

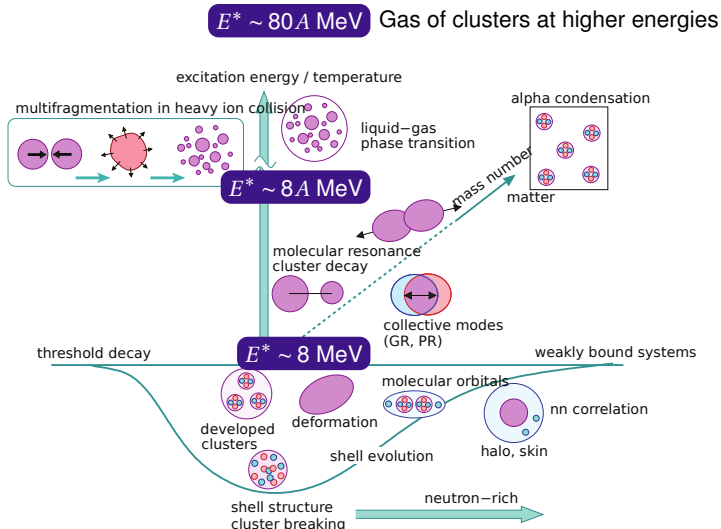
# Light clusters and light nuclei in collision dynamics

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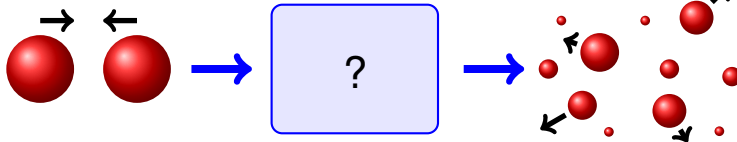
# Clustering phenomena in excited states of nuclear systems



Kanada-En'yo, Kimura, Ono, Prog. Theor. Exp. Phys. 2012 01A202 (2012)

# Importance of clusters in heavy-ion collisions

Collisions of two nuclei (e.g., Xe + Sn at 50 MeV/nucleon,  $b \approx 0$ )

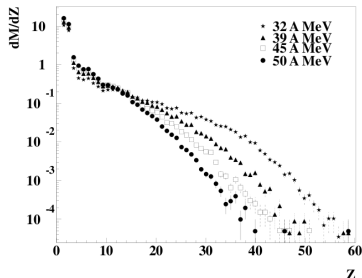


Partitioning of protons

	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	$\approx 10\%$	21%
$\alpha$	$\approx 20\%$	20%
d, t, $^3\text{He}$	$\approx 10\%$	40%
$A > 4$	$\approx 60\%$	18%

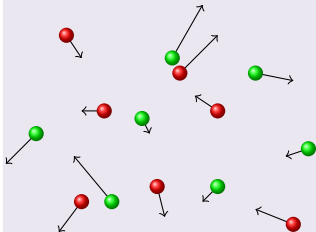
INDRA data, Hudan et al., PRC67 (2003) 064613.

FOPI data, Reisdorf et al., NPA 848 (2010) 366.



Light-cluster correlations are important in late times, at least.

Transport models  $\approx$  Nucleons move almost independently in the mean field.



Description by  $f_n(\mathbf{r}, \mathbf{p})$  and  $f_p(\mathbf{r}, \mathbf{p})$ , ignoring correlations.

Probability to find a cluster in a snapshot:

$$|\langle \Psi_d | \psi_n \psi_p \rangle|^2 = \iiint F_d(\mathbf{r}_1, \mathbf{p}_1, \mathbf{r}_2, \mathbf{p}_2) f_n(\mathbf{r}_1, \mathbf{p}_1) f_p(\mathbf{r}_2, \mathbf{p}_2) \frac{d\mathbf{r}_1 d\mathbf{p}_1 d\mathbf{r}_2 d\mathbf{p}_2}{(2\pi\hbar)^6}$$

$$\Psi_d(\mathbf{r}_1, \mathbf{r}_2) = \psi_d^{\text{int}}(\mathbf{r}_1 - \mathbf{r}_2) \psi_d^{\text{cm}}\left(\frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)\right)$$

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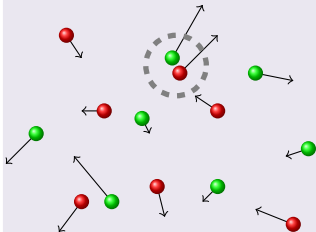
Unfortunately,  $|\langle \Psi_d | \psi_n \psi_p \rangle|^2$  is time dependent.

- Coalescence prescription: Assume that the transport calculation is correct until “freeze out”, and then clusters are suddenly emitted with these probabilities.

e.g. L.W. Chen, et al., NPA 729 (2003) 809.

- More dynamical approaches with cluster correlations:
  - Should predict when each cluster is formed.
  - Should calculate the fate of formed clusters.

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## BUU with clusters

Danielewicz and Bertsch, NPA 533 (1991) 712.

