Geometric approach to nuclear pasta phases

Sebastian Kubis
Cracow University of Technology

- Rigorous treatment of pasta phases in CLDM
- Stability of the lasagna phase

NySym17, Caen

Neutron star structure

g/cm^3

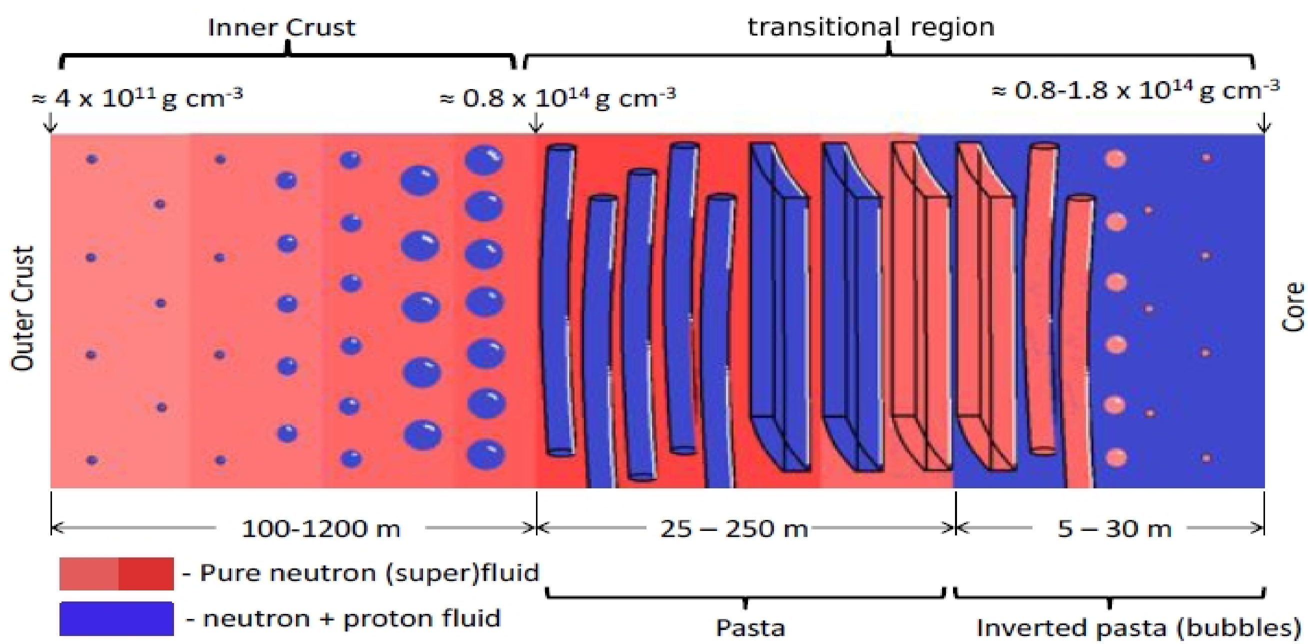
10^1 — atmosphere (ionized light elements: *C* , *O*, ..)

