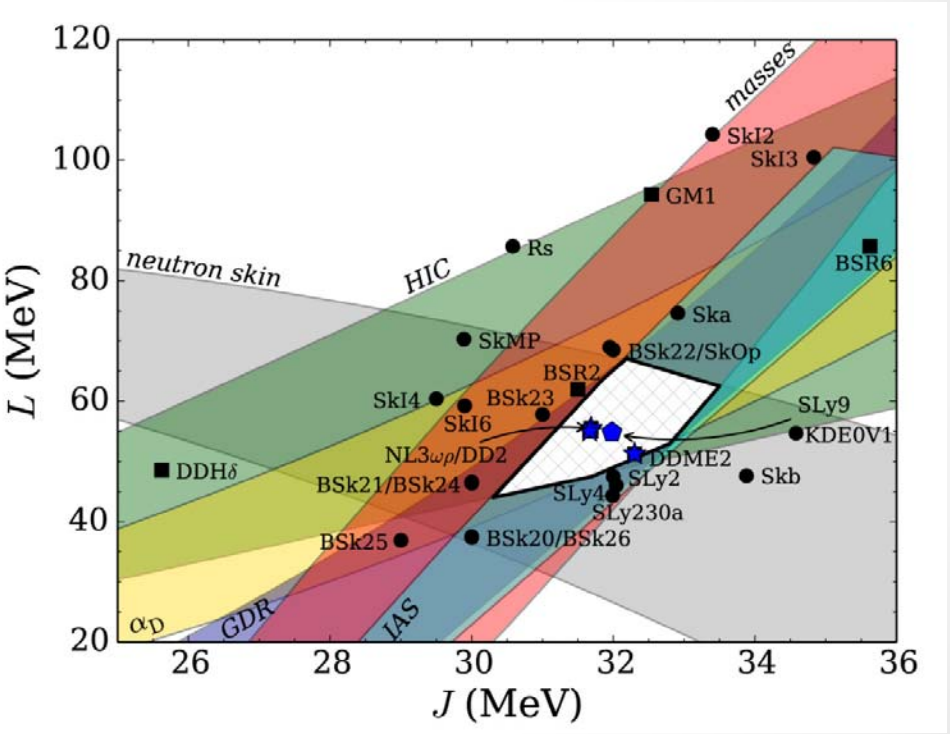
EoS constraints from a meta-modelling approach

