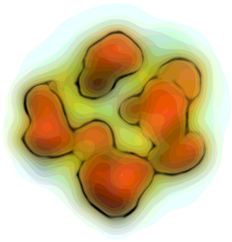
Inhomogeneity growth in two-component fermionic systems

P. Napolitani, M. Colonna

1) Can we apply Fermi-liquid theory of nuclear matter (*NM*) to heavy-ion collisions (*HIC*) in a one-body framework ?

2) Is the 3D solution of the BL eq. in NM giving fluctuations of correct amplitude in both isoscalar/isovector channels ?

3) New understanding of HIC ?



the full story :

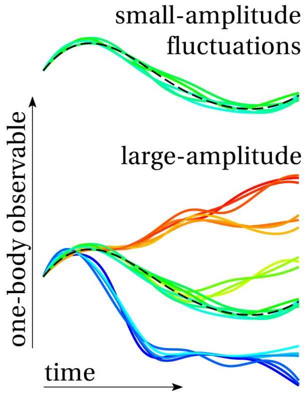
[Napolitani,Colonna arXiv :1705.08268 2017]

Describing large-amplitude dynamics of nucleonic systems

- *small amplitude-limit at low energy*
→ correlated channels, coherent states
[TDGCM : Reinhard,Goeke RepPrPh50(1987), Goutte *et al* PRC 71 (2005), Balian-Vénéroni (1981,1992), Simenel EPJA 48 (2012),...]

- *large-amplitude regimes at low energy* →
→ non-correlated states beyond single-part. picture [Lacroix arXiv :1504.01499 (2015)]

- *excitation beyond Pauli blocking* →
→ observables width spread, dissipation
[ETDHF : Wong,Tang PRL 40 (1978), Lacroix *et al* ProgPartNucPhys 52 (2004),...]



- *chaotic regime, bifurcation*
→ highly non-linear, possibly unstable
[STDHF / Boltzmann-Langevin : Reinhard,Suraud AnnPhys 216(1992), Slama,Reinhard,Suraud AnnPhys 355 (2015),...]
⇒ (1) Clusterisation from one-body density fluctuations
⇒ (2) n, p transport between fragments and the medium

Handling many-body correlations in mean-field extensions

- Pure mean-field equations are not valid in regions where instabilities, bifurcations, chaos are present

BBGKY (*Born Bogoliubov Green Kirkwood Yvon*) hyerarchy

for a 2-body interaction V_{ij} :

$$i\hbar\frac{\partial\rho_1}{\partial t} = [k_1,\rho_1] + \mathrm{Tr}_2[V_{12},\rho_{12}]$$

$$i\hbar\frac{\partial\rho_{12}}{\partial t} = [k_1+k_2+V_{12},\rho_{12}] + \mathrm{Tr}_3[V_{13}+V_{23},\rho_{123}]$$

kinetic equations

$$\dots \qquad \dots$$

$$i\hbar\frac{\partial\rho_{1\cdots k}}{\partial t} = \sum_{i=1}^{k}[k_i+\sum_{j<i}^{k}V_{ij},\rho_{1\cdots k}] + \sum_{i=1}^{k+1}\mathrm{Tr}_{k+1}[V_{ik+1},\rho_{1\cdots k+1}]$$

high coupling regimes

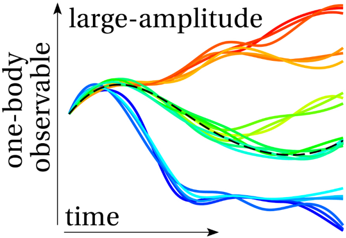
$$\dots \qquad \dots$$

($\rho_{1\dots k}$=many-body dens. matrix, k_i=Kin.E operator, Tr_{ij}=partial trace)

- aim : going from 2^{nd} *order* approx. to highly non-linear regimes

- higher-orders approximately recovered (rather than truncated) by handling several mean fields

[Lacroix *et al* EPJA52 (2016)]



Beyond 2nd order through a stochastic treatment

define a MF-trajectory subensemble $\{\rho_1^{(n)}; n = 1...\text{sub}\}$ over $\tau_{BL} = t - t_0$ with small fluctuations around the mean $\Leftrightarrow$ probability $\rho_1^{(n)}, \rho_2^{(n)}$ to find two nucleons 1, 2 at two config. points not all time decorrelated

