Status of HW3 — Pion Production

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Box Simulation Organizing Committee

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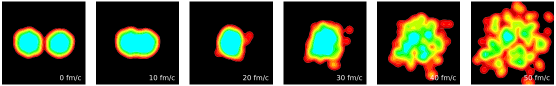
Yongjia Wang, Hermann Wolter, **Jun Xu**, Yingxun Zhang

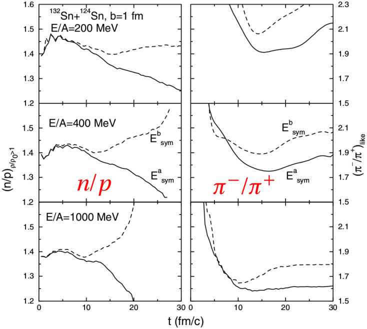
Symmetry energy and dynamics of nucleons and pions in HIC

Bao-An Li, PRL 88 (2002) 192701.



^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$, AMD calculation

Neutron-rich system ($N = 156$, $Z = 100$)



Symmetry Energy $S(\rho)$ $\Longleftrightarrow$ n/p $\Longleftrightarrow$ $\Delta^-/\Delta^0/\Delta^+/\Delta^{++}$ $\Longleftrightarrow$ π^-/π^+

- Currently, different predictions of π^-/π^+ ratio by different models.

  Different assumptions for nucleon dynamics, different cross sections, different potentials for Δ and π,...

- Do we agree when we start with the simplest case?

How the Box HW3 started

At the BNU meeting (June 2016), we started to discuss the box simulations with pions.

Summary by **Hermann Wolter**

A. HW1 & HW2

B. Box calculation of pion production

Two box configurations were proposed and it should be decided how to proceed.

1) **High temperature nuclear matter (Jun Xu)**
2) **Colliding nuclear matter (Hermann W)**

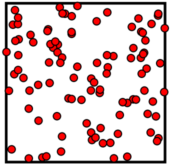
   Initialize the box with momenta taken from two Fermi spheres. . .

- (With the high temperature choice,) the spectra will still be dominated by the softer collisions (than in real heavy-ion collisions).

I think that before one can test pion production, one has to have the collisions and the blocking under control, i.e. part A.1. In pion production completely new physics enters into the box calculations (inelastic collisions, π and Δ potentials, width of the Δ, threshold effects). But the pion production depends first of all on the collision rate, and if this is not understood it is difficult to test the other.

Condition of Box Homework 3

Periodic box condition with $L = 20$ fm.



Initial condition

- Relativistic Boltzmann at $T = 60$ MeV
- $N + Z = 1280$ or $\rho = 0.16$ fm^{-3}

  asym $(N, Z) = (768, 512)$ or $\delta = 0.2$

  sym $(N, Z) = (640, 640)$ or $\delta = 0$

Common setup

- Only N, Δ and π, depending on Options.
- No mean field.
- $\sigma_{\text{elastic}} = 40$ mb (isotropic) for all $NN \to NN$, $N\Delta \to N\Delta$ and $\Delta\Delta \to \Delta\Delta$.
- No Pauli blocking.
- $M_n = M_p = 938$ MeV, $M_\pi = 139$ MeV, $M_\Delta^0 = 1232$ MeV.
- Run 1000 events (or 10 events with 100 test particles per nucleon).
- Run from $t = 0$ to $t = 150$ fm/c, with only $NN \to NN$ in the first 10 fm/c.

Progress of Box Homework 3

History

- Phase I: sent out on November 4, 2016; due on January 27, 2017.
- Phase II and Phase III: sent out on January 30, 2017; due on March 19, 2017.
- New Phase II: sent out on March 24, 2017.

Much progress during and after ICNT2017 and Transport 2017.