Coupled equations for  $f_n(\mathbf{r}, \mathbf{p}, t)$ ,  $f_p(\mathbf{r}, \mathbf{p}, t)$ ,  $f_d(\mathbf{r}, \mathbf{p}, t)$ ,  $f_t(\mathbf{r}, \mathbf{p}, t)$ ,  $f_h(\mathbf{r}, \mathbf{p}, t)$  are solved by the test particle method.

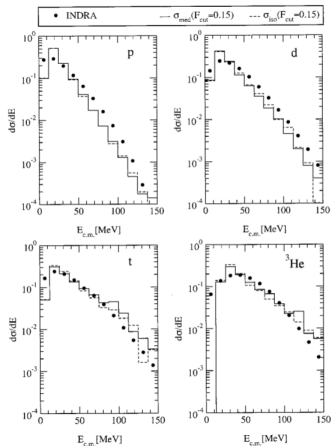
$$\frac{\partial f_n}{\partial t} + \mathbf{v} \cdot \frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}} \cdot \frac{\partial f_n}{\partial \mathbf{p}} = I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_p}{\partial t} + \mathbf{v} \cdot \frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}} \cdot \frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_d}{\partial t} + \mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}} \cdot \frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v} \cdot \frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}} \cdot \frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_h}{\partial t} + \mathbf{v} \cdot \frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}} \cdot \frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$




Renormalized cluster spectra

Kuhrts et al., PRC63(2001)034605.

# Antisymmetrized Molecular Dynamics (very basic version)

## AMD wave function



$$|\Phi(Z)\rangle = \det_{ij} \left[ \exp \left\{ -v \left( \mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{v}} \right)^2 \right\} \chi_{\alpha_i}(j) \right]$$

$$\mathbf{Z}_i = \sqrt{v} \mathbf{D}_i + \frac{i}{2\hbar\sqrt{v}} \mathbf{K}_i$$

$v$ : Width parameter =  $(2.5 \text{ fm})^{-2}$

$\chi_{\alpha_i}$ : Spin-isospin states =  $p \uparrow, p \downarrow, n \uparrow, n \downarrow$

## Equation of motion for the wave packet centroids $Z$

$$\frac{d}{dt} \mathbf{Z}_i = \{ \mathbf{Z}_i, \mathcal{H} \}_{\text{PB}} + (\text{NN collisions})$$

## $\{ \mathbf{Z}_i, \mathcal{H} \}_{\text{PB}}$ : Motion in the mean field

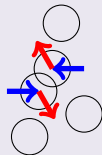
$$\mathcal{H} = \frac{\langle \Phi(Z) | H | \Phi(Z) \rangle}{\langle \Phi(Z) | \Phi(Z) \rangle} + (\text{c.m. correction})$$

$H$ : Effective interaction (e.g. Skyrme force)

## NN collisions

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i)$$

- $|V|^2$  or  $\sigma_{NN}$  (in medium)
- Pauli blocking

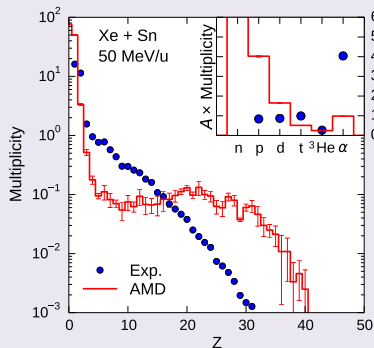
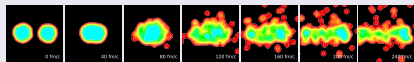


Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

# Failure of fragmentation and cluster production

AMD with usual NN collisions (very basic version)

## Central Xe + Sn at 50 MeV/u



## Partitioning of protons (experimental data)

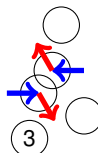
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INDRA data, [Hudan et al., PRC67 \(2003\) 064613.](#)

FOPi data, [Reisdorf et al., NPA 848 \(2010\) 366.](#)



# NN collisions without or with cluster correlations

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i)$$


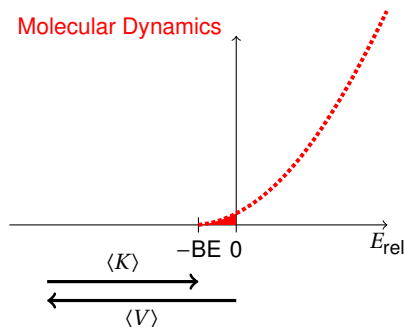
In the usual way of NN collision, only the two wave packets are changed.

$$\{ |\Psi_f\rangle \} = \{ |\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\dots)\rangle \}$$

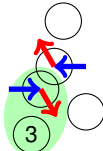
(ignoring antisymmetrization for simplicity of presentation.)