10^4 — e – degeneracy, Coulomb crystal of ^{56}Fe nuclei

10^7 — $\mu_e > m_n - m_p = 1.3$ MeV neutronization of nuclei

10^{11} — neutron drip → proton clusters in quasi-free neutron liquid

10^{13} — cluster deformation → pasta phases

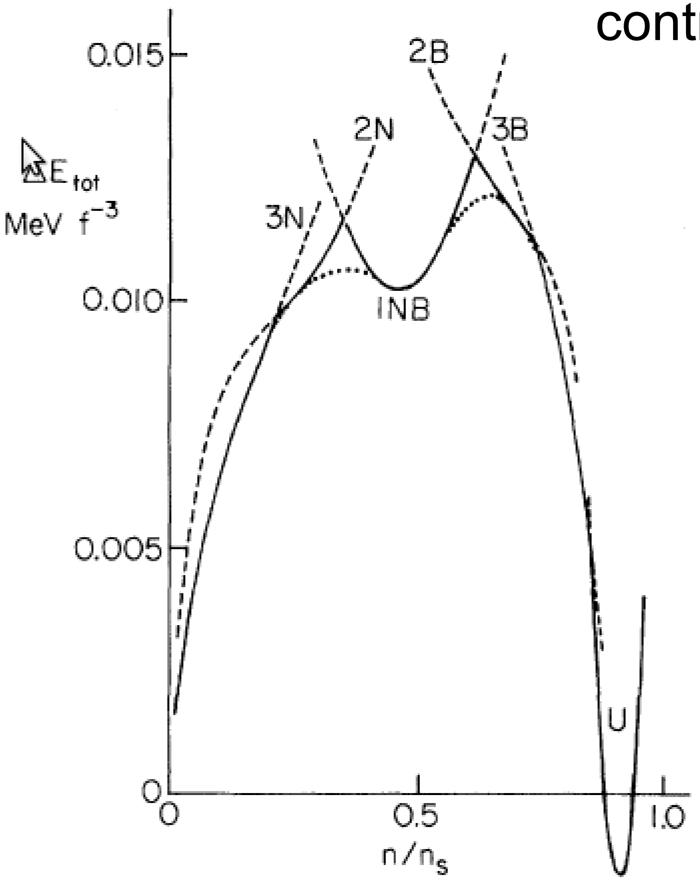


cartoon by W.G.Newton

Pasta appearance in the crust-core transition region

proton clusters in neutron gas = sharp boundary between two phases with opposite charge

$$\mathcal{E} = \mathcal{E}_{bulk}(w) + \Delta\mathcal{E}(shape)$$



contribution from surface and Coulomb energy

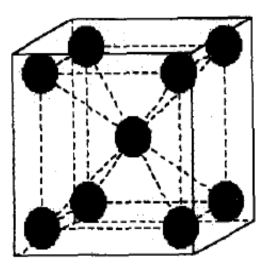
$$\Delta E(shape) = \mathcal{E}_C + \mathcal{E}_S$$

$$\mathcal{E}_C = \Delta\rho^2 f_d(w) \qquad \Delta\rho = \rho_+ - \rho_- = en_p$$

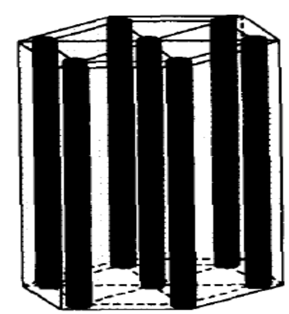
$$\mathcal{E}_S = \sigma \frac{wd}{r} \qquad w = \frac{V_P}{V_{cell}}$$