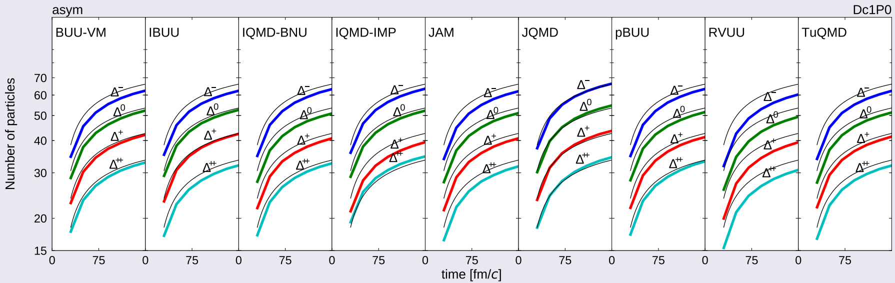
Participating codes

BUU-VM	Swagato Mallik
IBUU	Jun Xu
IQMD-BNU	Jun Su
IQMD-IMP	Zhao-Qing Feng
JAM	Natsumi Ikeno
JQMD	Tatsuhiko Ogawa
pBUU	Pawel Danielewicz
RVUU	Taesoo Song, Zhen Zhang
TuQMD	Dan Cozma

The first Option Dc1P0: Only one-way $NN \to N\Delta$ with a constant Δ mass

- Constant Δ mass $M_\Delta = 1.232$ GeV.
- Constant $\sigma(NN \to N\Delta) = 40$ mb for $\sqrt{s} > M_N + M_\Delta$. (isospin-averaged value)

Time evolution of numbers of nucleons and Deltas



Thin lines: the solutions of kinetic equation (by Jun Xu)

- The results of transport codes are lower (except for JQMD). Non-equilibrium effect?
- Agreement between transport codes is not perfect.

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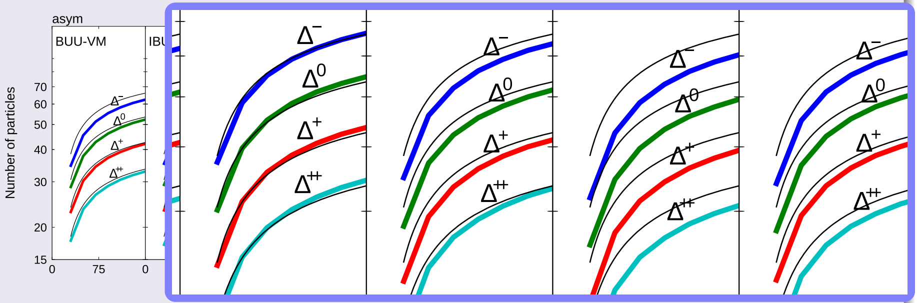

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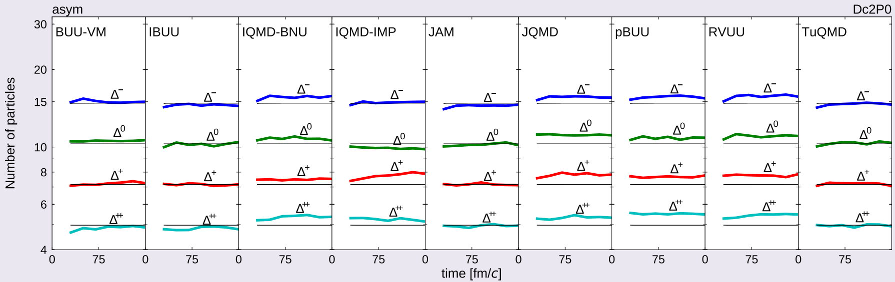
Option Dc2P0: Two-way $NN \leftrightarrow N\Delta$ with a constant Δ mass

Detailed balance: $\sigma(N\Delta \to NN) = \frac{1}{g} \frac{p_{NN}^2}{p_{N\Delta}^2} \sigma(NN \to N\Delta)$ (diverges at the threshold)

Number of Δ



- Four kinds of Δ are equally spaced in the log scale, and the spacing $\approx n/p$ ratio.
- Compared with kinetic theory, Num(Δ) is large in IQMD-BNU, JQMD, pBUU and RVUU.