Phase space or the density of states for two nucleon system

Molecular Dynamics



# NN collisions without or with cluster correlations

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## Extension for cluster correlations

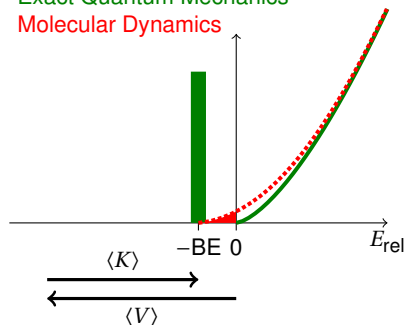
Include correlated states in the set of the final states of each NN collision.

$$\{ |\Psi_f\rangle \} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\dots)\rangle, \dots$$

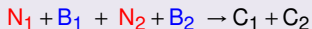
Phase space or the density of states for two nucleon system

Exact Quantum Mechanics

Molecular Dynamics



# NN collisions with cluster correlations



- $N_1, N_2$  : Colliding nucleons
- $B_1, B_2$  : Spectator nucleons/clusters
- $C_1, C_2$  :  $N, (2N), (3N), (4N)$  (up to  $\alpha$  cluster)

## Transition probability

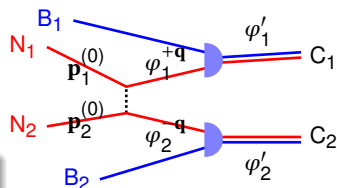
$$W(\text{NBNB} \rightarrow \text{CC}) = \frac{2\pi}{\hbar} |\langle \text{CC} | V | \text{NBNB} \rangle|^2 \delta(E_f - E_i)$$

$$vd\sigma \propto |\langle \varphi'_1 | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi'_2 | \varphi_2^{-\mathbf{q}} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\text{rel}}^2 dp_{\text{rel}} d\Omega$$

$|M|^2 = |\langle \text{NN} | V | \text{NN} \rangle|^2$ : Matrix elements of NN scattering

$\Leftarrow (d\sigma/d\Omega)_{\text{NN}}$  in medium (or in free space)

Similar to Danielewicz et al.,  
NPA533 (1991) 712.



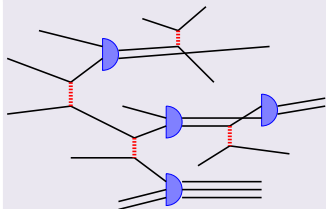
$$\mathbf{p}_{\text{rel}} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\text{rel}} \hat{\Omega}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$

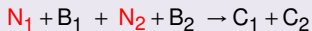
$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q} \cdot \mathbf{r}_{N_1}) \varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q} \cdot \mathbf{r}_{N_2}) \varphi_2^{(0)}$$

## NN collisions with cluster correlations (more explanations)



For each NN collision, cluster formation is considered.



$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle CC | V_{NN} | NBNB \rangle|^2 \delta(E_f - E_i)$$

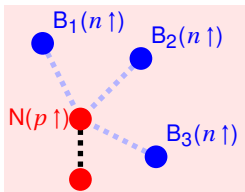
Ono, J. Phys. Conf. Ser. 420 (2013) 012103

Ikeno, Ono et al., PRC 93 (2016) 044612

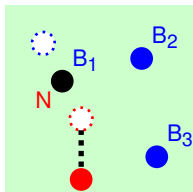
- We always have a Slater determinant of nucleon wave packets. A cluster in the final states is represented by placing wave packets at the same phase space point.
- Consequently the processes such as  $d + X \rightarrow n + p + X'$  and  $d + X \rightarrow d + X'$  are automatically taken into account.
- No parameters have been introduced to adjust individual reactions.
- There is an option to suppress clusters depending on the medium.
- There are many possibilities to form clusters in the final states. Non-orthogonality of the final states should be carefully handled.

# Construction of Final States

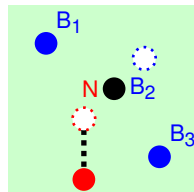
Clusters (in the final states) are assumed to have  $(0s)^N$  configuration.



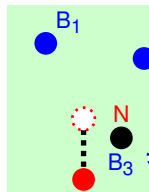
$|\Phi^{\mathbf{q}}\rangle$   
After  $\mathbf{p}^{(0)} \rightarrow \mathbf{p}^{(0)} + \mathbf{q}$



$|\Phi'_1\rangle$   
 $N + B_1 \rightarrow C_1$



$|\Phi'_2\rangle$   
 $N + B_2 \rightarrow C_2$



$|\Phi'_3\rangle$   
 $N + B_3 \rightarrow C$

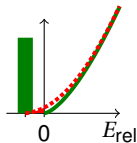
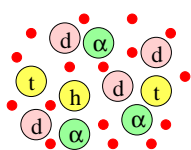
Final states are not orthogonal:  $N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}$

The probability of cluster formation with one of  $B$ 's:

$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N_{ij}^{-1} \langle \Phi'_j|, \quad P = \langle \Phi^{\mathbf{q}} | \hat{P} | \Phi^{\mathbf{q}} \rangle \neq \sum_i |\langle \Phi'_i | \Phi^{\mathbf{q}} \rangle|^2$$

- $\left\{ \begin{array}{l} P \\ 1 - P \end{array} \right. \Rightarrow$  Choose one of the candidates and make a cluster.
- $\left\{ \begin{array}{l} P \\ 1 - P \end{array} \right. \Rightarrow$  Don't make a cluster (with any  $n\uparrow$ ).