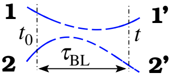
$\rho_{12}^{(n)}$ recovers some correlations of upper BBGKY orders, so that :

$$\rho_{12}^{(n)}(t_0) = \overbrace{\widetilde{\Omega}_{12}\mathcal{A}_{12}(\rho_1^{(n)}(t_0)\rho_2^{(n)}(t_0))\widetilde{\Omega}_{12}^{+}}^{\textit{collisional correlations}} + \overbrace{\delta\rho_{12}^{(n)}(t_0)}^{\text{fluctuations}} \; ;$$

fluctuations $\longrightarrow$ introduces fluct. around the collision integral

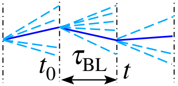
$$\langle \delta\rho_{12}^{(n)}(t_0) \rangle_{\tau_{BL}} = 0 \; ;$$

$$\langle \delta\rho_{12}^{(n)}(t_0)\delta\rho_{12}^{(n)}(t') \rangle_{\tau_{BL}} = \textbf{gain} + \textbf{loss}$$



(diffusion matrix : $G_{12} = V_{12}\widetilde{\Omega}_{12} \rightarrow$ NN diff. cross section $|G_{12}|^2 \sim d\sigma/d\Omega$)

- if $\delta\rho_{12}^{(n)}(t_0) = 0 \Rightarrow \rho_1^{(n)}, \rho_2^{(n)}$ fully decorrelated $\rightarrow$ 2nd order truncation without fluctuations $\rightarrow$ (quantum) Boltzmann
- else $\Rightarrow \delta\rho_{12}^{(n)}(t_0)$ is an intermittent source of fluctuation seeds $\rightarrow$ it can be exploited as a stochastic source



Obtaining an exploitable description

For one mean-field trajectory n in τ_{BL} :

Stochastic-TDHF scheme

average coll. term

$$i\hbar \frac{\partial \rho_1^{(n)}}{\partial t} \approx [k_1^{(n)} + V_1^{(n)}, \rho_1^{(n)}] + \bar{I}_{coll}^{(n)} + \delta I_{coll}^{(n)}$$

fluctuating coll. term

after τ_{BL}, it yields $\rho_1^{(n)} \rightarrow \{\rho_1^{(n_\lambda)}; \lambda = 1, ...\text{sub}_\lambda\}$

[Reinhard,Suraud AnnPhys 216 (1992) ; AnnPhys 355 (2015)
Lacombe,Reinhard,Suraud,Dinh AnnPhys 373 (2016)]

Wigner tr. →

Boltzmann-Langevin

$f^{(n)}$: distribution functions
→ Fermi stat. at equilibrium

$$\frac{\partial f^{(n)}}{\partial t} = \{h^{(n)}, f^{(n)}\} + I_{UU}^{(n)} + \delta I_{UU}^{(n)}$$

Markovian contrib. :

$$\langle \delta I_{UU}^{(n)}(\mathbf{r},\mathbf{p},t)\delta I_{UU}^{(n)}(\mathbf{r}',\mathbf{p}',t')\rangle = $$
$$= \mathbf{gain} + \mathbf{loss} = 2\mathcal{D}(\mathbf{r},\mathbf{p};\mathbf{r}',\mathbf{p}',t')\delta(t-t')$$

[Ayik,Grégoire PLB212(1988) ; NPA513(1990)
Colonna,Chomaz,Randrup NPA567(1994)]

Boltzmann-Langevin One Body

$$\frac{\partial f^{(n)}}{\partial t} - \{h^{(n)}, f^{(n)}\} = I_{UU}^{(n)} + \delta I_{UU}^{(n)} = g\int \frac{d\mathbf{p}_b}{h^3} \int W(\text{AB}\leftrightarrow\text{CD})\, F(\text{AB}\rightarrow\text{CD})\, d\Omega$$

transition rate — occupancy

$$W(\text{AB}\leftrightarrow\text{CD}) = |v_A - v_B| \frac{d\sigma}{d\Omega} \; ; \quad F(\text{AB}\rightarrow\text{CD}) = \Big[(1-f_A)(1-f_B)f_C f_D - f_A f_B (1-f_C)(1-f_D)\Big]$$

A, B, C, D : extended equal-isospin phase-space portions of size=nucleon imposed by the variance $f(1-f)$ in h^3 cells at equilibrium

