

Constraining the symmetry energy based on relativistic point coupling interactions and excitations in finite nuclei

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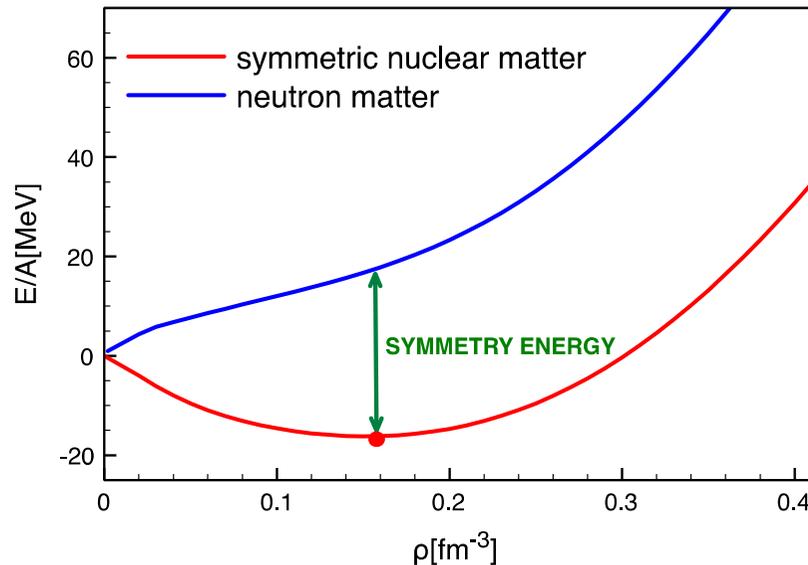
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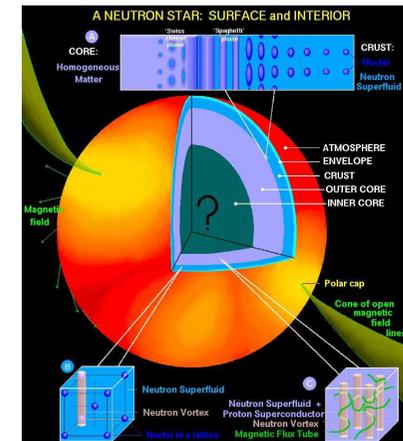
CENTER OF EXCELLENCE FOR THE THEORY OF QUANTUM AND COMPLEX SYSTEMS AND LIE ALGEBRA REPRESENTATION

INTRODUCTION

- Nuclear matter equation of state (EOS) plays important role in nuclear physics and astrophysics
- EOS of neutron matter is essential to understand the physics of neutron stars and binary mergers



- Symmetry energy $S(\rho)$ describes the increase in the energy of the $N \neq Z$ system as protons are turned into neutrons;
- It is important for understanding the properties of neutron-rich matter and neutron rich nuclei
- Constraining $S(\rho)$ by data on finite nuclei near the saturation density



- Neutron star properties
- Core collapse supernovae
- Nucleosynthesis

SYMMETRY ENERGY

- Nuclear matter equation of state:

$$E(\rho, \delta) = E_{SNM}(\rho) + E_{sym}(\rho)\delta^2 + \dots$$

$$\rho = \rho_n + \rho_p \quad \delta = \frac{\rho_n - \rho_p}{\rho}$$

- Symmetry energy term:

