R, T) = \int dr \ e^{-ipr} \langle \hat{\psi}^\dagger_H(R - r/2, T) \hat{\psi}_H(R + r/2, T) \rangle$$

where $\langle \cdot \rangle \equiv \langle \Psi | \cdot | \Psi \rangle$ and $|\Psi\rangle$ describes the initial state.

Evolution driven by a Hamiltonian. Interaction Hamiltonian:

$$\hat{H}^1 = \frac{1}{2} \int dx \ dy \ \hat{\psi}^\dagger(x) \hat{\psi}^\dagger(y) v(x - y) \hat{\psi}(y) \hat{\psi}(x),$$
Evolution Contour

\[
\langle \hat{O}_H(t_1) \rangle = \langle T^a \left[ \exp \left( -i \int_{t_1}^{t_0} dt' \, \hat{H}_I^1(t') \right) \right] \hat{O}_I(t_1) \rangle \\
\times \langle T^c \left[ \exp \left( -i \int_{t_0}^{t_1} dt' \, \hat{H}_I^1(t') \right) \right] \rangle \\
= \langle T \left[ \exp \left( -i \int_{t_0}^{t_1} dt' \, \hat{H}_I^1(t') \right) \right] \hat{O}_I(t_1) \rangle,
\]

Expectation value expanded perturbatively in terms of \( V \) and noninteracting 1-particle Green's functions on the contour

\[
iG_0(x, t, x', t') = \langle T \left[ \hat{\psi}_I(x, t) \hat{\psi}_I^\dagger(x', t') \right] \rangle
\]
Single-Particle Evolution

Wigner function corresponds to a particular case of the Green’s function on contour:

\[
f(p; R, T) = \int dr \ e^{-i pr} (\mp i) G^<(R + r/2, T, R - r/2, T)
\]

If we find an equation for \(G\), this will also be an equation for \(f\).

Dyson eq. from perturbation expansion:

\[
G = G_0 + G_0 \Sigma G
\]
Outcome of Evolution

Formal solution of the Dyson eq:

\[ \mp i G^<(x, t; x', t') = \int dx_1 \, dt_1 \, dx'_1 \, dt'_1 \, G^+(x, t; x_1, t_1) \]
\[ \times (\mp i) \Sigma^<(x_1, t_1; x'_1, t'_1) \, G^- (x, t; x_1, t_1) \]

and

\[ \mp i \Sigma^<(x, t; x', t') = \langle \hat{\psi}^*(x', t') \hat{\psi}(x, t) \rangle_{irred} \]

where the source \( j \) is

\[ \hat{j}(x, t) = \left[ \hat{\psi}(x, t), \hat{H}^1 \right] \]
Quasiparticle Limit

Under slow spatial and temporal changes in the system, the Green’s function expressible in terms of the Wigner function $f$ and 1-particle energy $\epsilon_p$

$$\mp iG^<(x, t; x', t') \approx \int dp f(p; \frac{x + x'}{2}, \frac{t + t'}{2}) e^{i(p(x-x') - \epsilon_p(t-t'))}$$

Then also Boltzmann eq:

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_p}{\partial p} \frac{\partial f}{\partial r} - \frac{\partial \epsilon_p}{\partial r} \frac{\partial f}{\partial p} = -i\Sigma^<(1 - f) - i\Sigma^> f$$

$$\mp i\Sigma^< : \quad = \quad \begin{array}{c}
\begin{array}{c}
\bigcirc \\
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\end{array}
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\end{array}
\end{array}$$
2-Particle Green’s Function

Transport eq. for deuterons ($A = 2$) from the eq. for 2-ptcle Green’s function??

$$iG_2^< = \langle \hat{\psi}^\dagger(x_1' t') \hat{\psi}^\dagger(x_2' t') \hat{\psi}(x_2 t) \hat{\psi}(x_1 t) \rangle$$

For the contour function:

$$G_2 = G + G v G_2$$

where $G$ – irreducible part of $G_2$ (w/o two 1-ptcle lines connected by the potential $v$; anything else OK)