[Napolitani,Colonna PLB726 2013 ; arXiv :1705.08268 2017]

The same scheme for NM and open systems

simplified SKM* [Guarnera,Colonna,Chomaz PLB373 (1996)]

$$\frac{E_{pot}}{A}(\rho) = \frac{A}{2}u + \frac{B}{\sigma+1}u^{\sigma} + \frac{C_{surf}}{2\rho}(\nabla\rho)^2 + \frac{1}{2}C_{sym}(\rho)u\beta^2$$

$A = -356$ MeV, $B = 303$ MeV, $\sigma = 7/6 \rightarrow K = 200$ MeV (soft) ;

$C_{sym}(\rho) = 32$(asy-stiff)/ $= \rho_{sat}(482 - 1638\rho)$MeV(asy-soft);

- momentum dependence omitted

residual term

- E-dependent free σ_{NN} or screened

BLOB) $\delta I \rightarrow$ fluct. in full phase space

we may compare to :

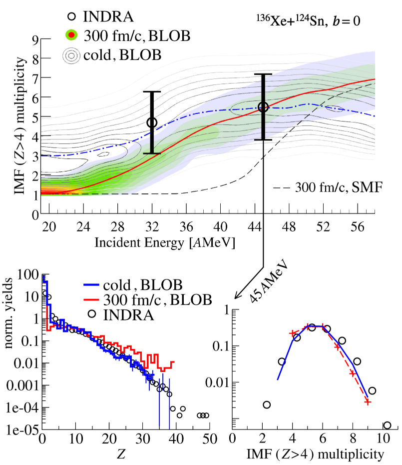
SMF) $\delta I \rightarrow$ separately treated as a stochastic force related to U_{ext}

$\Rightarrow$ fluctuations projected on spacial ρ

HIC$\Rightarrow$ fragment observables

consistent with exp. data in BLOB $\longrightarrow$

an example :



[Colonna,Napolitani,Baran Nuc.ClusterCorrelations...(2017),

data from INDRA : PRC86 (2012), EPJA 50 (2014)]

The same scheme for NM and open systems

simplified SKM* [Guarnera,Colonna,Chomaz PLB373 (1996)]

$$\frac{E_{pot}}{A}(\rho) = \frac{A}{2}u + \frac{B}{\sigma+1}u^{\sigma} + \frac{C_{surf}}{2\rho}(\nabla\rho)^2 + \frac{1}{2}C_{sym}(\rho)u\beta^2$$

$A=-356$ MeV, $B=303$ MeV, $\sigma=7/6 \rightarrow K=200$ MeV (soft) ;

$C_{sym}(\rho) = 32$(asy-stiff)/ $= \rho_{sat}(482 - 1638\rho)$MeV(asy-soft);

- momentum dependence omitted

residual term

- E-dependent free σ_{NN} or screened

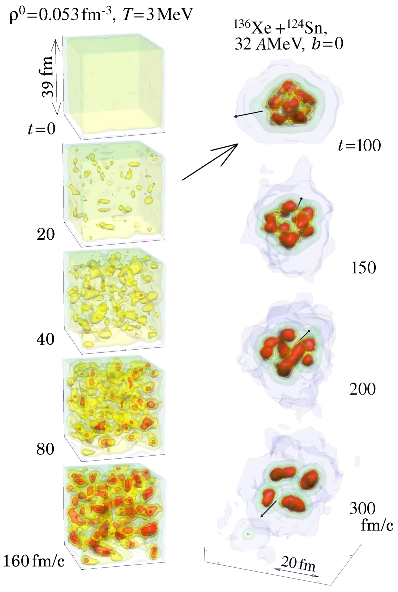
BLOB) $\delta I \rightarrow$ fluct. in full phase space

SMF) $\delta I \rightarrow$ fluctuations projected

HIC⇒ fragment observables consistent with exp. data in BLOB

NM⇒ *are fluctuation amplitudes also consistent with analytic expectations ?*

→ check in initially homogeneous NM ⟶ (Fermi-Dirac at $\rho^0, T=3$MeV)



Fluctuations in nuclear matter and the BL equation

- Small disturbance in uniform matter around the mean trajectory f^0 :

q=s) *isoscalar*, n, p in phase : $\delta f^{\mathrm{s}} = (f_{\mathrm{n}} - f_{\mathrm{n}}^0) + (f_{\mathrm{p}} - f_{\mathrm{p}}^0)$

q=v) *isovector*, n, p out of phase : $\delta f^{\mathrm{v}} = (f_{\mathrm{n}} - f_{\mathrm{n}}^0) - (f_{\mathrm{p}} - f_{\mathrm{p}}^0)$

$\delta f(\mathbf{r},\mathbf{p},t) \ll f^0(\mathbf{p},t)$

$\Rightarrow$ fluctuation ρ_k^{q} of variance $(\sigma_k^{\mathrm{q}})^2$ at equil.