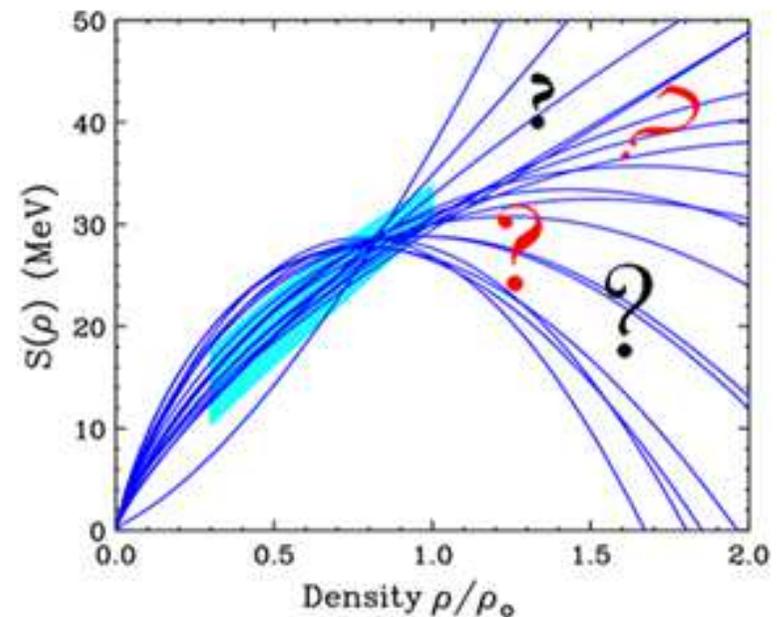
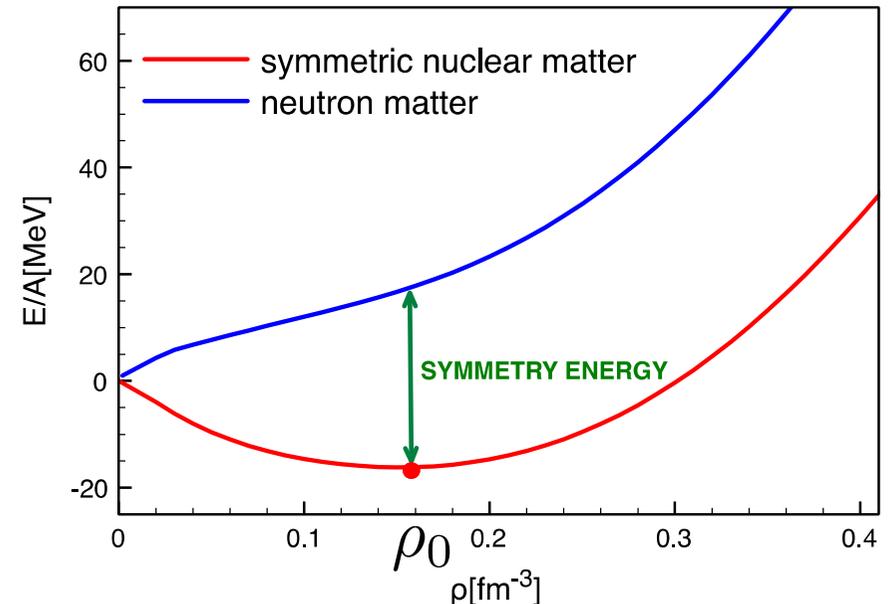
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \left. \frac{dS_2(\rho)}{d\rho} \right|_{\rho_0}$$

J – symmetry energy at saturation density

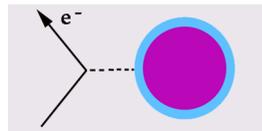
L – slope of the symmetry energy
(related to the pressure of neutron matter)



NEUTRON SKINS AND THE SYMMETRY ENERGY

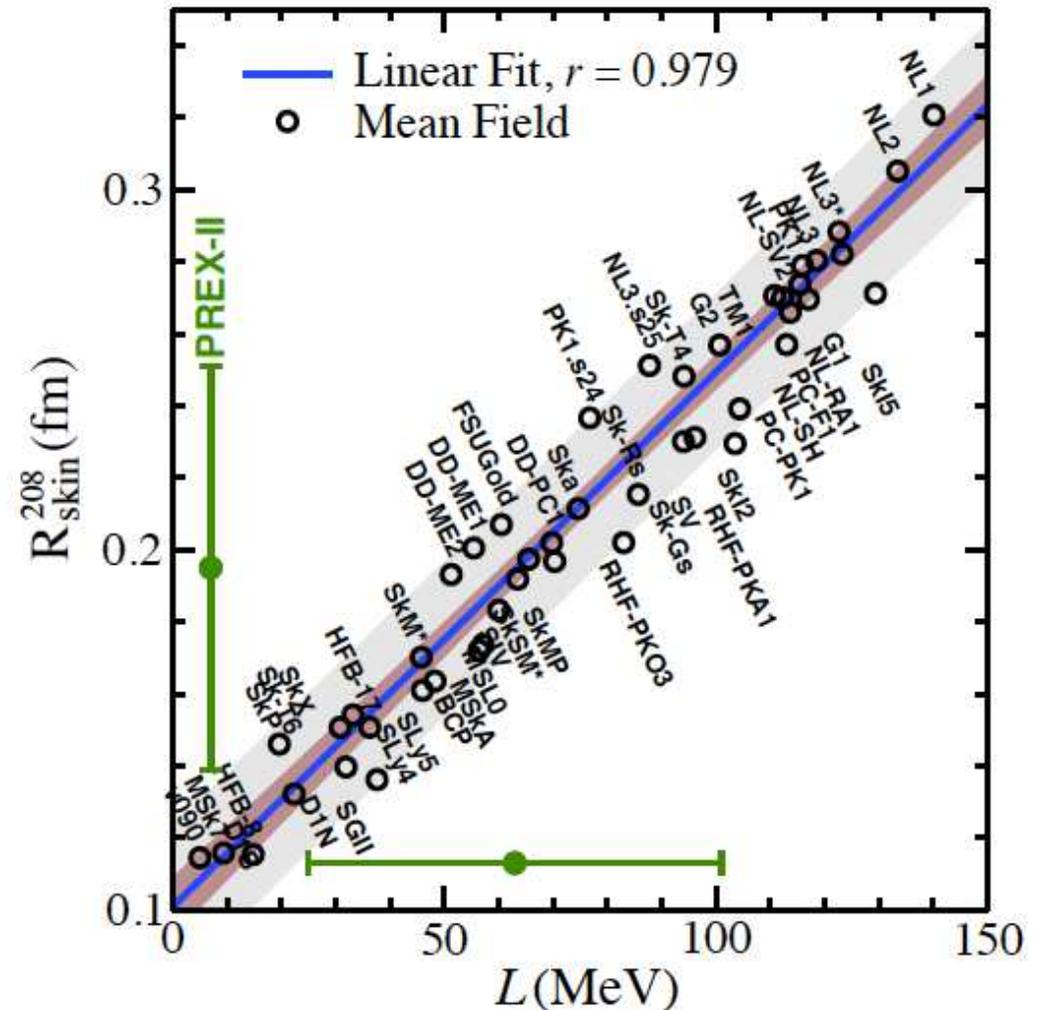
- In nuclei, thickness of the neutron skin $r_{np} = r_n - r_p$ depends on the pressure of neutron matter $P_{PNM} \sim L$
- the size of r_{np} increases with pressure as neutrons are pushed out against surface tension
- The pressure of neutron matter $P_{PNM} \sim L$ is poorly constrained
- Parity violating electron scattering - Lead Radius Experiment (PREx) @ JLab:

$$R_n - R_p = 0.33^{+0.16}_{-0.18}$$



Abrahamyan et al. PRL 108, 112502 (2012)

- PREx II, CREX



X. Roca-Maza et al., PRL106, 252501 (2011)
J. Piekarewicz, arXiv:1502.01559 (2015)

COLLECTIVE EXCITATIONS AND THE SYMMETRY ENERGY

- There are various (isovector) modes of collective excitations in nuclei that provide constraints on the neutron skin thickness, with recent experimental data available

- **Isovector giant dipole resonances**

- **Dipole polarizability:** A. Tamii et al., PRL 107, 062502 (2011)

$$\alpha_D \sim m_{-1}$$

D.M. Rossi et al., PRL 111, 242503 (2013)

T. Hashimoto et al., Phys. Rev. C 92, 031305(R) (2015)

- **Pygmy dipole resonances:** A. Carbone et al., PRC 81, 041301(R) (2010)
A. Klimkiewicz et al., PRC 76, 051603(R) (2007)

- **Anti-analog GDR:** A. Krasznahorkay et al., PLB 720, 428 (2013)

- **Isovector giant quadrupole resonances:** S.S. Henshaw, M.W. Ahmed, et al, PRL 107, 222501 (2011)

- ...

- The aim: exploit collective excitations to constrain the symmetry energy J & L

SELF-CONSISTENT RELATIVISTIC MEAN FIELD MODEL

- Relativistic point coupling model ~~σ~~ ~~ω~~ ~~ρ~~
- The basis is an effective Lagrangian with four-fermion (contact) interaction terms; isoscalar-scalar, isoscalar-vector, isovector-vector, derivative term

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma \cdot \partial - m)\psi \\ & - \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\ & - \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ & - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A \frac{(1 - \tau_3)}{2}\psi \end{aligned}$$

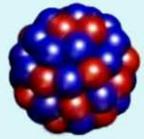
- many-body correlations encoded in density-dependent coupling functions that are motivated by microscopic calculations but parameterized in a phenomenological way
- Extensions: pairing correlations in finite nuclei [T. Niksic, et al., Comp. Phys. Comm. 185, 1808 \(2014\)](#).
 - Relativistic Hartree-Bogoliubov model (e.g. with separable form of the pairing interaction [Y. Tian et al., PLB 676, 44 \(2009\)](#).)
- In the small amplitude limit, self-consistent quasiparticle random phase approximation (QRPA) is used to compute nuclear excitations, etc.