Ravenhall ' 83

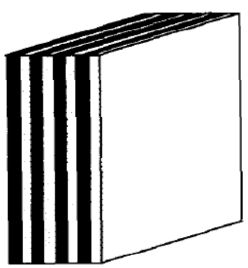
d=3 - gnocchi

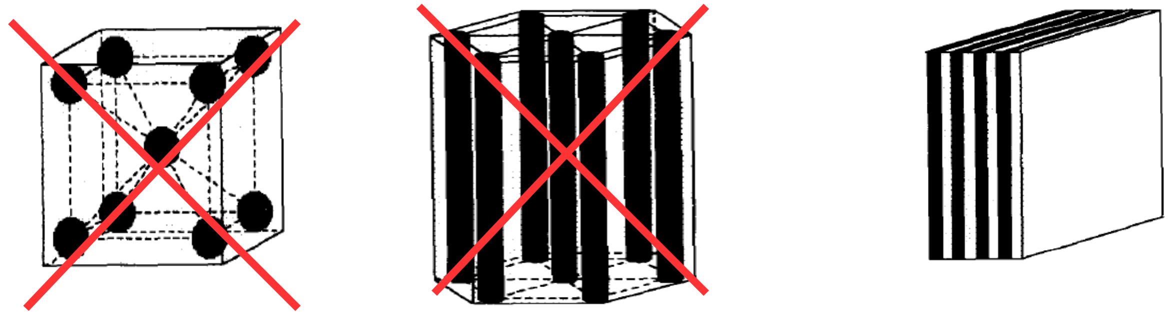


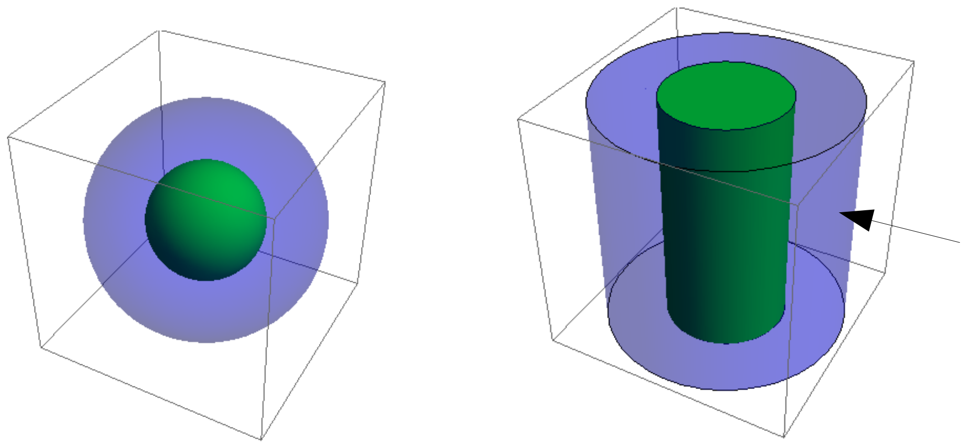
d=2 - spaghetti



d=1 - lasagna







Fixed geometry !

$\vec{E} = 0$ at cell boundary

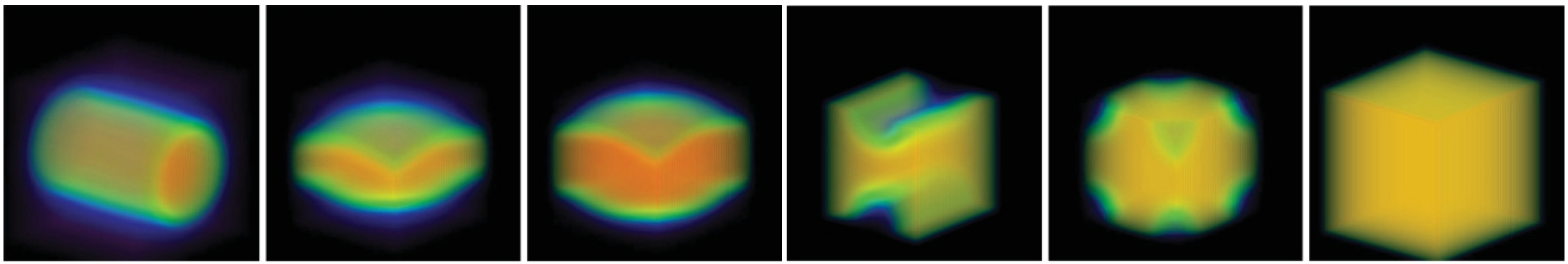
no interactions between neighboring clusters:

- Deformations ?
- Transition between different shapes ?
- Stability analysis of such structures ?