- BL eq. for a fluctuating field U', in symmetric uniform matter, at low T ($\Rightarrow I_{\mathrm{UU}} = 0$), applied to the disturbance δf^{q} (1^{st} order approx) :

stable modes : *Fluctuation-dissipation th.*

spacial density correlation variance in ΔV at equilibrium (T)

$$(\sigma_k^{\mathrm{q}})^2 = \frac{T}{F^{\mathrm{q}}(k)} \; ; \quad (\sigma_{\rho^{\mathrm{q}}})^2 = \frac{T}{\Delta V}\left\langle \frac{1}{F^{\mathrm{q}}(k)} \right\rangle_{\mathbf{k}}$$

related to free-energy dens. curvature

$\Rrightarrow$ *nucl. matter :* check $(\sigma_{\rho^{\mathrm{v}}})^2$ versus Symmetry E

$\Rrightarrow$ *open-system analogy :* isotopic distributions

unstable modes

$$(\sigma_k^{\mathrm{q}})^2(t) \approx D_k \tau_k (e^{2\frac{t}{\tau_k}} - 1) + (\sigma_k^{\mathrm{q}})^2(t{=}0) e^{2\frac{t}{\tau_k}}$$

both continuous and initial fluctuation seeds yield an exponential growth τ_k

[Colonna *et al* PRC47(1993) NPA567 (1994)]

$\Rrightarrow$ *nucl. matter :* check instability growth rates

$\Rrightarrow$ *open-system analogy :* clusterisation

Isovector fluctuations in two-component nuclear matter

Isovector behaviour ($q \to v$, $\rho^v = \rho_n - \rho_p$) in BL in stable uniform matter

- scalar terms suppressed in $U^v \to$ stable conditions at all ρ_0

fluctuation-dissipation th.

$$(\sigma_{\rho^v})^2 = \frac{T}{\Delta V}\left\langle \frac{1}{F^v(k)} \right\rangle_{\mathbf{k}} \longrightarrow F^v_{\text{eff}} = \frac{T}{2\Delta V}\frac{\rho^0}{(\sigma_{\rho^v})^2} = \frac{T}{2\Delta V}\frac{\rho^0}{\langle[\delta\rho_n(\mathbf{r}) - \delta\rho_p(\mathbf{r})]^2\rangle} \propto E_{\text{sym}}$$

[Colonna PRL110 (2013)]

SMF ($\delta I \to$ projection) :

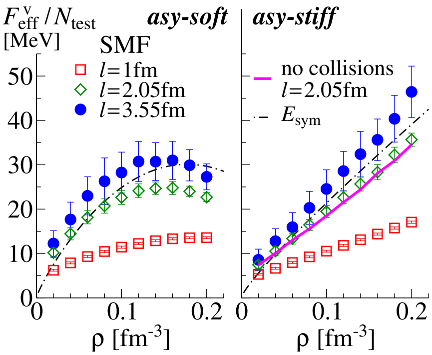
- solved with N_{test} test particles and fluctuation from MF noise

$\Rightarrow F^v_{\text{eff}} \approx N_{\text{test}} E_{\text{sym}}$

- same result without residual term

$\Rightarrow$ explicit iv term are missing in the iv channel

[Napolitani,Colonna arXiv:1705.08268 (2017)]

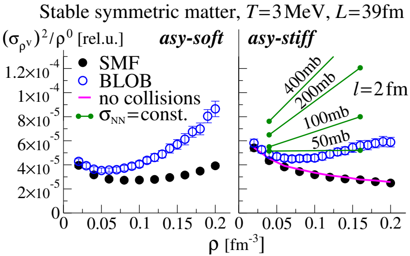


l : edge of periodic box (surface effects if too small)