CONSTRAINING THE FUNCTIONAL

- The model parameters $\mathbf{p} = (p_1, \dots, p_n)$ are constrained directly by many-body observables using χ^2 minimization

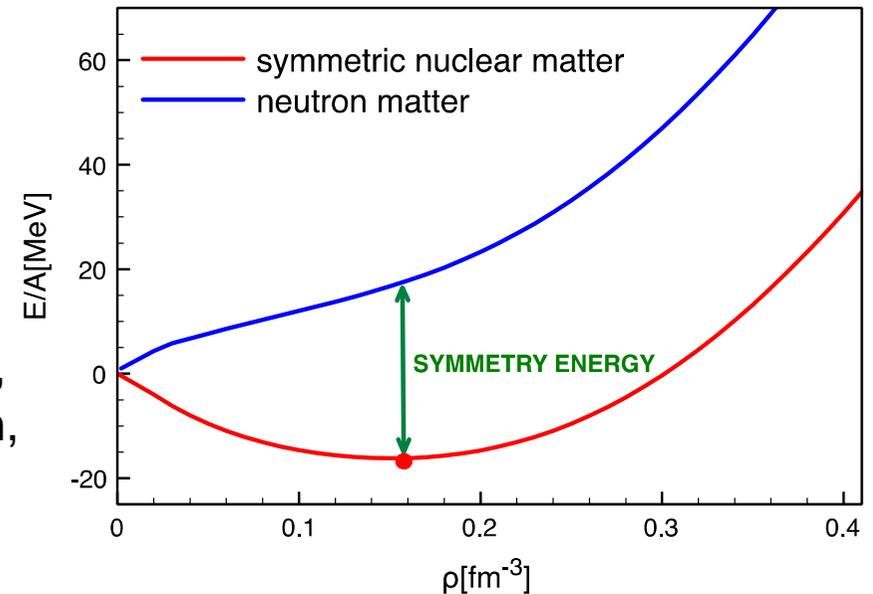
$$\chi^2(\mathbf{p}) = \sum_{i=1}^m \left(\frac{\mathcal{O}_i^{\text{theo.}}(\mathbf{p}) - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

- Calculated values can be compared to experimental, observational, and pseudo-data



properties of finite nuclei – binding energies, charge radii, diffraction radii, surface thicknesses, pairing gaps, etc.,...

- nuclear matter properties** – equation of state, binding energy and density at the saturation, symmetry energy J & L, incompressibility...



- Isovector channel of the EDF is weakly constrained by exp. data such as binding energies and charge radii. Possible observables for the isovector properties:
neutron radii, neutron skins, dipole polarizability, pygmy dipole strength, neutron star radii