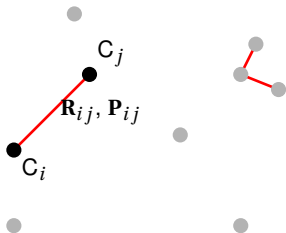
# Correlations to bind several clusters



Clusters may form a loosely bound state.

e.g.,  ${}^7\text{Li} = \alpha + t - 2.5 \text{ MeV}$

Need more probability of  $|\alpha + t\rangle \rightarrow |{}^7\text{Li}\rangle$



**Step 1** Clusters (and nucleons)  $C_i$  and  $C_j$  are *linked*,

- if  $C_i$  is one of the 4 clusters closest to  $C_j$ , and  $(i \leftrightarrow j)$ ,
- and if the distance is  $1 \text{ fm} < |\mathbf{R}_{ij}| < 5 \text{ fm}$ ,
- and if they are slowly moving away,  $\mathbf{P}_{ij}^2 / 2\mu_{ij} < 8 \text{ MeV}$  and  $\mathbf{R}_{ij} \cdot \mathbf{P}_{ij} > 0$ .

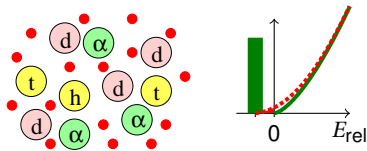
**Step 2** Linked clusters (CC) are identified. Following steps are taken only for CC with mass number  $6 \leq A \leq 9$ .

**Step 3** Transition of the internal state of CC by eliminating the internal momentum

$$\mathbf{P}_i \rightarrow 0 \quad \text{for } i \in \text{CC in the c.m. of CC}$$

**Step 4** Energy conservation.

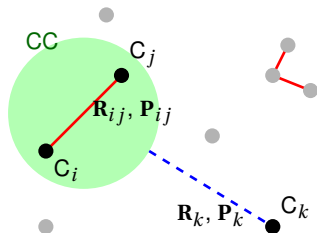
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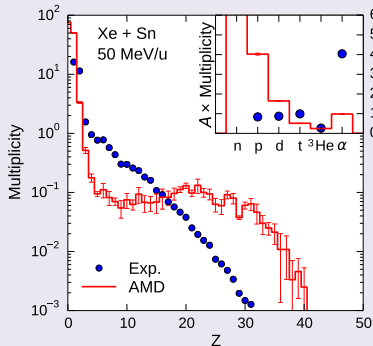
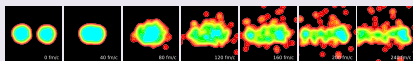
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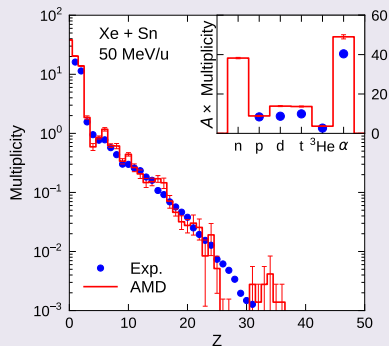
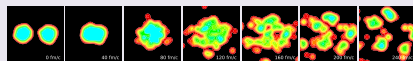
**Step 4** Energy conservation.

# Effect of cluster correlations: central Xe + Sn at 50 MeV/u

## Without clusters



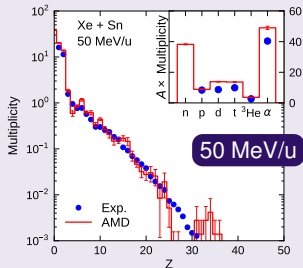
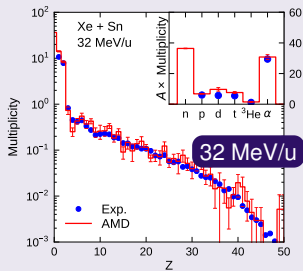
## With clusters



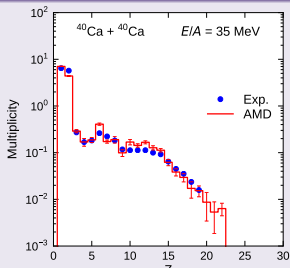


# Results for multifragmentation in central collisions

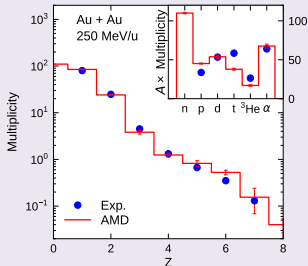
## Xe + Sn



## Ca + Ca at 35 MeV/u



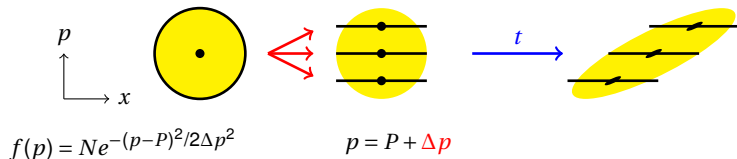
## Au + Au at 250 MeV/u



Data: Hudan et al., PRC 67 (2003) 064613.  
 Hagel et al., PRC 50 (1994) 2017.  
 Reisdorf et al., NPA 848 (2010) 366.