Development in other approaches to pasta phases
Hartree-Fock

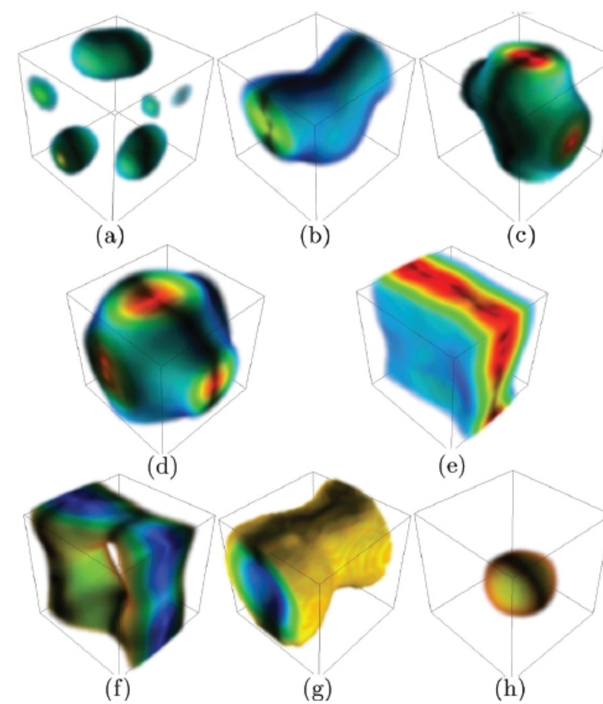
Periodic boundary conditions

$n_B = 0.04\ \mathrm{fm}^{-3}$ $n_B = 0.11\ \mathrm{fm}^{-3}$



W.G. Newton '09

variety of structures…
not only rods and slabs

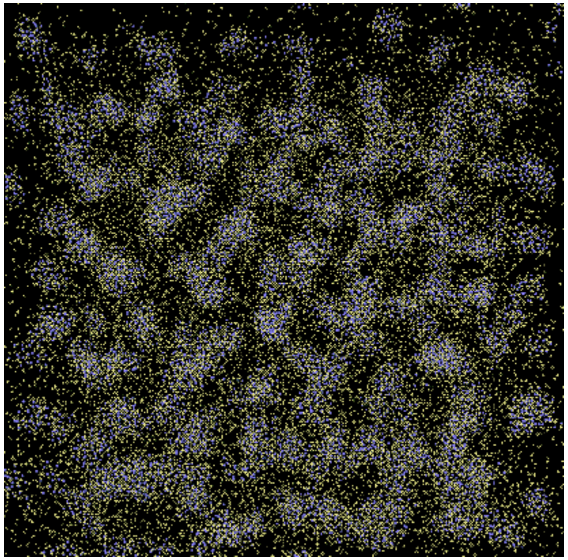


Temperature ??

Time-Dependent HF
Schuetrumpf `13

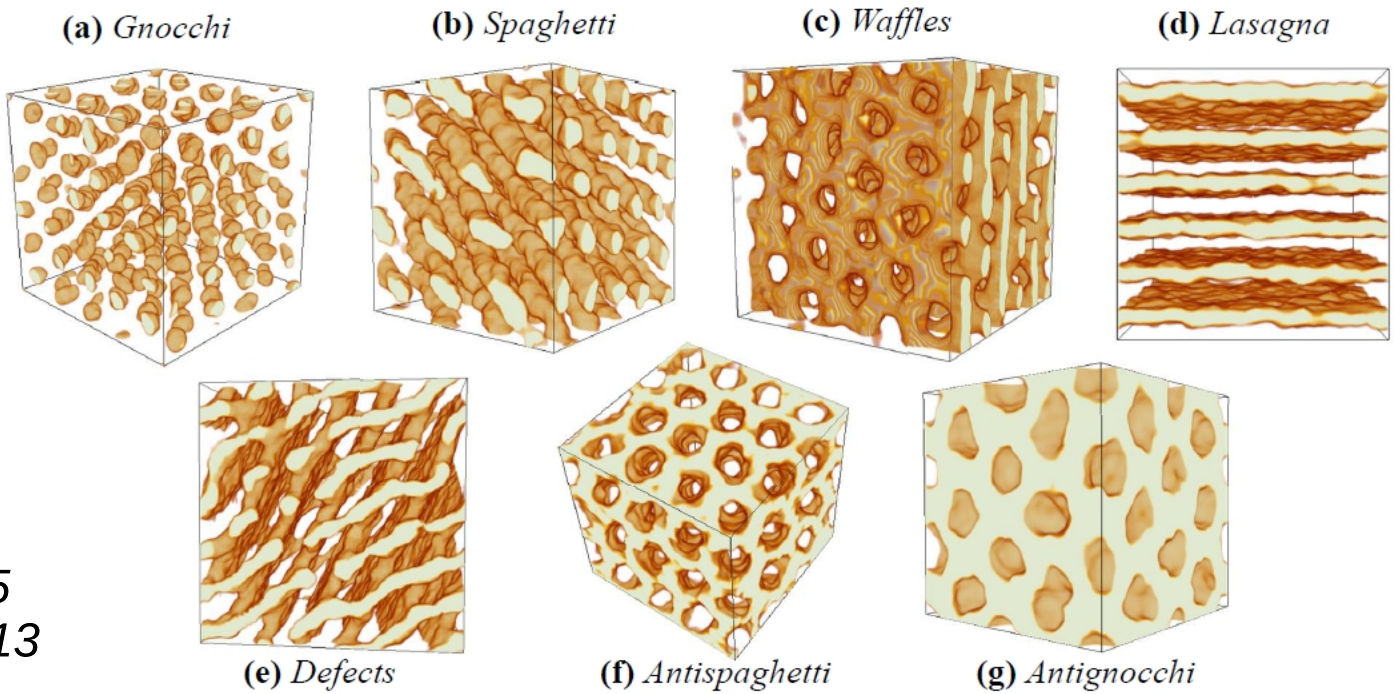
Development in other approaches to pasta phases

Molecular Dynamics



$$V_{NN}(r) \sim e^{-r^2/\Lambda} \; , \quad \sim \frac{\alpha}{r} e^{-r/\lambda}$$