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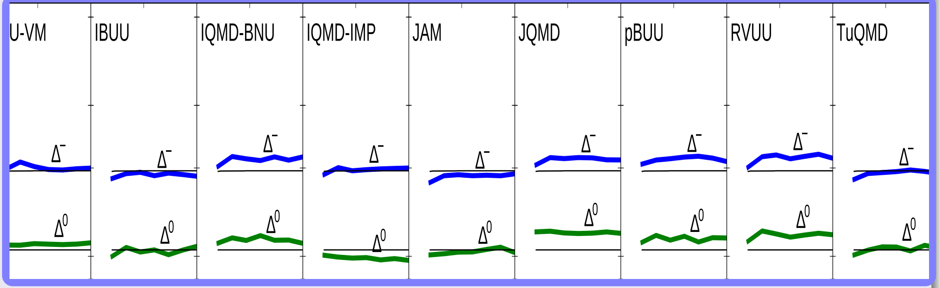

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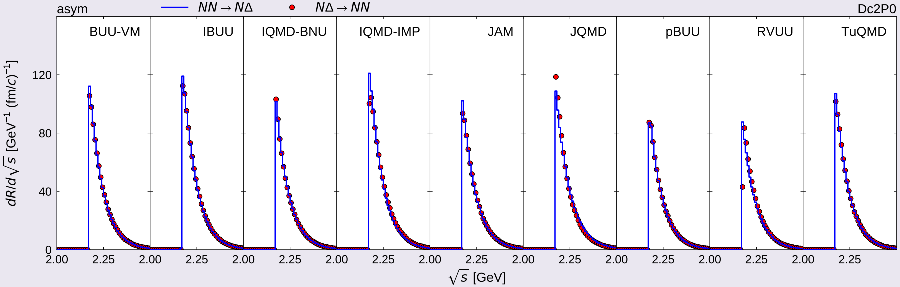
Distribution of $\sqrt{s}$ for $NN \to N\Delta$ and $N\Delta \to NN$



- Forward and reverse reactions don't balance in some codes.
- $(N\Delta \to NN) < (NN \to N\Delta)$ may be expected for the lowest bin due to the finite box size, i.e. $\sigma < (1 \sim 3)\pi(L/2)^2$.

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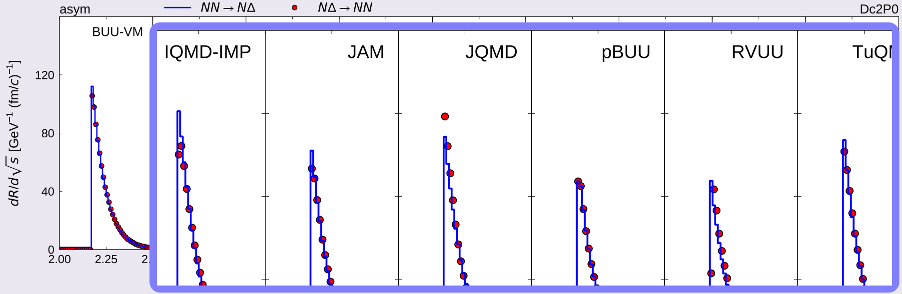

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Option Db and Dd

$\sigma(NN \to N\Delta)$ in the Bertsch-DasGupta paper.

$$\sigma(NN \to N\Delta) = \frac{(\sqrt{s} - 2M_N - M_\pi)^2}{(0.015\ \text{GeV}^2) + (\sqrt{s} - 2M_N - M_\pi)^2} \times 20\ \text{mb} \qquad \text{for } \sqrt{s} > 2M_N + M_\pi$$

The Δ mass $m \in [M_N + M_\pi,\ \sqrt{s} - M_N]$ should be sampled from a Breit-Wigner form.

$$P(m) = \frac{A(m)p(s,m)m}{\int_{M_N+M_\pi}^{\sqrt{s}-M_N} A(m')p(s,m')m'\mathrm{d}m'}, \qquad A(m) = \frac{4{M_\Delta^0}^2\Gamma_\Delta}{(m^2 - {M_\Delta^0}^2)^2 + {M_\Delta^0}^2\Gamma_\Delta^2}$$

with $\Gamma_\Delta = 0.115$ GeV and $p(s,m)$ being the momentum of the final $\Delta(m)$ in the c.m. frame.
(Option Db2 took another Breit-Wigner form.)