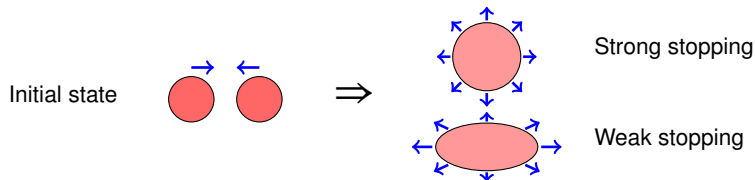
## Transition from a wave packet to a plane wave

Each wave packet has a momentum width. E.g., it is an important part of the Fermi motion.



The momentum fluctuation  $\Delta p$  is given to a wave packet when it is '**emitted**', following [Ono and Horiuchi, PRC53 \(1996\) 845](#) [a simple version of wp splitting].

- For a formed cluster, the momentum fluctuation is given to its center-of-mass motion.
- Total momentum and energy conservation.
- A particle is regarded as '**emitted**' when there is no other particles around it in phase space within the radius  $(\Delta r, \Delta v) = (3.5 \text{ fm}, 0.25c)$ .
- Consistency with the method of the zero-point energy correction.



## A quantity to represent stopping

$$R_E = \frac{\sum(E_x + E_y)}{2\sum E_z}, \quad \Sigma \text{ for all charged products } (Z \geq 1)$$

Stopping should depend on

- Inmedium NN cross sections
- Treatment of Pauli blocking
- Effective interaction
- How to select events

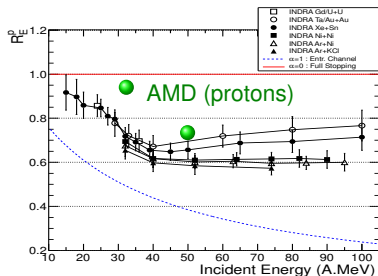
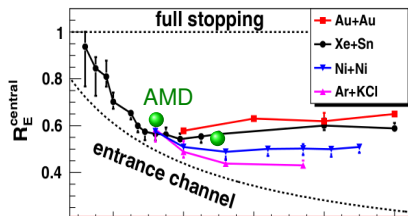
# Problem of Stopping

$$R_E = \frac{\sum(E_x + E_y)}{2\sum E_z}, \quad \Sigma \text{ for all } (Z \geq 1)$$

Data: [Lehaut et al., PRL104 \(2010\) 232701.](#)

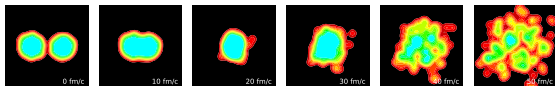
$$R_E^p = \frac{\sum(E_x + E_y)}{2\sum E_z}, \quad \Sigma \text{ for protons}$$

Data: [O. Lopez et al., PRC 90 \(2014\) 064602.](#)

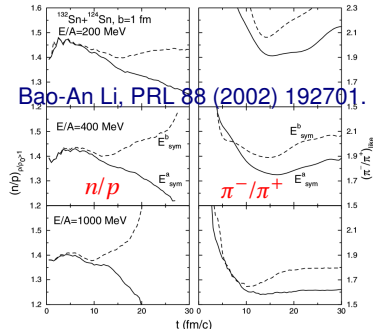


# Collisions at $\sim 300$ MeV/nucleon

- High density ( $\sim 2\rho_0$ )
- Pion production



$^{132}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 300$  MeV,  $b \sim 0$ , AMD calculation  
Neutron rich system ( $N = 156$ ,  $Z = 100$ )



Bao-An Li, PRL 88 (2002) 192701.

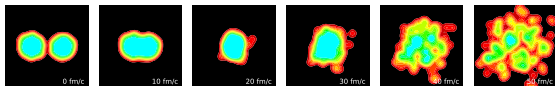
## Simple picture

$$\text{Symmetry Energy } S(\rho) \iff n/p \iff \frac{NN \leftrightarrow N\Delta}{\Delta^- / \Delta^0 / \Delta^+ / \Delta^{++}} \iff \frac{\Delta \leftrightarrow N\pi}{\pi^- / \pi^+}$$

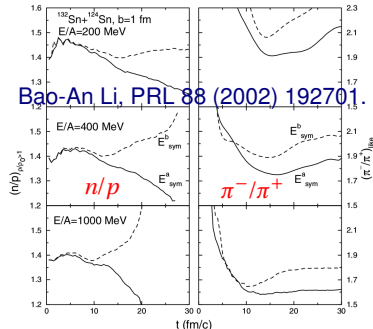
Is the simple expectation  $(n/p)^2 \approx \pi^- / \pi^+$  valid in HIC?