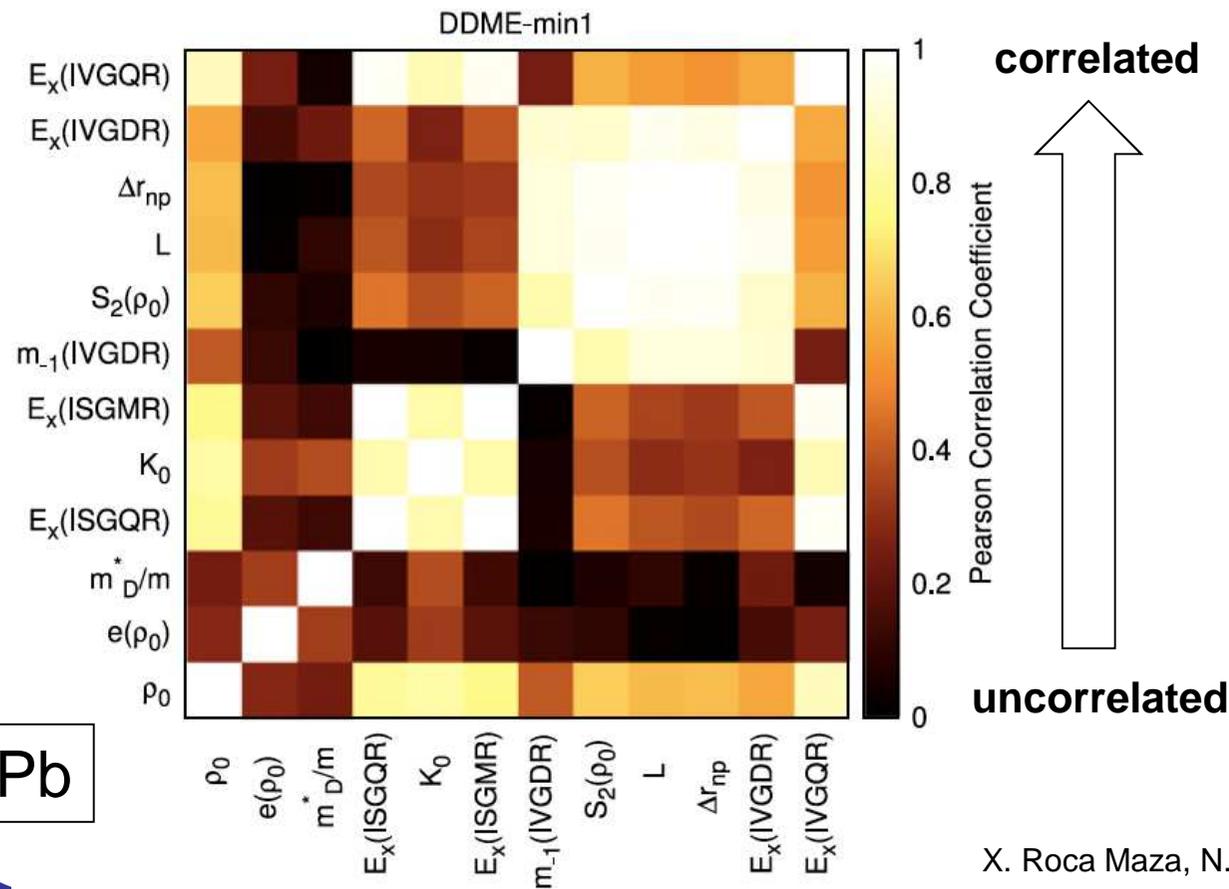
CORRELATIONS: NUCLEAR MATTER vs. PROPERTIES OF NUCLEI

- Covariance analysis in the EDF framework**

information on relevant correlations and statistical uncertainties

- Pearson product-moment correlation coefficient**

provides a measure of the correlation (linear dependence) between two variables A and B



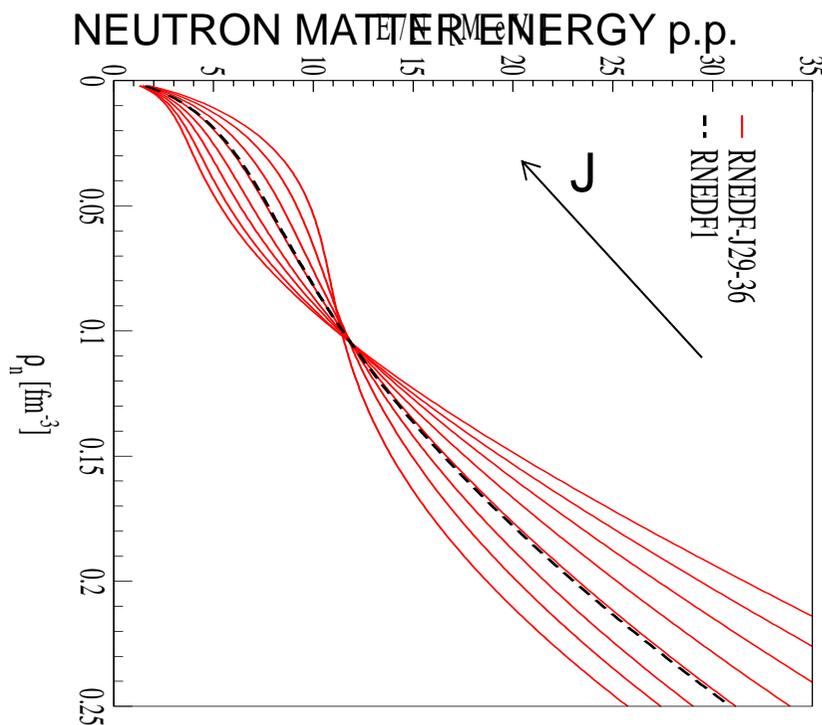
Correlation matrix between nuclear matter properties and properties for ^{208}Pb

- neutron skin thickness, properties of giant resonances,...