In terms of retarded Green’s function $G_2^<$:

$$iG_2^< = (1 + G_2^+ v) iG^< (1 + v G_2^-)$$
Deuteron Quasiparticle Limit

In the limit of slow spatial and temporal changes, deuteron contribution to the 2-ptcle Green’s function:

\[ iG_2^< = \langle \hat{\psi}^\dagger(x'_1 t') \hat{\psi}^\dagger(x'_2 t') \hat{\psi}(x_2 t) \hat{\psi}(x_1 t) \rangle \]

\[ \simeq \int dp \ f_d(p, R, T) \ \phi_d^*(r') \ \phi_d(r) \ e^{i p \left( \frac{x_1 + x_2}{2} - \frac{x'_1 + x'_2}{2} \right)} \ e^{-i \epsilon_d(t - t')} \]

where \( R = \frac{1}{4} (x_1 + x_2 + x'_1 + x'_2) \), \( r = x_1 - x_2 \)

\( \phi_d \) and \( f_d \) – internal wave function and cm Wigner function

\( \cdots \equiv \text{continuum} \)

Transport eq from integral quantum eq of motion:

\[ \frac{\partial f_d}{\partial T} + \frac{\partial \epsilon_d}{\partial p} \frac{\partial f_d}{\partial R} - \frac{\partial \epsilon_d}{\partial R} \frac{\partial f_d}{\partial p} = \mathcal{K}^< (1 + f_d) - \mathcal{K}^> f_d \]
Wave Equation

From Green’s function eq, the equation for wavefunction:

\[
(\epsilon_d(P) - \epsilon_N(P/2 + p) - \epsilon_N(P/2 - p)) \phi_d(p)
\]

\[
- (1 - f_N(P/2 + p) - f_N(P/2 - p)) \int dp' \nu(p - p') \phi_d(p') = 0
\]

In zero-temperature matter, discrete states lacking over a vast range of momenta
Cluster Production & Absorption

?? Production & absorption rates:

\[ iK^< = \phi^* v \, iG^< v \phi \]

Leading contribution

\[ K^< = \int \, dr \, dr' \, \phi_d^* \, v \, \langle \hat{\psi}^{\dagger}(x_1', t') \, \hat{\psi}(x_1, t) \rangle \langle \hat{\psi}^{\dagger}(x_2', t') \, \hat{\psi}(x_2, t) \rangle \, v \, \phi_d \]

Leading-order in the quasiparticle expansion: neutron & proton come together and make a deuteron.

If system approximately uniform and stationary, the process not allowed by energy-momentum conservation.

Process possible in a mean field varying in space, but, in nuclear case, the high-energy production rate low – tested in Glauber model.
First correction to the pure 1-ptcle state, from a coupling to p-h excitations, yields a contribution to the d-production due to 3-nucleon collisions.

Still more nucleons involved in production of heavier clusters.
Deuteron Production

Detailed balance:

\[ |M_{npN \rightarrow Nd}|^2 = |M_{Nd \rightarrow Nnp}|^2 \propto d\sigma^{Nd \rightarrow Nnp} \]

Thus, production can be described in terms of breakup.

Problem: Breakup cross section only known over limited range of final states - Interpolation/extrapolation needed

Impulse approximation works at high incident energy
Renormalized Impulse Approximation

Renormalization factor for squared matrix element to get breakup cross section right as a function of energy

$$d\sigma^{N_{d}\rightarrow N_{n}p} \propto F \sigma_{NN} |\phi_d(p)|^2$$
Single-Particle Spectra

proton & deuteron inclusive spectra

histograms: calculations using
\[ |\mathcal{M}^{npN \rightarrow Nd}|^2 = |\mathcal{M}^{Nd \rightarrow npN}|^2 \propto d\sigma_{Nd \rightarrow npN} \]
and \( \langle f \rangle < 0.2 \) cut-off for deuterons
A = 3 Particles + Tests

$A = 3$-ptcles from 4N collisions

Christiane Kuhrts: solving finite-$T$ Galitski-Feynman (GF) and modified (in-medium) Alt-Grassberger-Sandhas eqs

solid lines: finite-$T$ GF for cross-sections and existence
dashed lines: free cross sections + $\langle f \rangle$ cut-off

symbols: INDRA data $^{129}$Xe + $^{119}$Sn at 50 MeV/nucleon

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Cluster Yields and Entropy

Compression in central reactions accompanied by heating. Is the matter heated as much as expected for shock compression??