J. Margueron, F. Gulminelli and R.Casali, ArXiV/nucl-th:1708.06894.
ArXiV/nucl-th:1708.06895.

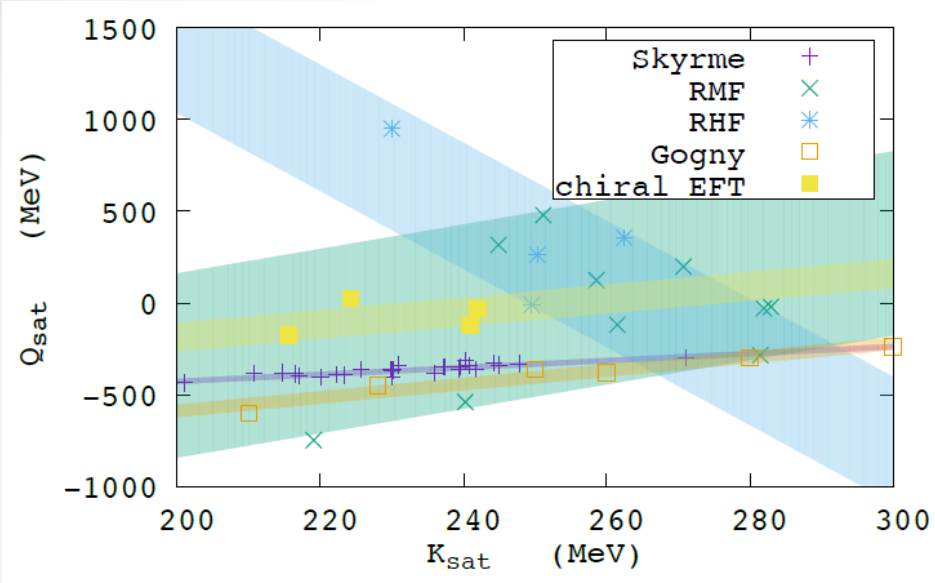
EoS and empirical constraints

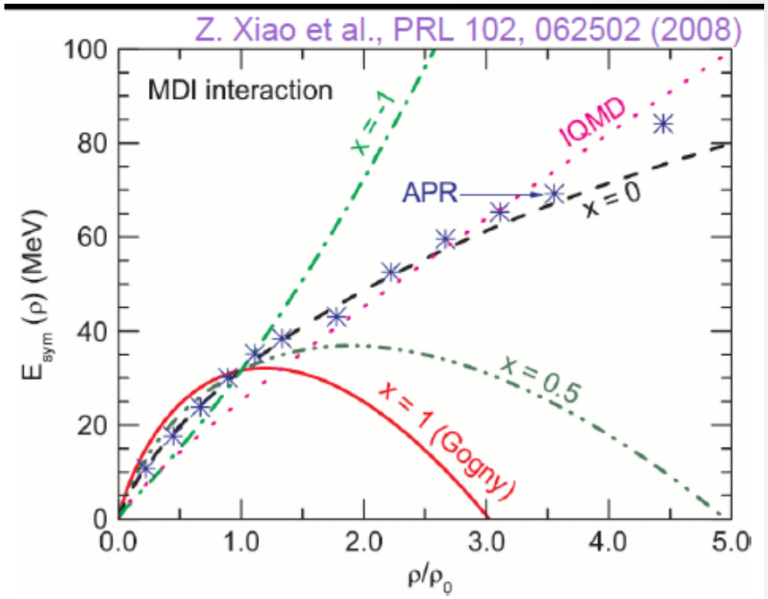
- EoS ⇔ set of empirical parameters $P_k = \partial^k e/\partial\rho^k$ ex: E_{sym} ={J,L,K_{sym} , Q_{sym} , Z_{sym}…}
- DFT models corresponding to different EoS are compared to exp.data
- $P_k \pm \Delta P_k$ determined fitting the model to the data
- Correlations among P_k are typically observed



M.Fortin et al, PRC 94,035804

Problem: model dependence





- DFT models assume a specific functional dependence: they have less parameters than the empirical set P_k
  - => emp.parameters are a-priori correlated
  - ⇒ not all possible density dependences are explored
  - ⇒ results are model dependent and extrapolations to high density not reliable

A meta-modelling approach

- An empirical functional that can reproduce most existing models and continuously interpolate among them

Taylor expansion around n_0

$$e_{NM}(n,\delta) = \sum_k \frac{1}{k!}\left(c_k^{IS} + c_k^{IV}\delta^2\right)\left(\frac{n-n_0}{3n_0}\right)^k$$

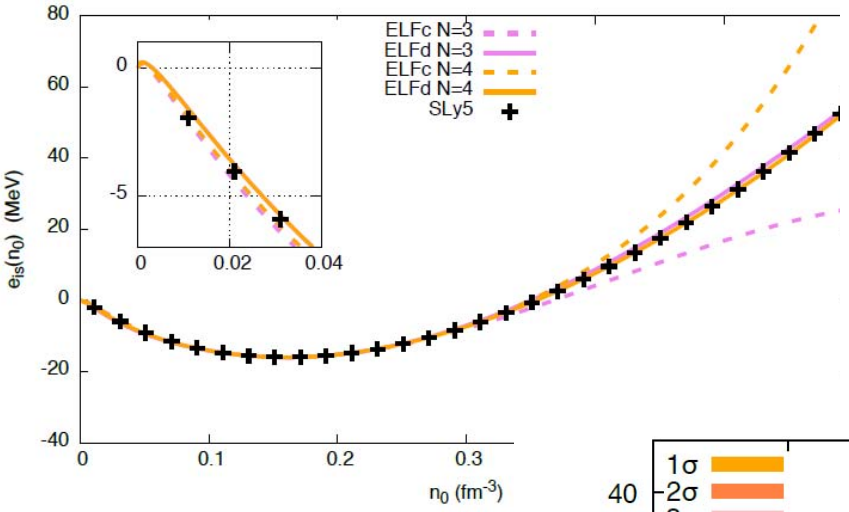
$$\begin{cases} n = n_p + n_n \\ \delta = (n_p - n_n)/n \end{cases}$$

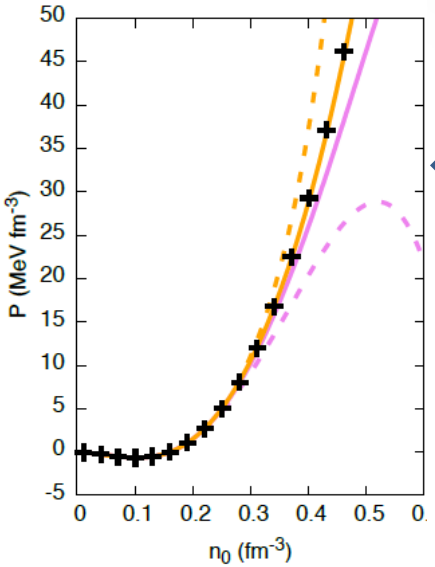
+ boundary conditions $n = 0$ & $n \to \infty$

+ explicit expression of the kinetic term (high order δ terms, effective mass)