^{208}Pb

VARIATION OF THE SYMMETRY ENERGY IN CONSTRAINING THE EDF

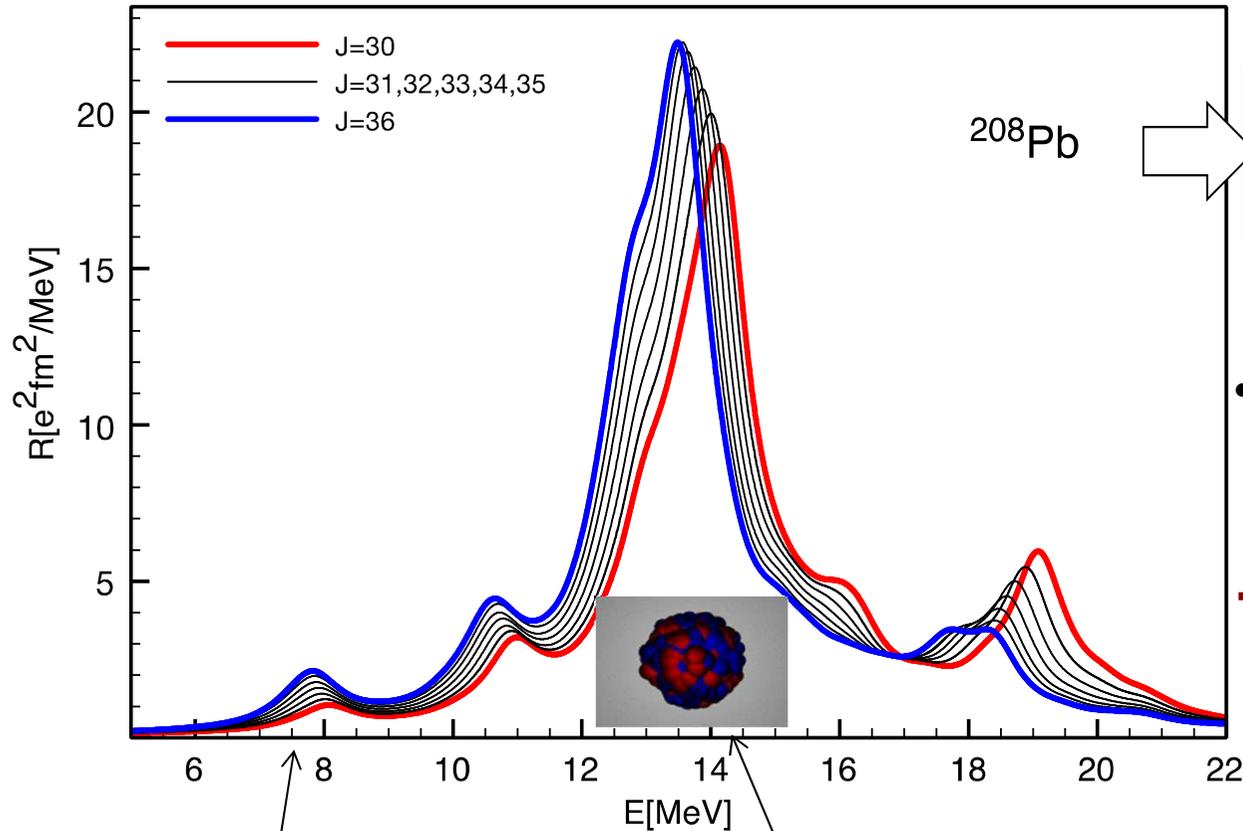
- Adjust the properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps)
- Establish a set of 8 relativistic point coupling interactions that span the range of values of the symmetry energy at saturation density: $J=29,30,\dots,36$ MeV
- Each interaction is determined independently using the same dataset supplemented with an additional constraint on J



J[MeV]	L[MeV]
29	31.9
30	37.0
31	44.1
32	52.5
33	62.2
34	72.3
35	83.4
36	94.3

CONSTRAINING THE SYMMETRY ENERGY

- Isovector dipole transition strength for ^{208}Pb using a set of relativistic point coupling interactions which vary the symmetry energy properties ($J=30,31,\dots,36$ MeV)



Pygmy strength

Isovector giant dipole resonance

- Isovector giant dipole resonance
- Pygmy dipole strengths
- Dipole polarizability ($\alpha_D \sim m_{-1}$)