Experimental measure of entropy: relative cluster yields

\[ E = T S - P V + \mu A \quad \Leftrightarrow \quad 3 A T / 2 \approx T S - A T + \mu A \]

as at freeze-out ideal gas and then

\[ \frac{S}{A} \approx \frac{5}{2} - \frac{\mu}{T} \]

In equilibrium

\[ \frac{N_d}{N_p} \propto \frac{\exp \left( \frac{2\mu}{T} \right)}{\exp \left( \frac{\mu}{T} \right)} \quad \Rightarrow \quad \frac{S}{A} \approx 3.9 - \log \left( \frac{N_d}{N_p} \right) \]
Validity of Entropy Determination

entropy per nucleon

\[ \frac{S}{A} \]

Nb + Nb 650 MeV/nucleon

Deuteron formula
directly from
dynamics

number of ejected nucleons

\[ N_p \]

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Collective Expansion

Is expansion viscous or isentropic? Is pressure carrying out work producing a collective expansion of matter?

\[
\langle E_x \rangle = \frac{3}{2} T + \frac{m_x \langle v^2 \rangle}{2}
\]

\[
= \frac{3}{2} T + A_x \frac{m_N \langle v^2 \rangle}{2}
\]

In isentropic expansion, average kinetic energy should increase with fragment mass.

Energy increases linearly!

-- data, Poggi et al

-- calculation

\[\text{Au+Au 250AMeV} \quad 60^\circ < \theta_{cm} < 90^\circ\]

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Head-On Au + Au (FOPI)

Rapidity Distribution

400 MeV/nucleon

proton, b=1 fm

MF$^N_{\pi}$

FOPI
$A = 3$ in Head-On Au + Au (FOPI)

Rapidity Distributions

$^{3}$H, $b=1\text{fm}$

$^{3}$He, $b=1\text{fm}$

400 MeV/nucleon
Semicentral Au + Au (FOPI)

Elliptic Flow

400 MeV/nucleon
Semicentral Au + Au (FOPI)

Elliptic Flow

400 MeV/nucleon
Future of Light-Cluster Production in Transport

Production rate for cluster of mass $A$:

$$\mathcal{K}^{<}(p_A) = \int dp'_1 \ldots dp'_{N'} dp_1 \ldots dp_{N-1} |\mathcal{M}_{1'+\ldots+N'}\rightarrow 1+\ldots+A|^2$$

$$\times \delta(p'_1 + \ldots + p'_{N'} - p_1 - \ldots - p_{N-1} - p_A)$$

$$\times \delta(\epsilon'_1 + \ldots + \epsilon'_{N'} - \epsilon_1 - \ldots - \epsilon_{N-1} - \epsilon_A)$$

$$\times f_1' \ldots f_{N'} (1 \pm f_1) \ldots (1 \pm f_{N-1})$$

Determination and sampling of separate $|\mathcal{M}|^2$ for every possible process... Potential nightmare! E.g.

$$N + \Delta \leftrightarrow d + \pi$$

$$d + d + N \leftrightarrow \alpha + N$$

AGS

e tc.

Any simplifications??
Simplified Matrix Elements

Batko, Randrup, Vetter
NPA536(92)786

$|\mathcal{M}|^2 \propto 1 \Rightarrow \text{Mini Fireball}$

??Too much dissipation??

Generalized coalescence:

$|\mathcal{M}|^2 \propto \theta(p_0 - \frac{p_A}{A} - p'_1) \cdots \theta(p_0 - \frac{p_A}{A} - p'_{N'})$

Branching?? Automation needed!
Conclusions

- Real-time many-body theory provides fundamentals for production of clusters in transport theory
- Few-body collisions or rapidly changing mean-field conditions are needed to spur cluster production
- Detailed balance must be obeyed for thermodynamic consistency
- Breakup data yield production rates in collisions
- Clusters emphasize collective motion and provide information on phase-space densities and entropy
- Production description needs to be simplified in extending reach of theory.

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