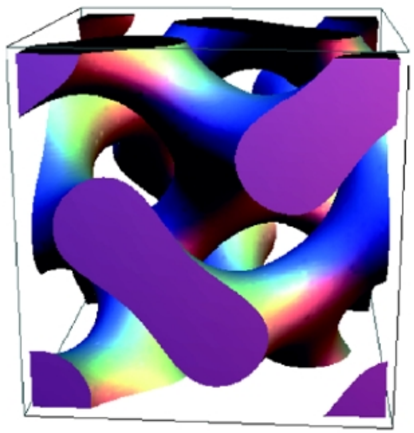
- two body potential for pointlike particles
- 50 000 nucleons in a box
- temperature ~ 1 MeV
- periodic boundary



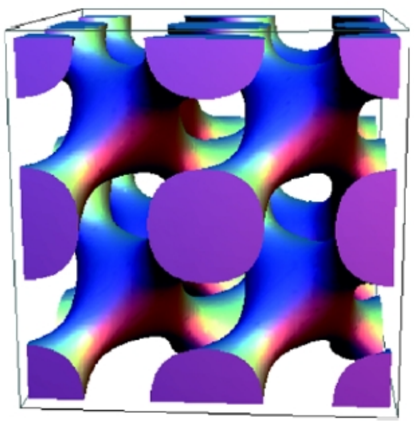
Horowitz '15
Schneider '13

Nontrivial structures in CLDM

Gyroid



Double-diamond

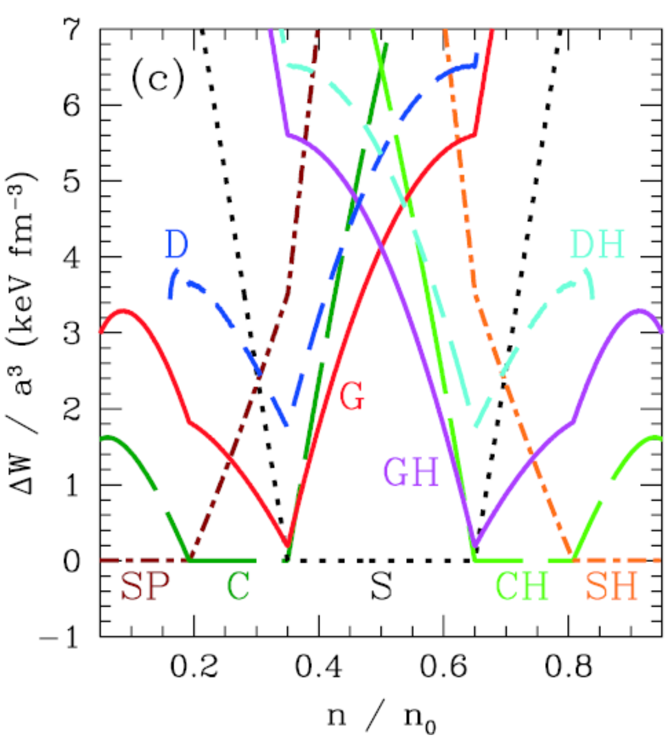


Minimal surfaces:

- mean curvature *H=0*

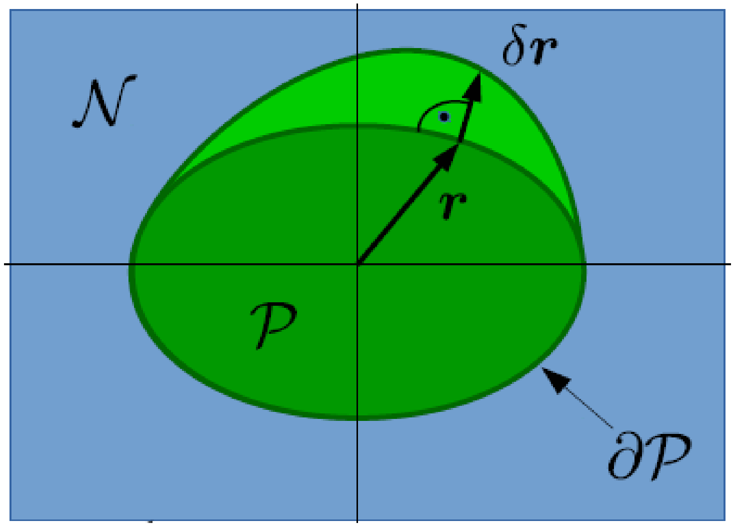
- shape imposed – not derived

Nakazato '09



energy always greater than
for the "traditional shapes"