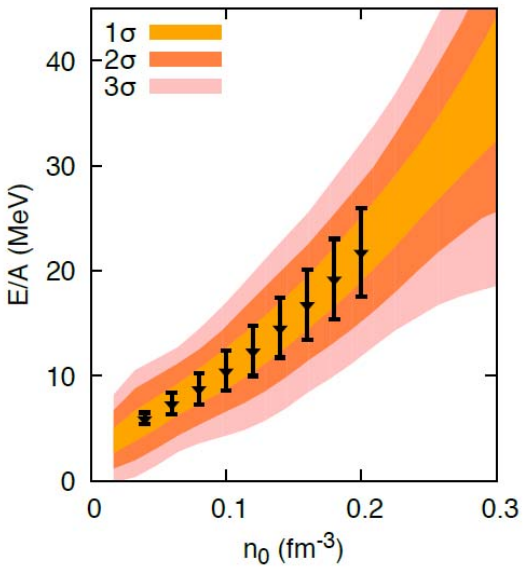
=> one-to-one correspondence between c_k and emp.parameters P_k

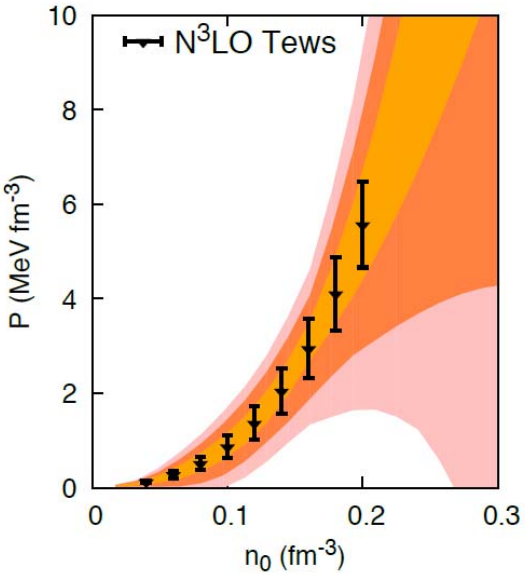
Reproduction of existing models





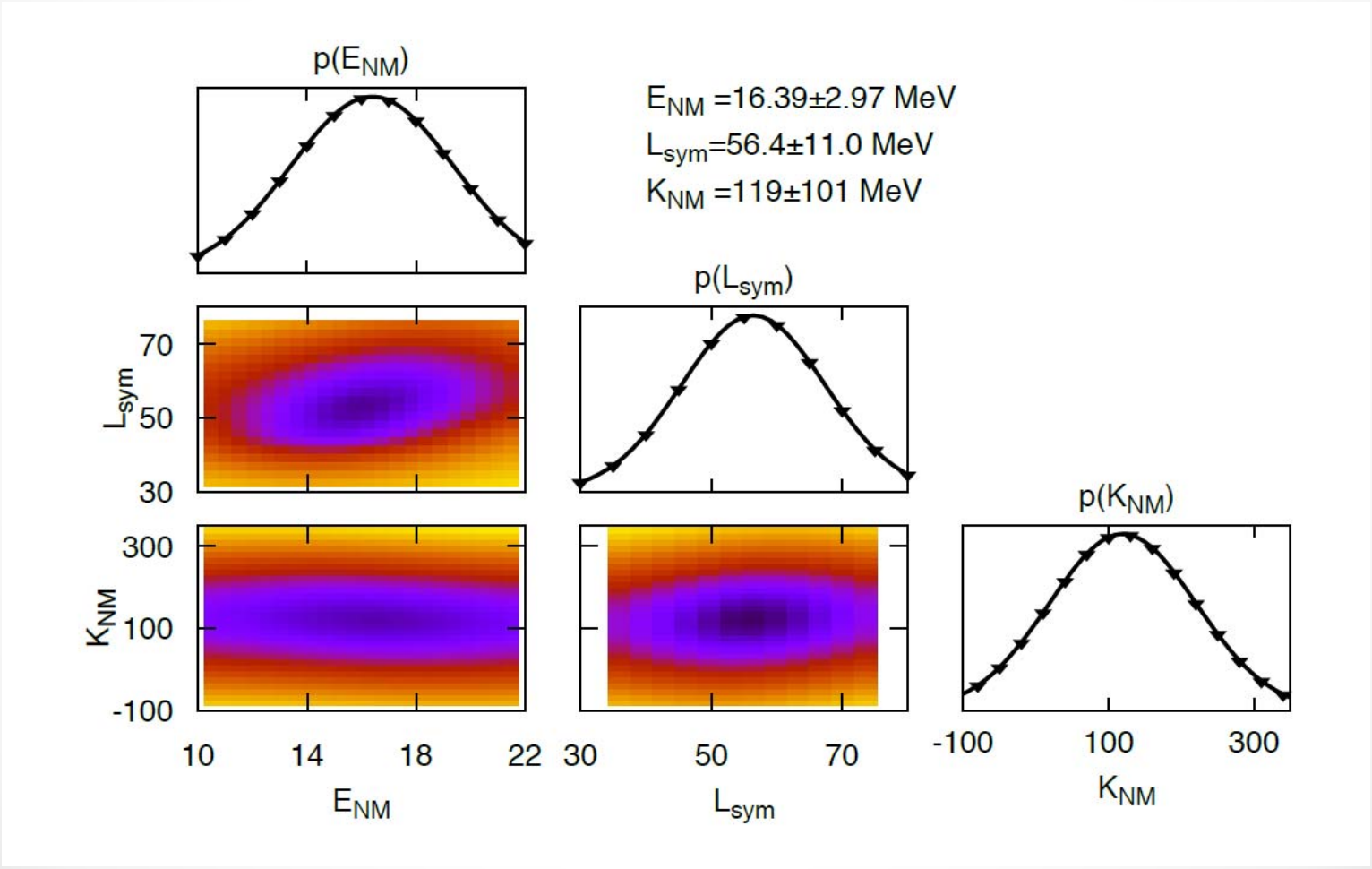
Sly5
E.Chabanat, P.Bonche, P.Hansel, J.Meyer, R.Schaeffer, NPA627(1997)710