Option Db2

Bad detailed balance

$$\sigma[N\Delta(m) \to NN] = \frac{1}{g}\frac{p_{NN}(s)^2}{p(s,m)^2}\sigma(NN \to N\Delta)$$

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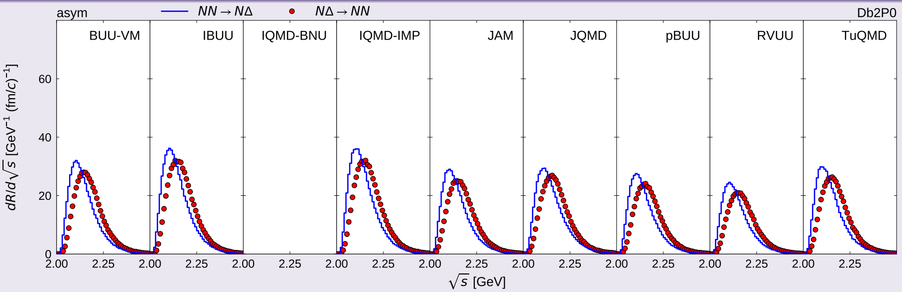
Option Dd2

Good detailed balance suggested by Danielewicz.

$$\sigma[N\Delta(m) \to NN] = \frac{1}{g} \frac{p_{NN}(s)^2 \cdot 2\pi m}{p(m,s) \int_{M_N+M_\pi}^{\sqrt{s}-M_N} A(m')p(s,m')m'\mathrm{d}m'} \sigma(NN \to N\Delta)$$

Bad detailed balance in Db2P0

Distribution of $\sqrt{s}$ for $NN \to N\Delta$ and $N\Delta \to NN$



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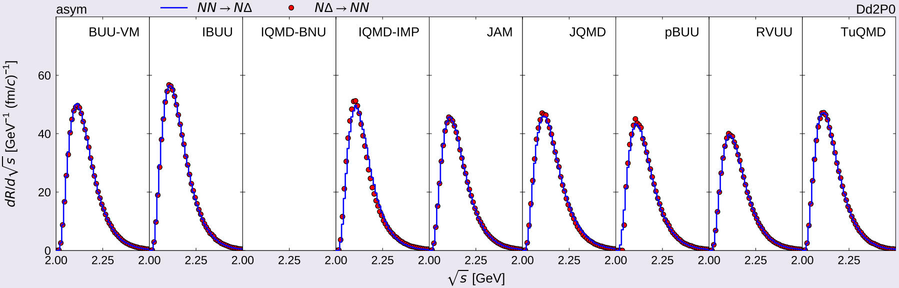

- In many codes, the balance is almost perfect.
- The reaction rates in different codes don't agree.
- In some codes, unbalance of forward and reverse reactions is visible.

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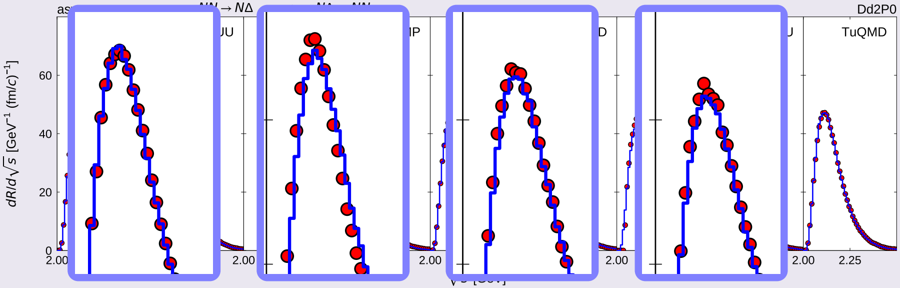