Rigorous treatment of shape variation



$$\mathcal{E}_C = \frac{1}{2V_{cell}} \int \Phi \, \rho \, d^3x \quad , \qquad \mathcal{E}_S = \frac{1}{V_{cell}} \sigma S$$

$$\nabla^2 \Phi = -4\pi\rho$$

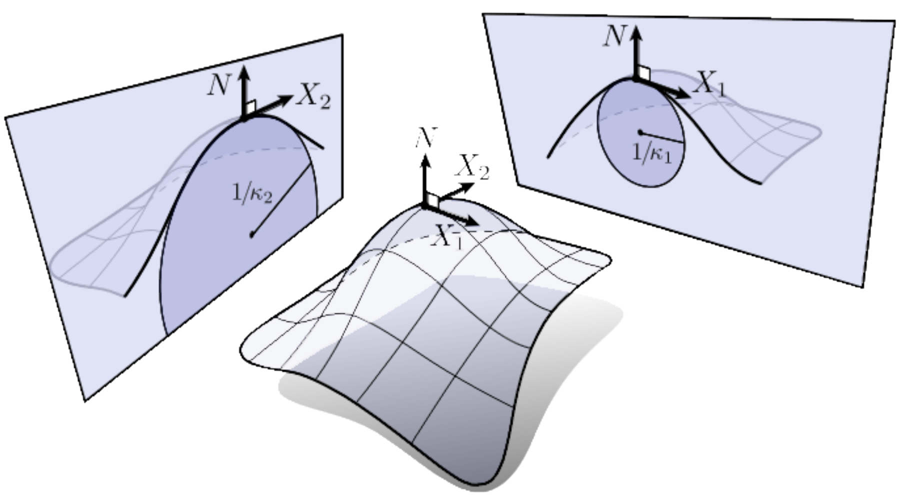
any deformation $\boldsymbol{\delta}\mathrm{r} \longrightarrow \boldsymbol{\delta\Phi}$

$$\delta(\mathcal{E}_{bulk} + \mathcal{E}_C + \mathcal{E}_S) = 0$$

modified Young-Laplace equation

$$P^{\mathcal{P}} - P^{\mathcal{N}} = -2\sigma H + \Delta\rho \, (\Phi - \langle \Phi \rangle_P)$$

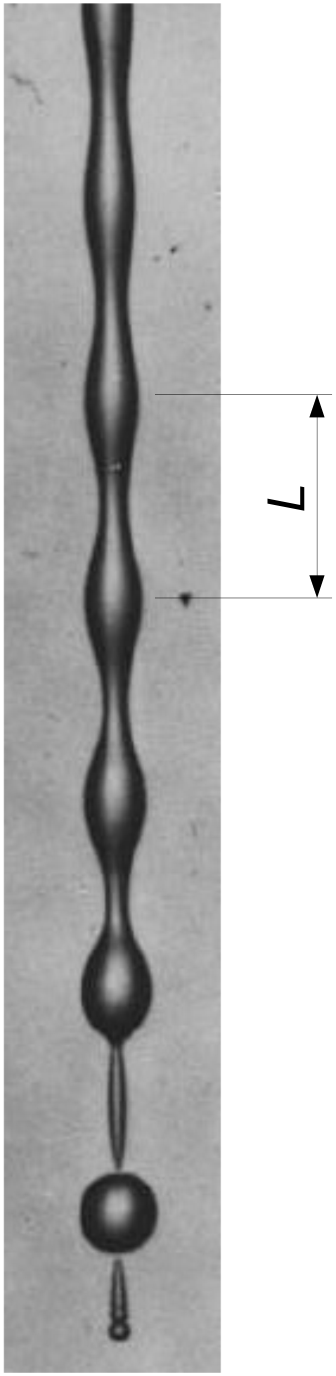
Kubis '16



$\Delta\rho = \rho_+ - \rho_- = e \, n_p$ – charge contrast

$H = \frac{1}{2}(\kappa_1 + \kappa_2)$ – mean curvature

mean curvature *H* prescribed by
pressure difference and the potential *Φ*



an example - instability of uncharged cylinder

$$P_{in} - P_{out} = \frac{2\sigma}{R_0}$$

perturbed cylinder $R_0 \to R_0 + \epsilon$

volume preserving deformation $\int \epsilon \, dS = 0$

$$\delta^2 \mathcal{E}_S = \frac{\sigma}{2} \int ((\nabla \epsilon)^2 - (\kappa_1^2 + \kappa_2^2)\, \epsilon^2) dS > 0$$

negative !