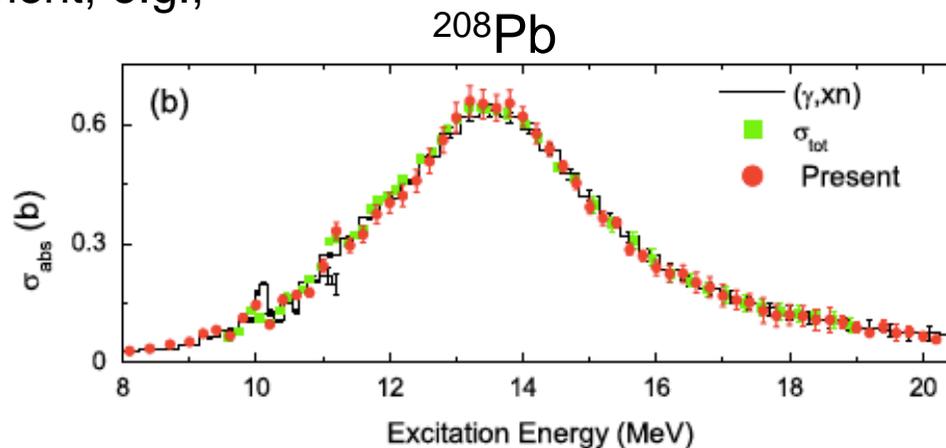
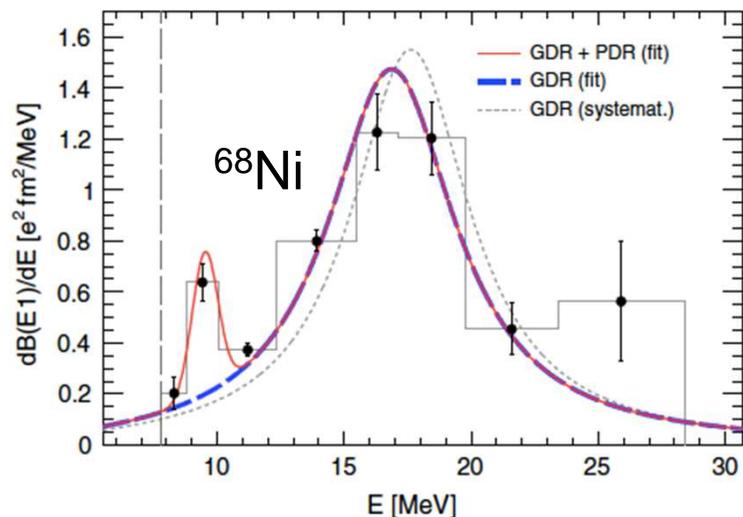
- The transition strength is sensitive on the properties of symmetry energy - (J,L)

→ Dipole response can be used to constrain effective nuclear interactions (isovector channel)

- There are exp. data available on the dipole response in nuclei (α_D , IVGDR, pygmy strength)

DIPOLE POLARIZABILITY AND SYMMETRY ENERGY

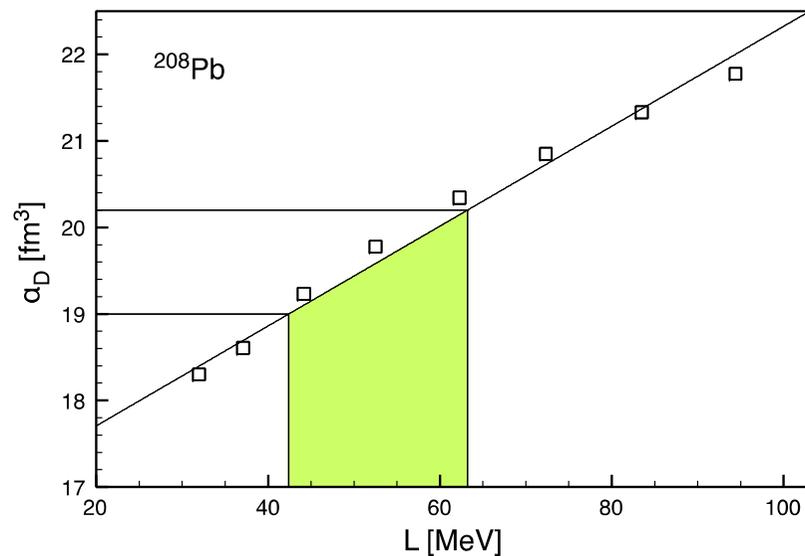
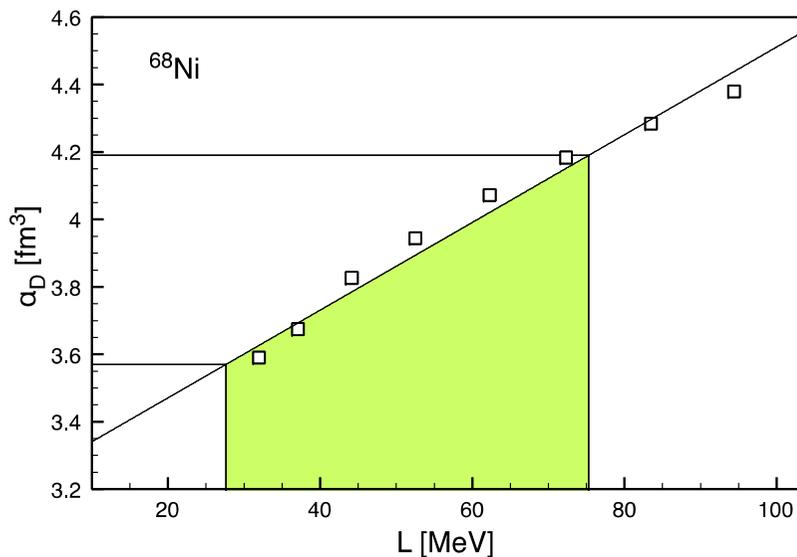
- Dipole polarizability from experiment, e.g.,