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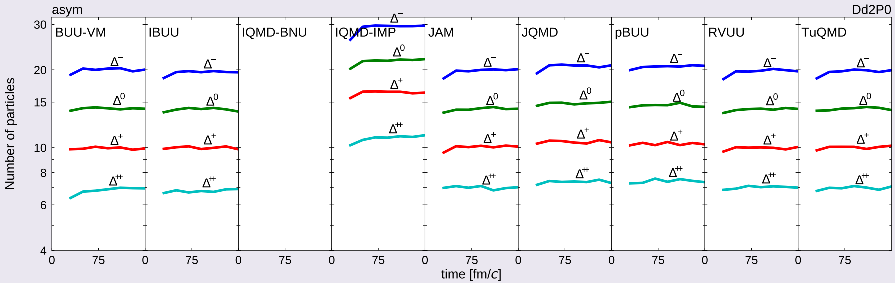
Option Dd2P0: Numbers of Δ

Number of Δ



- The agreement between different codes is relatively good.
- The number of Δ is larger in some codes. This is similar to the situation in Dc2P0.
- (We don't have reference lines of solutions of kinetic equation.)

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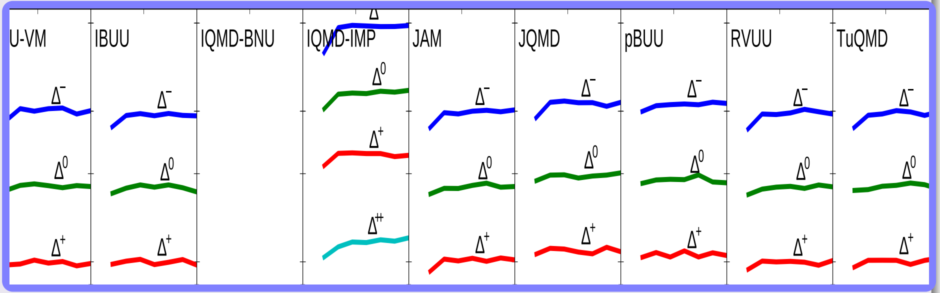

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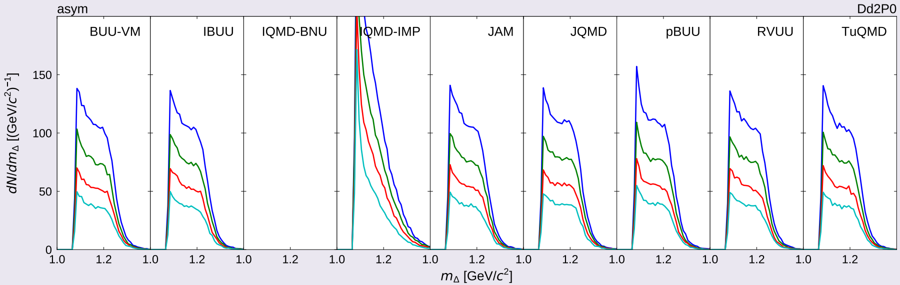
Dd2P0 ⇒ Dd2Pd: with pions

Mass distribution of existing Δ

Dd2P0 (no pions)



Pions in Dd2Pd

$$\Gamma[\Delta \to N\pi] = 0.115 \text{ GeV}$$

$$\sigma[N\pi \to \Delta] = \frac{4\pi}{3}\left(\frac{\hbar c}{p_{N\pi}}\right)^2 A(\sqrt{s}) \times \Gamma[\Delta \to N\pi] \qquad \text{(diverges at threshold)}$$

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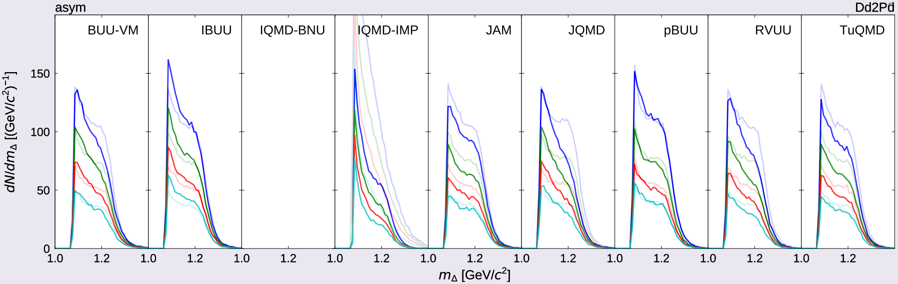