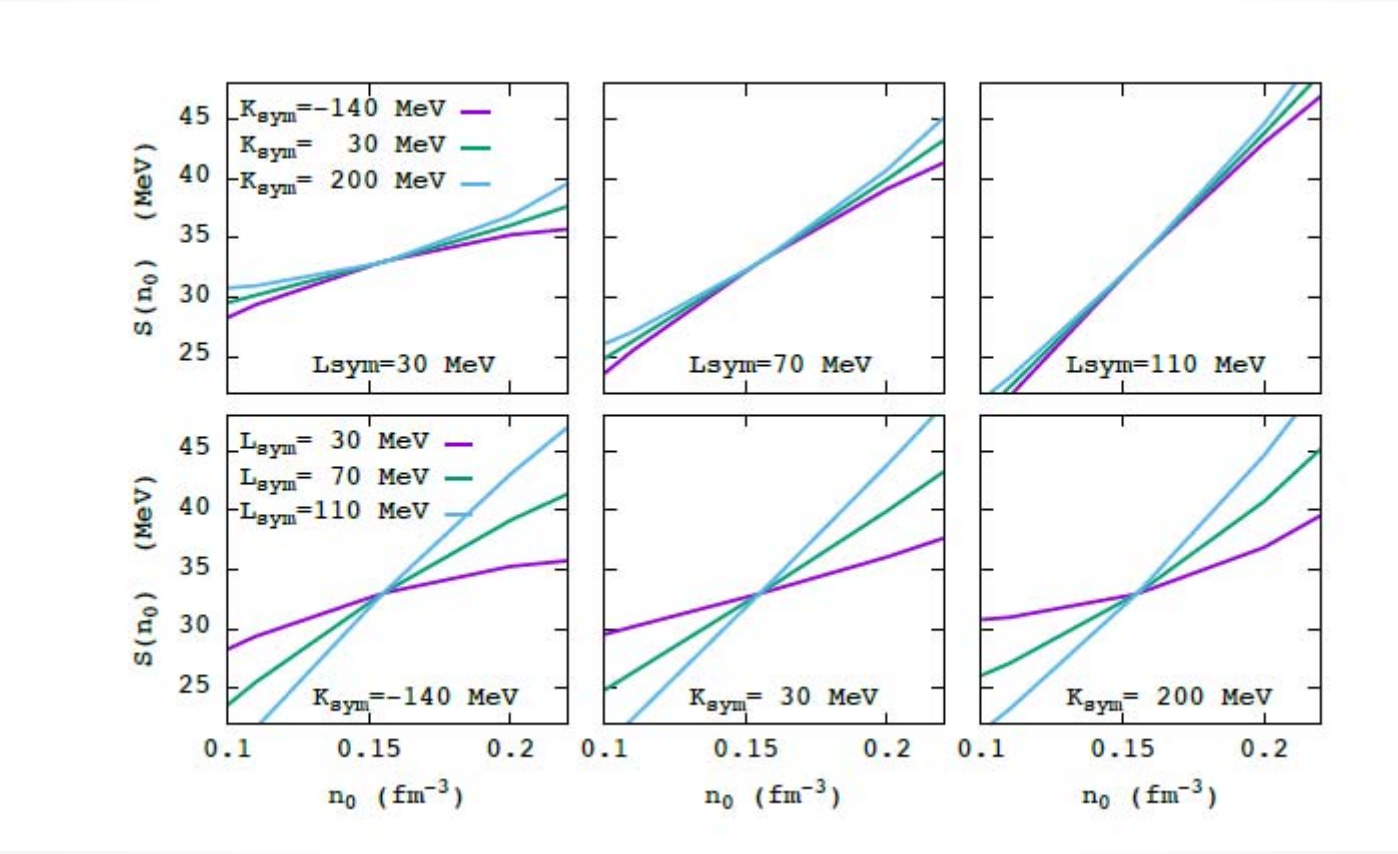


Chiral EFT
I.Tews, T.Kruger,K.Hebeler,A.Schwenk PRL110(2013)032504

Reproduction of existing models



Flexibility: symmetry energy



Varying P_k within their uncertainties: ~25 millions models

P_α	E_{sym} MeV	L_{sym} MeV	K_{sat} MeV	K_{sym} MeV	Q_{sat} MeV	Q_{sym} MeV	Z_{sat} MeV	Z_{sym} MeV
$P_{\alpha,1}$	32	60	230	-100	300	0	-500	-500
$P_{\alpha,2}$	2	15	20	100	400	400	1000	1000
Min	26	20	190	-400	-1300	-2000	-4500	-5500
Max	38	90	270	200	1900	2000	3500	4500
step	2	10	20	75	400	400	1000	1000
N	7	8	5	9	9	11	9	11