first unstable mode $-\nabla^2 \epsilon - \frac{1}{R_0^2}\, \epsilon = 0$ Jacobi eqaution

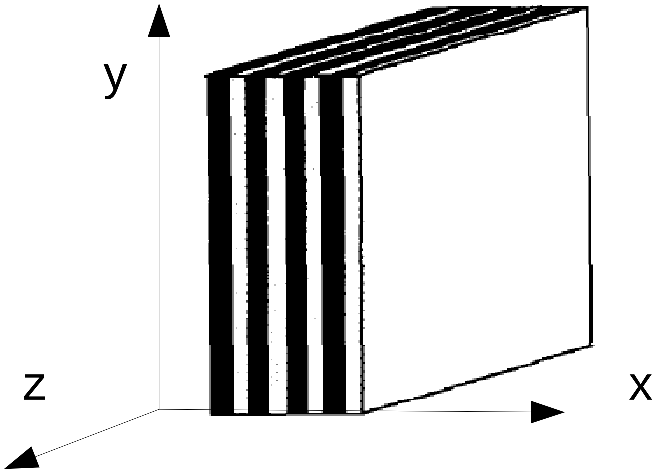
$\epsilon(z) \sim \sin(2\pi z/L)$ *L* – mode wavelength

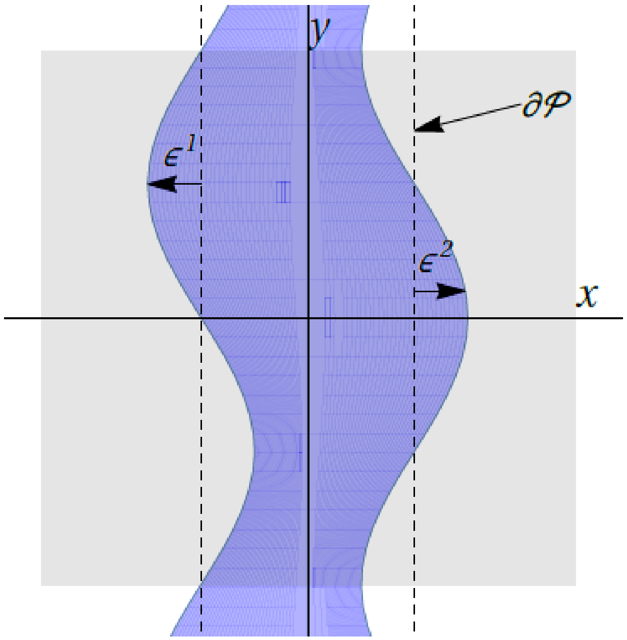
$\frac{2\pi R_0}{L} > 1$ liquid cylinder is always unstable

Rayleigh-Plateau 1878

Looking for nontrivial shapes - stability analysis

lasagna – the only one periodic solution in the CLDM





different modes for different faces

$$\delta^2(\mathcal{E}_{bulk} + \mathcal{E}_C + \mathcal{E}_S) > 0$$

$$\int \sigma((\nabla\epsilon)^2 - (\kappa_1^2 + \kappa_2^2)\ \epsilon^2) + \Delta\rho\ (\partial_n\Phi\ \epsilon^2 + \delta_\epsilon\Phi\ \epsilon)\ dS > 0$$ *Kubis '16*

“Virial theorem” $\mathcal{E}_S = 2\mathcal{E}_C$ $\delta^2\mathcal{E}$ deos not depend on $\sigma,\ \Delta\rho$

symmetry energy does not matter !