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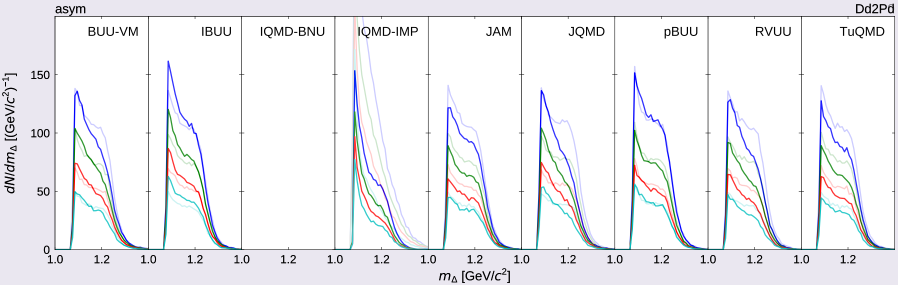
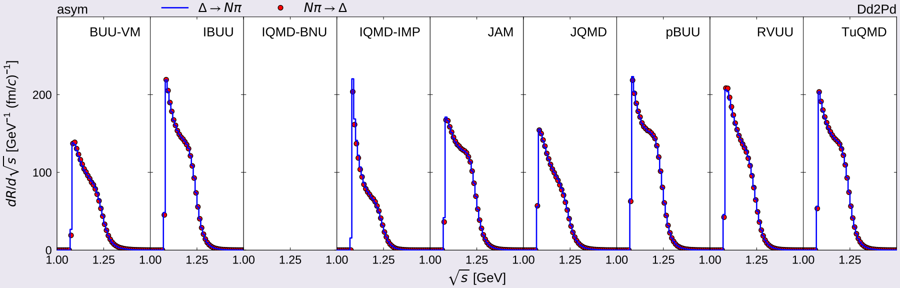


The changes of Δ mass distribution by pions [difficult to understand]

- High-mass part is mainly reduced. (BUU-VM, JQMD, RBUU)
- Overall reduction. (JAM, TuQMD)
- Low-mass Δ's increase. (IBUU)
- Almost no change (pBUU).

Option Dd2Pd — Balance of $\Delta \leftrightarrow N\pi$

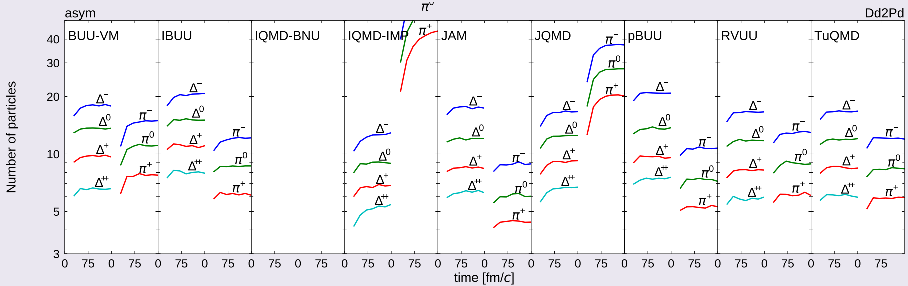
Distribution of $\sqrt{s}$ for $\Delta \rightarrow N\pi$ and $N\pi \rightarrow \Delta$



- The balance of $\Delta \leftrightarrow N\pi$ is OK in all the codes.
- Rate($\Delta \rightarrow N\pi$) $\approx \Gamma_\Delta \times$ Number(Δ).
  BUU-VM and JQMD seem to have Γ_Δ different from other codes. (time step issue?)

Option Dd2Pd — Number of Pions

Number of Δ and π



- JQMD: too many pions. (but it had the fewest pions in Db2Pb)
- BUU-VM, pBUU, JAM: unequal spacing between Δ's.
- JAM: relatively small number of pions.