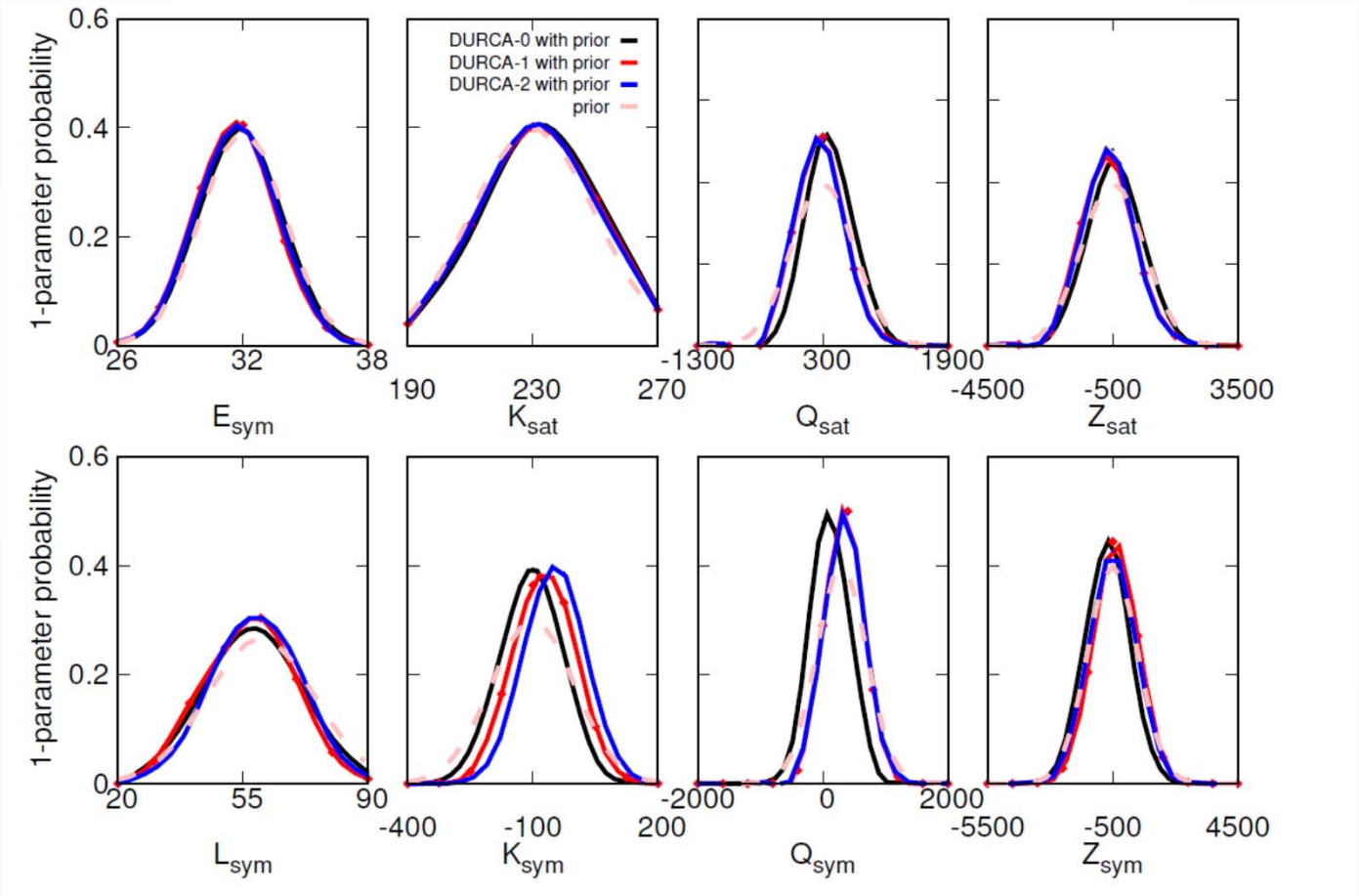
Present estimations of P_k and their uncertainty: a compilation