$$\delta^2\mathcal{E}(w, a, b, c) > 0$$

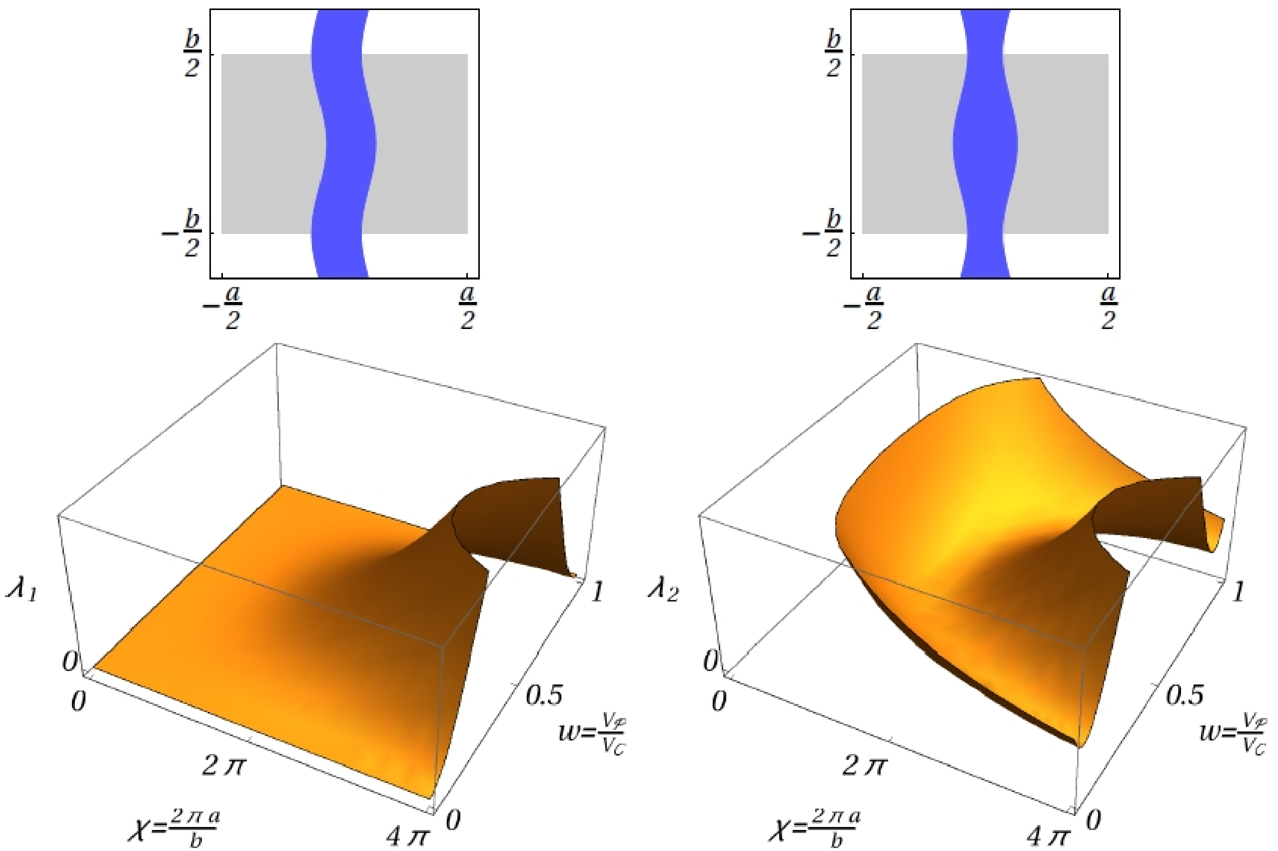
a - cell size
b, c – mode wavelengths in *y, z* directions
only the geometry is valid !

Looking for unstable modes for the slab

$\delta^2\mathcal{E} = \vec{\epsilon}\,\hat{M}\,\vec{\epsilon}^{\,T}$ $\quad \vec{\epsilon} \to \epsilon_{i,k,l}$ *l*-th face, *k,l* modes numbers for z and y directions

normal modes of $\hat{\mathsf{M}}$ $\rightarrow$ eigenvalues $\lambda_{\mathsf{j}}(\mathsf{w},\chi)$ stability functions

modes in different directions decouple



$$\chi = \frac{2\pi a}{b}$$

$$w = \frac{V_{\mathcal{P}}}{V_{cell}}$$

$\delta^2\mathcal{E} > 0$ for any mode sizes and volume fraction !

the slab locally stable → no “decay” channel ?

Conclusions

- nontrivial shapes in CLDM described by the extension of Young-Laplace equation

$$P^{\mathcal{P}} - P^{\mathcal{N}} = -2\sigma H + \Delta\rho\,\left(\Phi - \langle\Phi\rangle_P\right)$$

- stability of charged cluster – nontrivial question

$$\sigma(-\nabla^2\epsilon - \left(\kappa_1^2 + \kappa_2^2\right)\epsilon) + \Delta\rho\,\left(\partial_n\Phi\,\epsilon + \delta_\epsilon\Phi\right) = 0$$

- stability does not depend on symmetry energy

- lasagna phase is locally stable for any wavelength mode and volume fraction occupied by the cluster