Problem of prescriptions for collisions and decays

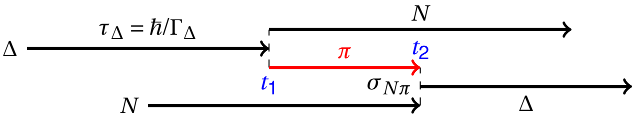
from Box Homework 1 paper

	Distance condition	Time condition
BUU-VM	$\pi d_\perp^{*2} < \sigma$	$\lvert t^*_{\text{coll}} - t^*_0 \rvert < \frac{1}{2}\Delta t$
IBUU	$\pi d_\perp^{*2} < \sigma$	$\lvert t^*_{\text{coll}} - t^*_0 \rvert < \frac{1}{2}\Delta t$
IQMD-BNU	$\pi d_\perp^{*2} < \sigma$	$\lvert t^*_{\text{coll}} - t^*_0 \rvert < \frac{1}{2}\Delta t$
IQMD-IMP	$\pi d_\perp^{*2} < \sigma$	$\lvert t^*_{\text{coll}} - t^*_0 \rvert < \frac{1}{2}\gamma\Delta t$
JAM	$\pi d_\perp^{*2} < \sigma$	$\bar{t}_{\text{coll}} \in [t_0,\ t_0 + \Delta t]$
JQMD	$d^*_\perp < b_{\text{max}},\ P = \sigma/\pi b^2_{\text{max}}$	$\lvert \bar{t}_{\text{coll}} - t_0 \rvert < \frac{1}{2}\Delta t$
pBUU	$i, j \in$ the same V_{cell} volume	$P = \frac{\sigma}{N_{\text{TP}}} \frac{1}{\gamma V_{\text{cell}}} v^*_{ij} \alpha \Delta t$
RVUU	$\pi d_\perp^{*2} < \sigma_{\text{max}}/N_{\text{TP}},\ P = \sigma/\sigma_{\text{max}}$	$\lvert t^*_{\text{coll}} - t^*_0 \rvert < \frac{1}{2}\Delta t$
TuQMD	$\pi d_\perp^{*2} < \sigma$	$t^*_{1-}, t^*_{2-} < t^*_{\text{coll}} < t^*_{1+}, t^*_{2+}$

Correct treatments of relativistic effects are more important here than in Box Homework 1.

What about the decay rate?

- $dP = \Gamma_\Delta dt$
- $\left[dP = \Gamma_\Delta dt\right]$ in the rest frame of Δ, namely $dP = \Gamma_\Delta dt/\gamma$



JAM

$\tau_\Delta = \hbar/\Gamma_\Delta = 1.72$ fm/c, $\sigma_{N\pi\to\Delta} \lesssim 200$ mb, $\Delta t = 0.5$ or 1.0 fm/c

- Dependence on Δt is found in RVUU and BUU-VM, though not very strong.
- Strong dependence on which is checked earlier, decays or collisions.

do istep = 1, nstep
call decays(Δt)
call collisons(Δt)
call output()
enddo

many codes

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JQMD

do istep = 1, nstep
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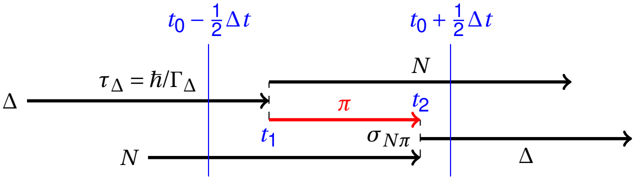
maybe optimal

Time step problem



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Summary of the current status

- 9 or 8 participating codes
- Some problem remains in the most simplest case Dc1P0 ($NN \to N\Delta$ only).
  - ...
- Relatively good agreement when $NN \leftrightarrow N\Delta$ is introduced.
  - Difference from the solution of kinetic theory.
  - Violation of detailed balance depending on the codes.
  - Some codes predict larger Num(Δ).
  - Some codes predict different Δ mass distributions from other codes.
- When pions are introduced ($\Delta \leftrightarrow N\pi$), any two of the codes don't agree. Why?
  - Related to the divergence of $\sigma(N\pi \to \Delta)$?
  - Time step issue? Which is tested earlier, collisions or decays?