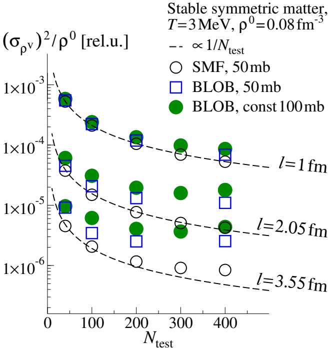
Isovector fluctuations in two-component nuclear matter

BLOB ($\delta I \rightarrow$ full ph. space) :

- larger iv variance, as a function of ρ^0
small $\rho^0 \rightarrow$ ineffective coll. correlations
large $\rho^0 \rightarrow$ longer path to convergence
[Napolitani,Colonna arXiv:1705.08268 (2017)]



- noise issue : smearing from approx. MF mapping [Reinhard,Suraud AnnPhy216 (1992)]
$\rightarrow N_{\text{test}}$ dependence. $\rightarrow$ *cured by* :
(1) *reviving NN correlations*
(larger coll. rate)
(2) *augmenting* N_{test}
$\Rightarrow$ still not enough in equilibrated NM
(low T, low $\rho \rightarrow$ rare collisions), but
better cured in early stages of HIC
(out of equilibrium, high coll. rate)



Sampling zero-sound propagation

Can a BL theory develop spontaneously isosc. fluct. of correct amplitude ?

- δI in homogeneous nucl. matter at low T ⇒ 0-sound collective modes
- if unstable conditions ⇒ amplification ⇒ catastrophic process
- if T increases in time ⇒ 0-to-1st-sound [Larionov *et al* PRC61 (2000), Kolomietz *et al* (1996)]

⇒ early times and small temperature restriction

- *linear-response* approx. → assuming small deviations from f^0

⇒ *dispersion relation* from *linearised Vlasov* (no residual terms) :

$$1 = \frac{g}{h^3}\frac{\partial U_k}{\partial \rho}\int \frac{\partial f^0}{\partial \epsilon}\frac{\mathbf{k}\cdot\mathbf{p}/m}{\omega_k + \mathbf{k}\cdot\mathbf{p}/m}\mathrm{d}\mathbf{p}$$

at $T = 0$ integration restricted to the Fermi surface

self-consistency ⟵

$$\omega_k f_k + \mathbf{k}\cdot\frac{\mathbf{p}}{m}f_k - \frac{\partial f^0}{\partial \epsilon}\frac{\partial U_k}{\partial \rho}\mathbf{k}\cdot\frac{\mathbf{p}}{m}\rho_k = 0$$

at $T = 0$ eigenmodes f_k depend on states near the Fermi level ϵ_F

⇓

dispersion relation

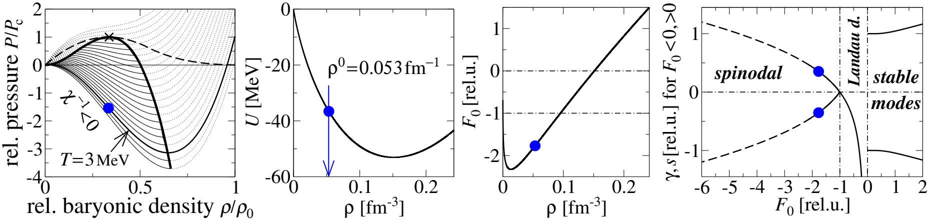
$$-\frac{1}{F_0} = 1 - \frac{s}{2}\ln\left(\frac{s+1}{s-1}\right), \quad \text{sound velocity} \quad s = \frac{\omega_k}{k v_F}$$

(v_F: Fermi velocity)

Landau parameter $F_0(\mathbf{k}) = \frac{3}{2}\frac{\rho^0}{\epsilon_F}\frac{\partial U_k}{\partial \rho}$

[Landau SovPhJETP 5 (1957)
Khalatnikov, Abrikosov SovPhJETP 6 (1958)
Colonna, Chomaz, Randrup NPA567 (1994) ; PhRep 389 (2004)]

Instabilities in zero-sound conditions



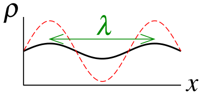
conditions of mechanical instability

negative incompressibility $\chi^{-1} \equiv \rho \dfrac{\partial P}{\partial \rho} = \dfrac{2}{3}\rho\epsilon_F[1 + F_0(k=0)] < 0 \quad \Rightarrow F_0(k=0) < -1$