Model (N_α)	der. order	E_{sat} MeV 0	E_{sym} MeV 0	n_{sat} fm^{-3} 1	L_{sym} MeV 1	K_{sat} MeV 2	K_{sym} MeV 2	Q_{sat} MeV 3	Q_{sym} MeV 3	Z_{sat} MeV 4	Z_{sym} MeV 4	m^*_{sat}/m -	$\Delta m^*_{sat}/m$ -
Phenomenological approaches													
Skyrme	Average	-15.88	30.25	0.1595	47.8	234	-130	-357	378	1500	-2219	0.73	0.08
(16)	σ	0.15	1.70	0.0011	16.8	10	66	22	110	169	617	0.10	0.24
Skyrme	Average	-15.87	30.82	0.1596	49.6	237	-132	-349	370	1448	-2175	0.77	0.127
(35)	σ	0.18	1.54	0.0039	21.6	27	89	89	188	510	1069	0.14	0.310
RMF	Average	-16.24	35.11	0.1494	90.2	268	-5	-2	271	5058	-3672	0.67	-0.09
(11)	σ	0.06	2.63	0.0025	29.6	34	88	393	357	2294	1582	0.02	0.03
RHF	Average	-15.97	33.97	0.1540	90.0	248	128	389	523	5269	-9956	0.74	-0.03
(4)	σ	0.08	1.37	0.0035	11.1	12	51	350	237	838	4156	0.03	0.01
Ab-initio approaches													
APR	Average	-16.0	33.12	0.16	50.0	270	-199	-665	923	337	-2053	1.0	0.0
(1)	σ	-[†]	0.30	-[†]	1.3	2	13	30	67	94	125	-[†]	-[†]
χ-EFT	Average	-15.16	32.01	0.171	48.1	214	-172	-139	-164	1306	-2317	-	-
Drischler 2016	σ_{tot}	1.24	2.09	0.016	3.6	22	40	104	234	214	379	-	-
(7)	Min	-16.92	28.53	0.140	43.9	182	-224	-310	-640	901	-2961	-	-
	Max	-13.23	34.57	0.190	53.5	242	-108	24	96	1537	-1750	-	-

Filtering the models: constraints from neutron star physics

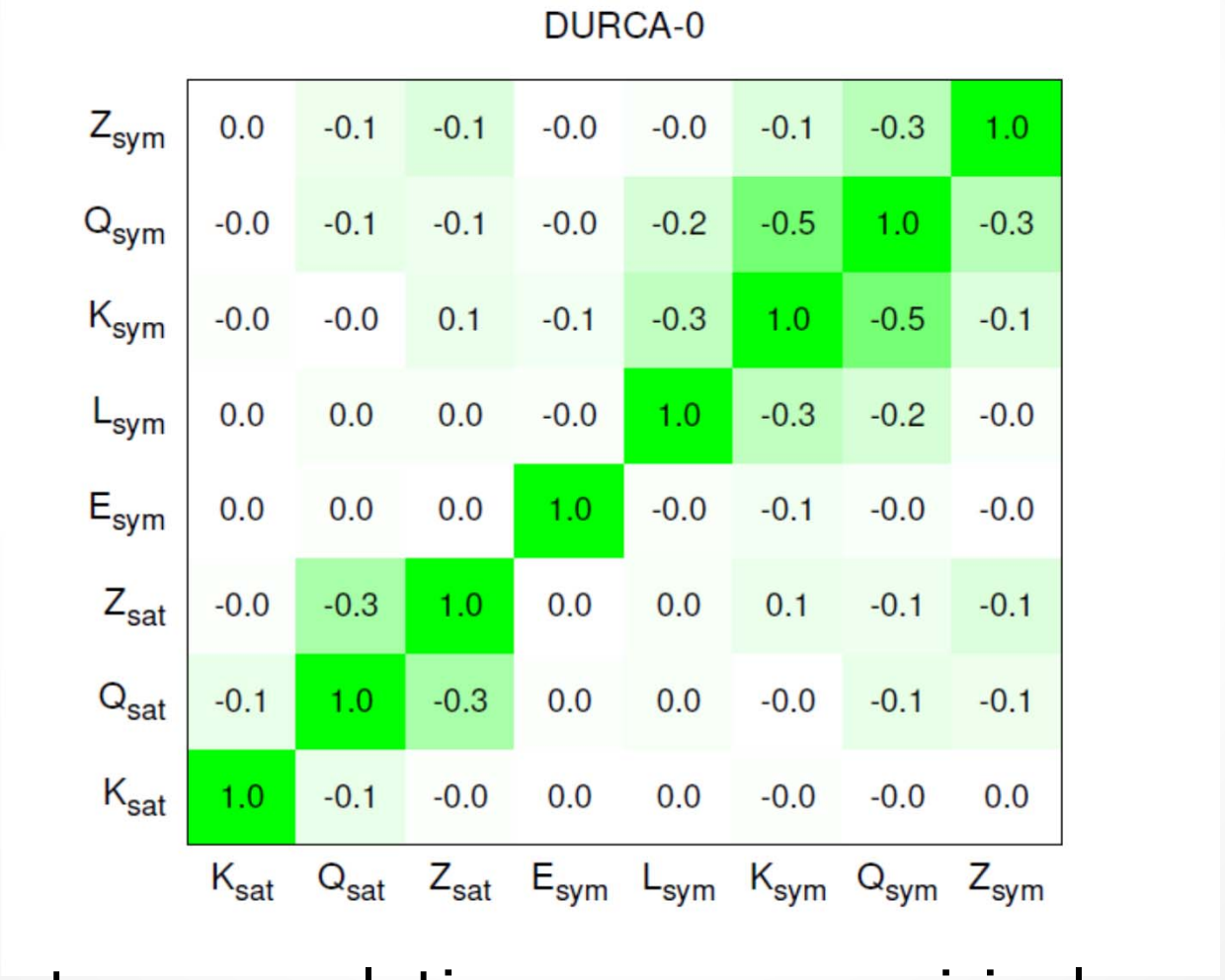
- Causality: $0 < v_s < c$
- NS stability: $\nabla p > 0$ for $n > n_0$ in β-equilibrium
- $E_{sym} > 0$
- $M_{max}>2M_o$
- Limit on DURCA:
  - No DURCA up to $2M_o$ – DURCA0
  - DURCA only for $M>1.8M_o$ – DURCA1
  - DURCA only for $M>1.6M_o$ – DURCA2