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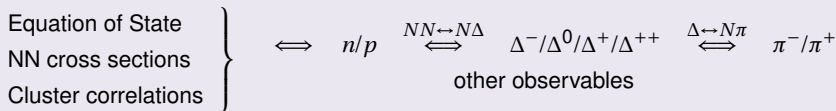


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Bao-An Li, PRL 88 (2002) 192701.

## More complicated picture

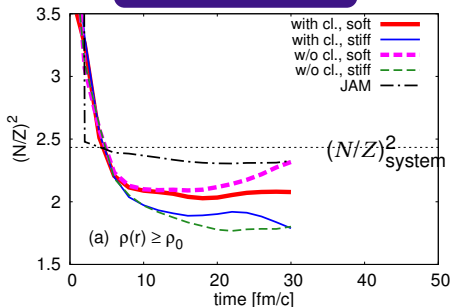


Is the simple expectation  $(n/p)^2 \approx \pi^- / \pi^+$  valid in HIC?

# Does $\Delta$ production agree with n/p dynamics?

Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612. (updated)

$(N/Z)^2$  @ high density



Different n/p dynamics  $\leftarrow$  different  $E_{\text{sym}}(\rho)$  and cluster correlations

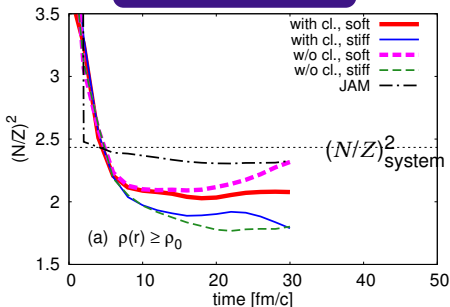
Four AMD+JAM calculations and a JAM calculation

- Solid lines: with clusters; **Soft** ( $L = 46$  MeV) and **Stiff** ( $L = 108$  MeV)
- Dashed lines: without clusters; **Soft** ( $L = 46$  MeV) and **Stiff** ( $L = 108$  MeV)
- Dot-dashed line: JAM (without mean field)

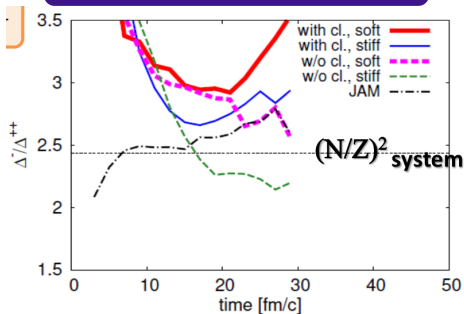
# Does $\Delta$ production agree with n/p dynamics?

Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612. (updated)

$(N/Z)^2$  @ high density



Rate( $nn \rightarrow p\Delta^-$ ) / Rate( $pp \rightarrow n\Delta^{++}$ )



Different n/p dynamics  $\leftarrow$  different  $E_{\text{SYM}}(\rho)$  and cluster correlations

Four AMD+JAM calculations and a JAM calculation

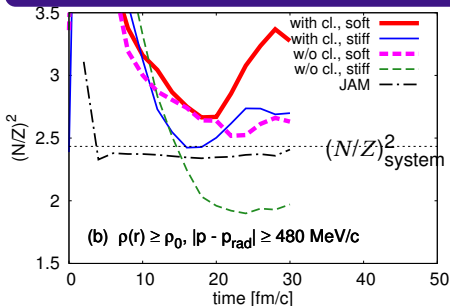
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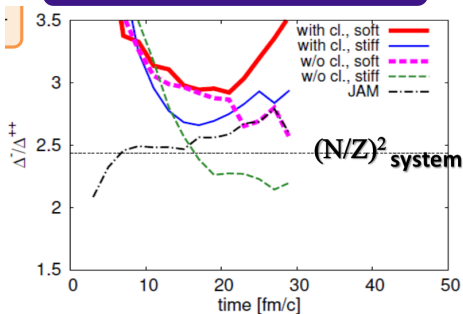
# Does $\Delta$ production agree with n/p dynamics?

Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612. (updated)

$(N/Z)^2$  @ high density and high momentum



Rate( $nn \rightarrow p\Delta^-$ ) / Rate( $pp \rightarrow n\Delta^{++}$ )



Different n/p dynamics  $\leftarrow$  different  $E_{\text{SYM}}(\rho)$  and cluster correlations

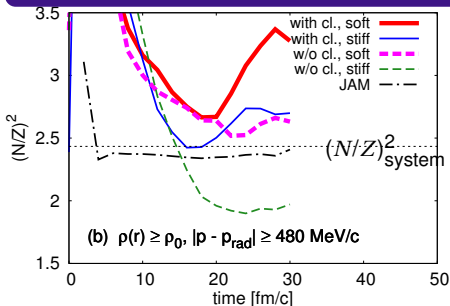
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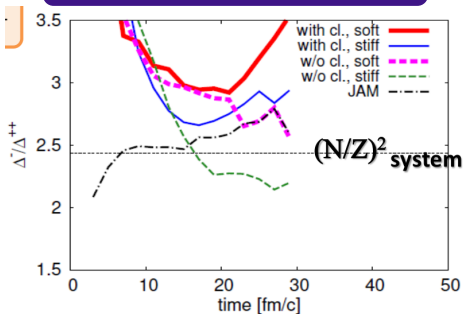
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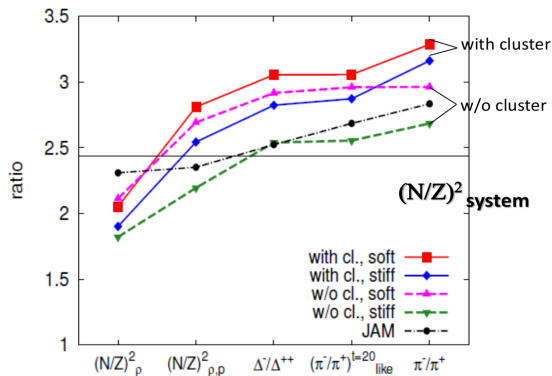
Rate( $nn \rightarrow p\Delta^-$ ) / Rate( $pp \rightarrow n\Delta^{++}$ )



## Nucleons $\Rightarrow$ $\Delta$ production

- $\Delta$  production sees the nucleons in the **high-density and high-momentum** region of phase space.
- $\Delta$  production is also affected by the nucleons in the **low-momentum** region of phase space.  $\Rightarrow$  Ikeno's talk on Wednesday.

# Summary of ratios, for $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/nucleon

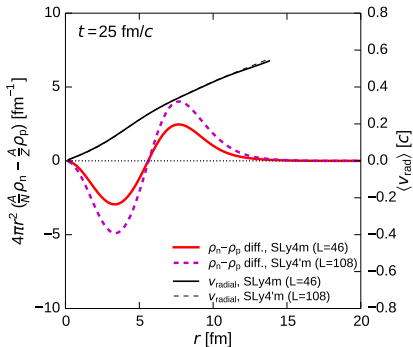


Ikeno, Ono, Nara, Ohnishi,  
 PRC 93 (2016) 044612  
 (updated)

$(N/Z)^2_{\rho}$ ,  $(N/Z)^2_{\rho,p}$ ,  $\Delta^-/\Delta^{++}$ :  
 representative values integrated  
 (or averaged) over time.

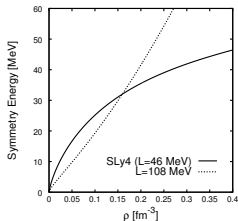
- $(N/Z)^2$  in high- $\rho$  and high- $p$  region  $\approx (\pi^-/\pi^+)_{\text{like}}$  at  $t = 20 \text{ fm}/c$  ... Final  $\pi^-/\pi^+$   
~30% reduction of  $E_{\text{Sym}}$  effect
- Effects of clusters: Larger ratios, and weaker dependence on  $E_{\text{Sym}}$

# N/Z Ratio in $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/u (AMD with clusters)

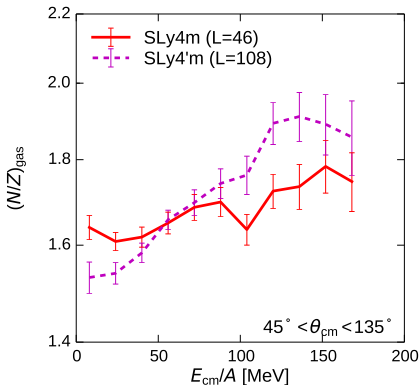
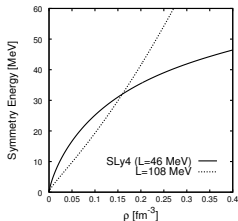
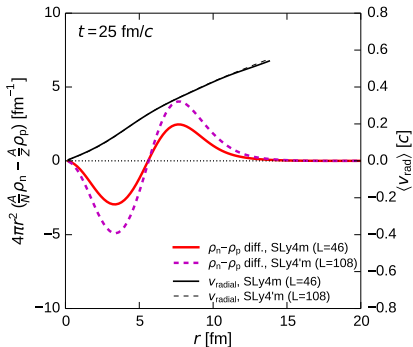


Pions have to go through the exterior region of the expanding system.