[Pomaranchuk SovPhJETP 8 (1959)]

$\Rightarrow$ imaginary solutions $\gamma = is$ from $1 + \dfrac{1}{F_0(k)} = \gamma \arctan\dfrac{1}{\gamma} \rightarrow |\gamma| = \dfrac{|\omega_k|}{kv_F} = \dfrac{1}{\tau_k k v_F}$

$\Rightarrow$ disturbances of wave number k get amplified with a growth time τ_k and a growth rate $\Gamma_k = 1/\tau_k$



Response intensity at zero-sound conditions, analytic

ultraviolet cutoff

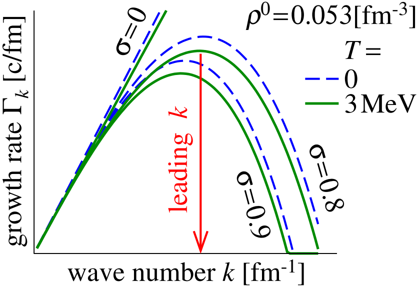
- small k : linear $\Gamma \propto k$ evolution (the more matter to be relocated, the longer it takes)
- large k : small λ excluded as a function of the interaction range

→ Gaussian smearing σ in $\mathcal{R}$ space

$$U \rightarrow U \otimes g(k), \text{ with } g(k) = \mathrm{e}^{-\frac{1}{2}(k\sigma)^2}$$

[Colonna,Chomaz PRC49 (1994) ; Kolomietz,Shlomo PRC60 (1999)]

⇒ Γ spectra display a *leading k mode* (the fastest growing disturbance)



leading k ⇒ most probable spinodal fragment size

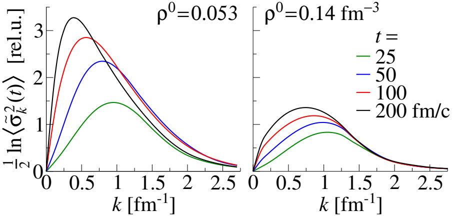
low, finite T

To fully explore the EOS (HIC ...) finite T should be included

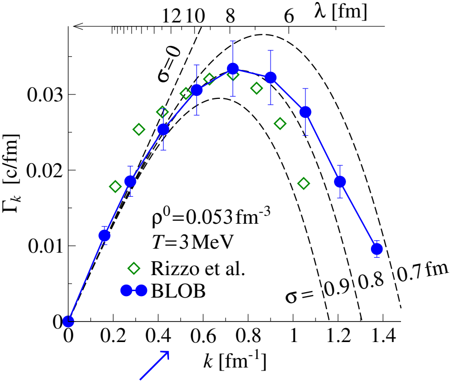
- Low-T expansion of chem.pot. $\eta(T > 0) \rightarrow \Gamma$ reduced

Response intensityat zero-sound conditions, BLOB

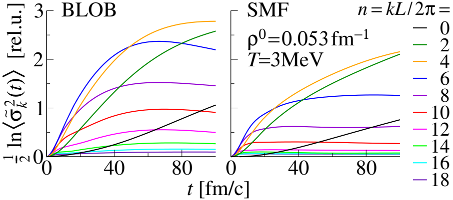
(asy-stiff, σ_{NN}=free+cut50mb, $l=1$fm)



$$\Gamma_k = \frac{1}{2}\frac{\partial}{\partial t}\ln < \tilde{\sigma}_k^2(t) >_{\text{paths}}$$



- $\rho^0 = 0.053\text{fm}^{-3} \rightarrow$ amplification $\Longrightarrow$
- $\rho^0 = 1.4\text{fm}^{-3} \rightarrow$ Landau damping with significant amplitude
- evolution : earlier saturation in BLOB, large k combine into small k :



leading λ~8to9fm, τ_k~200fm/c

$\Rightarrow$ Fragments arise with a size $A \approx \rho^0\lambda^3$, in the region of neon, separating at around 200fm/c

Isospin of emerging fragments in HIC

BLOB calculations in open systems :

$N' - Z'$ distr. for forming clusters around C and Ne, before and during fragment formation,