Posterior distribution of emp. parameters



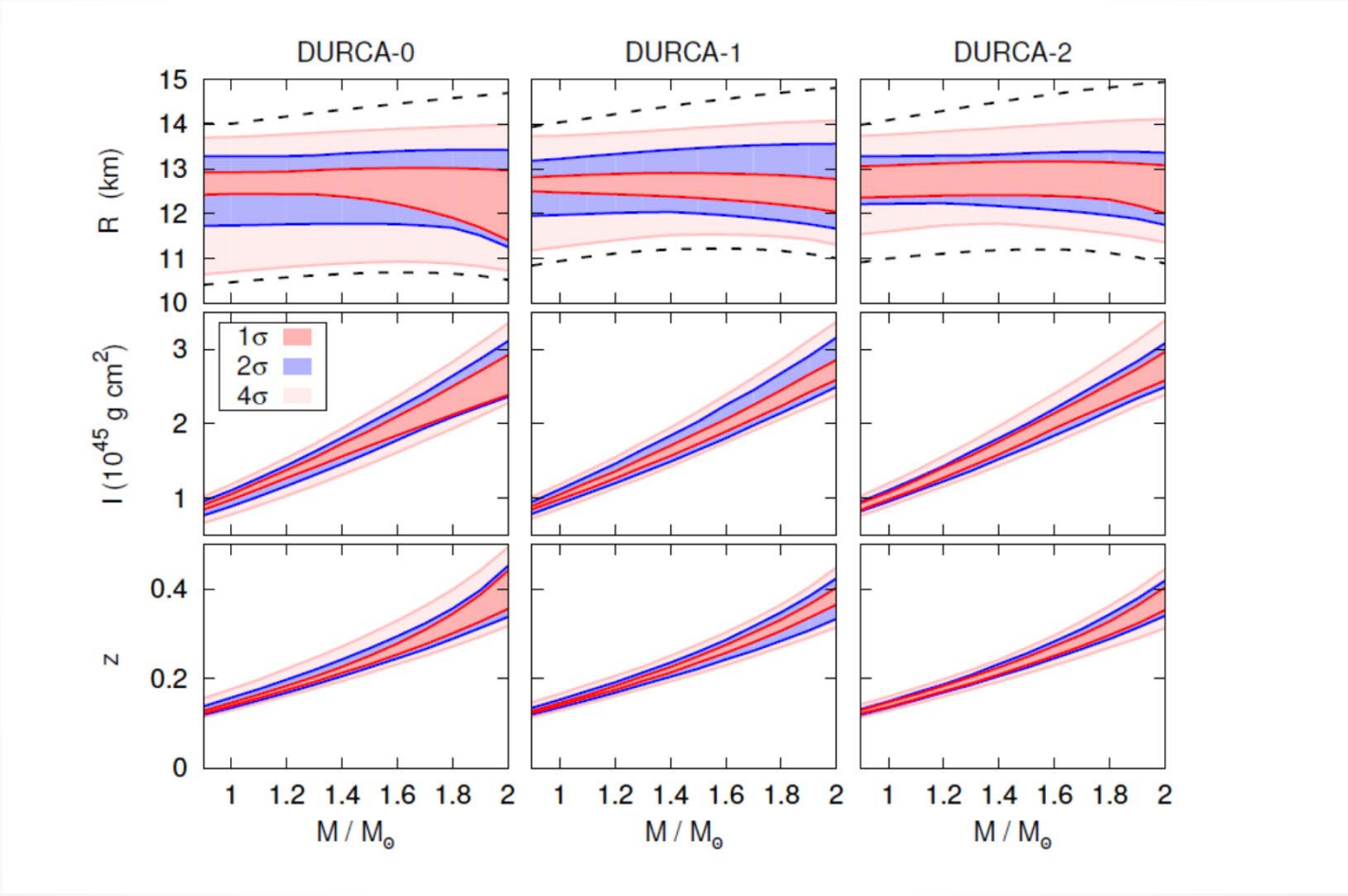
- => Almost no constraint from NS!