- For stiff symmetry energy,
  - The interior region is less neutron rich.  
 $\pi^- / \pi^+ \searrow \searrow \searrow$
  - The exterior region is more neutron rich.  
 $\pi^- / \pi^+ \nearrow$



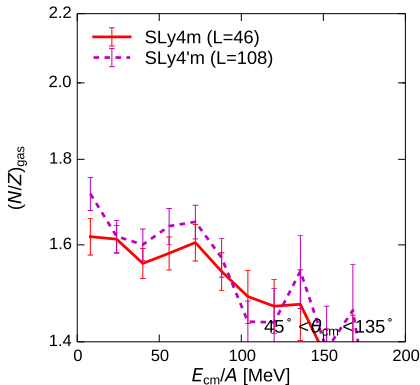
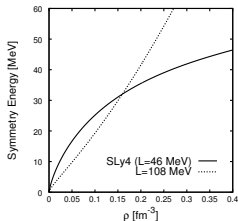
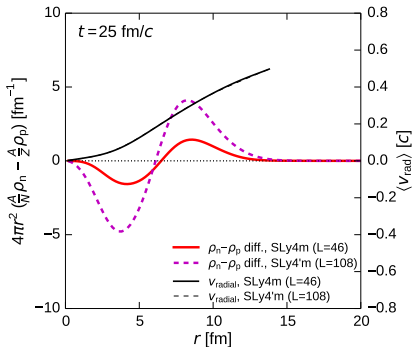
# N/Z Ratio in $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/u (AMD with clusters)



$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

$N/Z$  of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.

# $N/Z$ Ratio in $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/u (AMD without clusters)

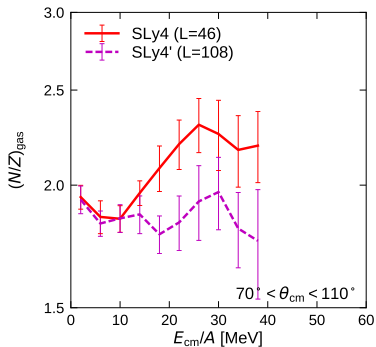


$$\left( \frac{N}{Z} \right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

$N/Z$  of spectrum of emitted particles is NOT similar to the neutron-proton density difference at the compression stage.

## Central collisions calculated by AMD with clusters

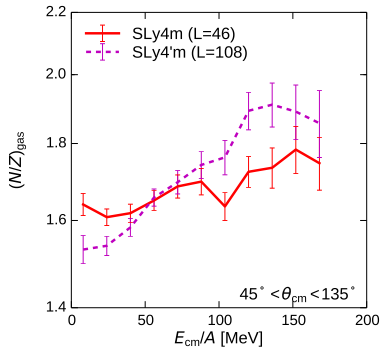
$^{124}\text{Sn} + ^{124}\text{Sn}$  at  $E/A = 50$  MeV



Low density effect

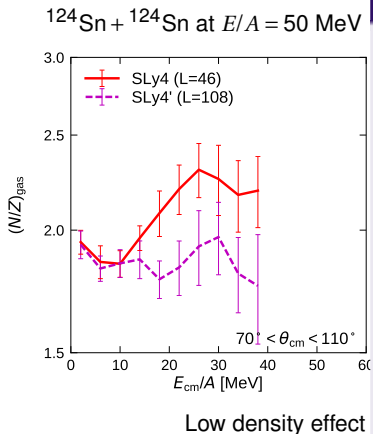


$^{132}\text{Sn} + ^{124}\text{Sn}$  at  $E/A = 300$  MeV



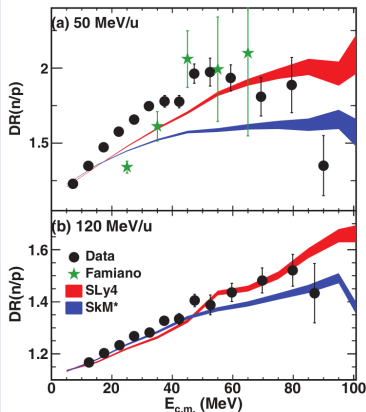
High density effect

Central collisions calculated by AMD with clusters



Low density effect

ImQMD result



Coupland et al., PRC94(2016)011601(R)

“Probing effective nucleon masses...”



- Recent extensions of AMD
  - Light clusters, produced at  $NN$  collisions
  - Correlations between light clusters, to produce light nuclei
  - Momentum fluctuation in Gaussian wave packet
- Fragment size distributions in various collisions can be reproduced reasonably well.
- Cluster correlations (as well as the symmetry energy) influence the dynamics of compression and expansion of the two-component (n&p) system.
- The effects are reflected in some observables, such as the  $\pi^-/\pi^+$  ratio and the  $n/p$  spectral ratio.

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- The effects are reflected in some observables, such as the  $\pi^-/\pi^+$  ratio and the  $n/p$  spectral ratio.
- TODO: More consistent description of various reactions and observables, such as the energy dependence of stopping observables and the particle spectra.