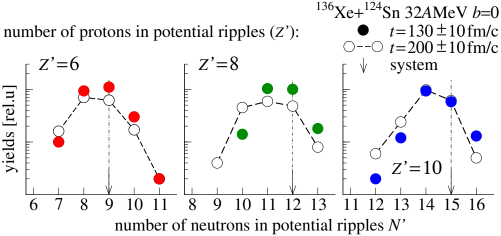
$$Y \approx \exp[-(\delta^2/A')\,C_{sym}(\rho)/T]$$

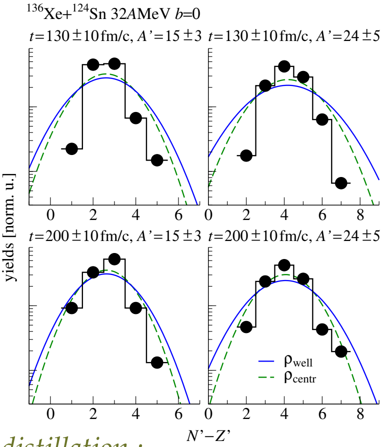
- underextimated but compensated by

1) fluct. built out of equilibrium

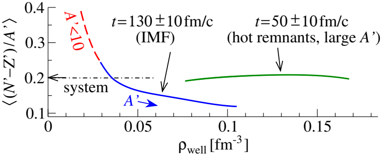
→ higher coll.rate

2) particle evaporation





distillation :

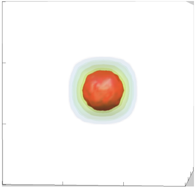


Some BLOB highlights from spallation to HIC

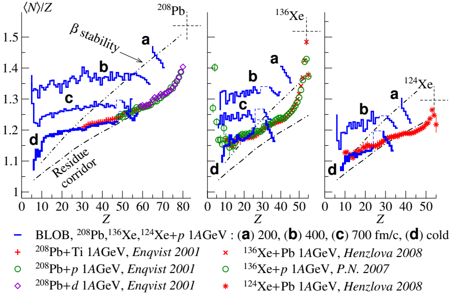
Fast *binary splits from re-aggregation*,

different from asymm. fission in ^{136}Xe+p1*A*GeV :

[Napolitani,Colonna PRC92 034607 (2015)]



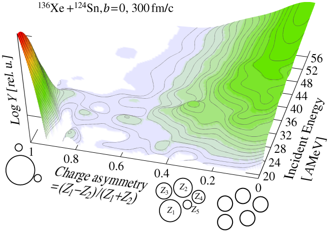
Isospin "thermometry" in spallation :



[Napolitani,Colonna PRC92 034607 (2015)]

Bimodality in central HIC at Fermi E

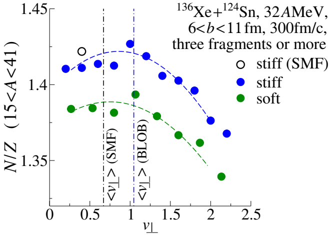
[Napolitani, Colonna PLB726 382 (2013)]



Neck formation and isospin migration

$$D^{\rho}_{iv}\nabla\rho \propto \frac{\partial S(\rho)}{\partial\rho} I\nabla\rho$$

[Colonna, P.N.,Baran Nuc.Cluster Correl...(2017),



Baran2005 ; Colonna2006 ; DiToro2006 ; Rizzo2008 ; ...]

Conclusions

BLE :

- No a priori assumption on the degree of equilibration
- BLOB : one-body theory based on a full solution of the BLE

Application to NM :

- BLOB : correct connection between MF pot. and unstable/stable is/iv modes tested in NM

Application to Open systems :

→ tracking a variety of dynamical trajectories

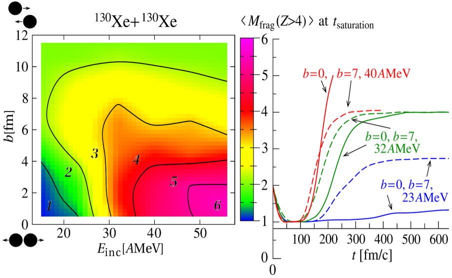
→ Fragment formation and recombination + isovector properties reasonably described over a large time interval

Dissipative HIC with BLOB

Fermi E :

large-ampl. phase-space fluct. + in-medium dissipation + one-body collective behaviour ⇒

- fragments→ thresholds,
  variety of mechanisms



- isospin currents as a function of t

Central / peripheral (BLOB) :