Correlation matrix



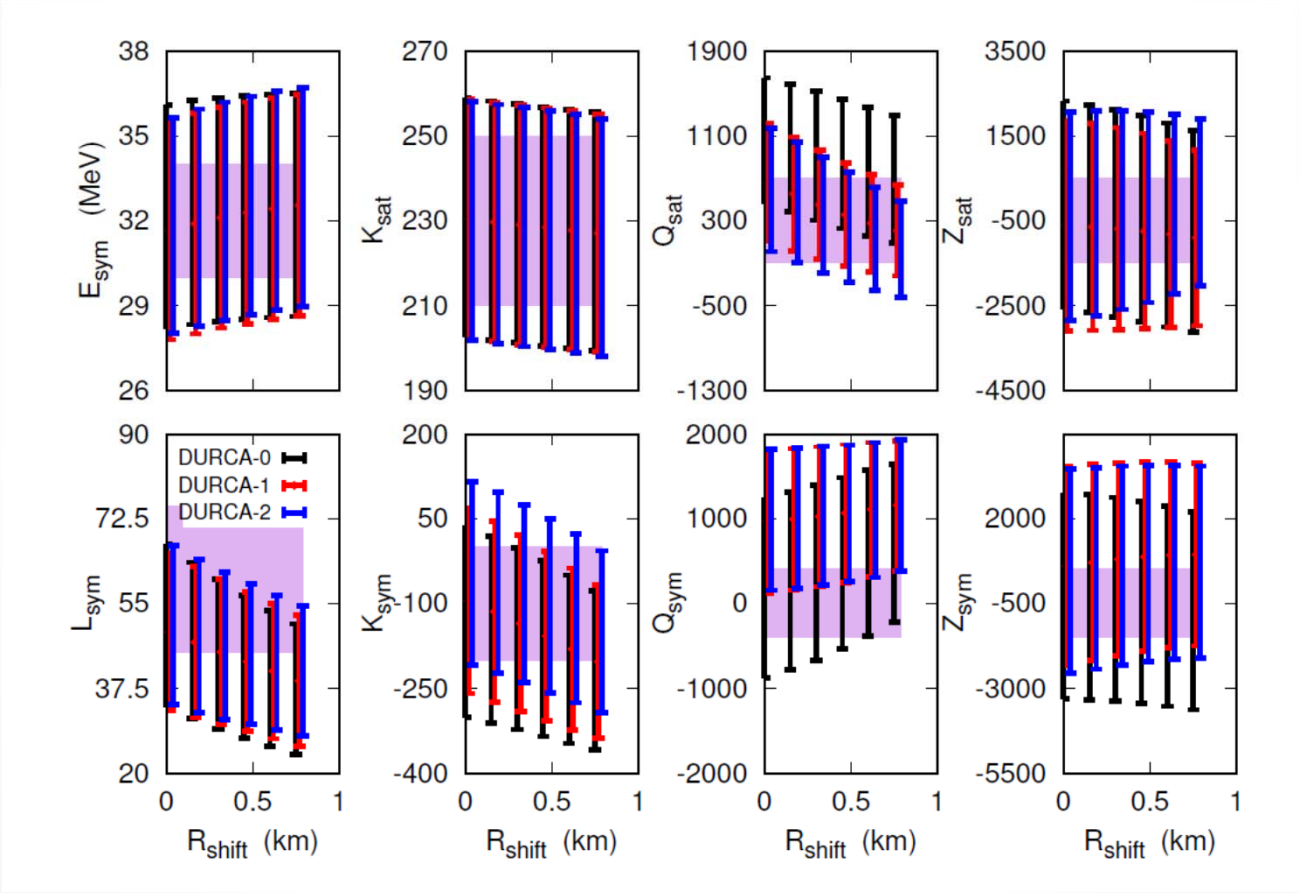
=> Almost no correlation among empirical parameters!

Predictions on astro observables



- => 11,5<R<13,5 at 2σ independent of the mass!

Inversion problem



=> A shift in the R(M) observation would considerably improve our knowledge of Lsym and Ksym!

Conclusions

- Meta-modelling empirical EoS able to reproduce the different nuclear models and interpolate between them
- Bayesian determination of parameters with flat or gaussian prior

- **Third order derivatives still largely unconstrained**
- **ALMOST NO CORRELATION AMONG EMP.PARAMETERS**
- **Quantitative predictions on NS radii**
- **Many more results in nucl-th:1708.06894. &1708.06895**
- **Constraints from mass and radii in Debarati's talk!**