D. Rossi et al, PRL 111, 242503 (2013)

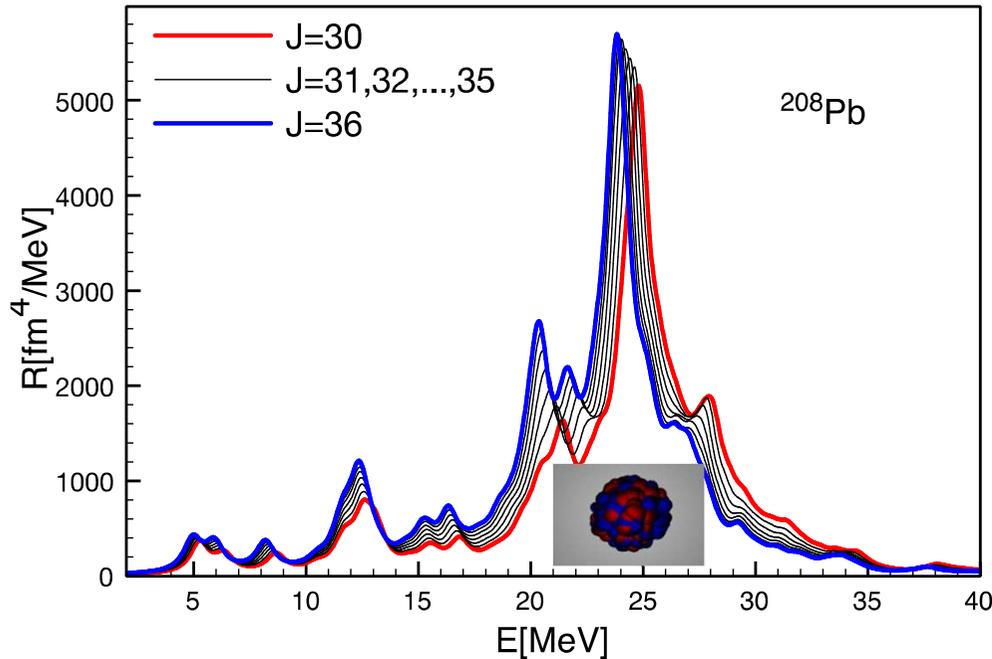
A. Tamii et al., PRL 107, 062502 (2011)

- constraining the slope of the symmetry energy using relativistic point-coupling interactions



GIANT QUADRUPOLE RESONANCES IN ^{208}Pb AND SYMMETRY ENERGY

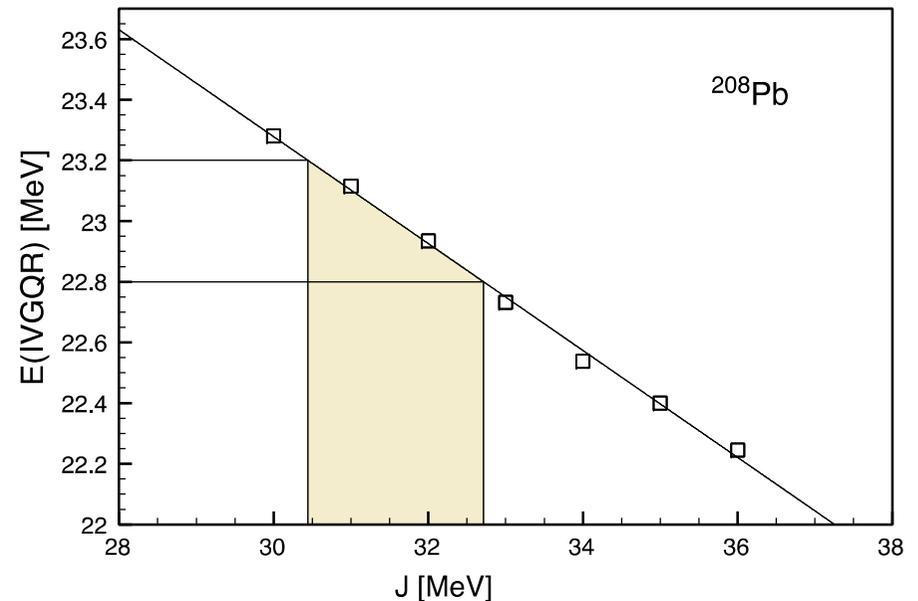
- Isovector quadrupole transition strength for ^{208}Pb using a set of relativistic point coupling interactions which vary the symmetry energy properties ($J=30,31,\dots,36$ MeV)



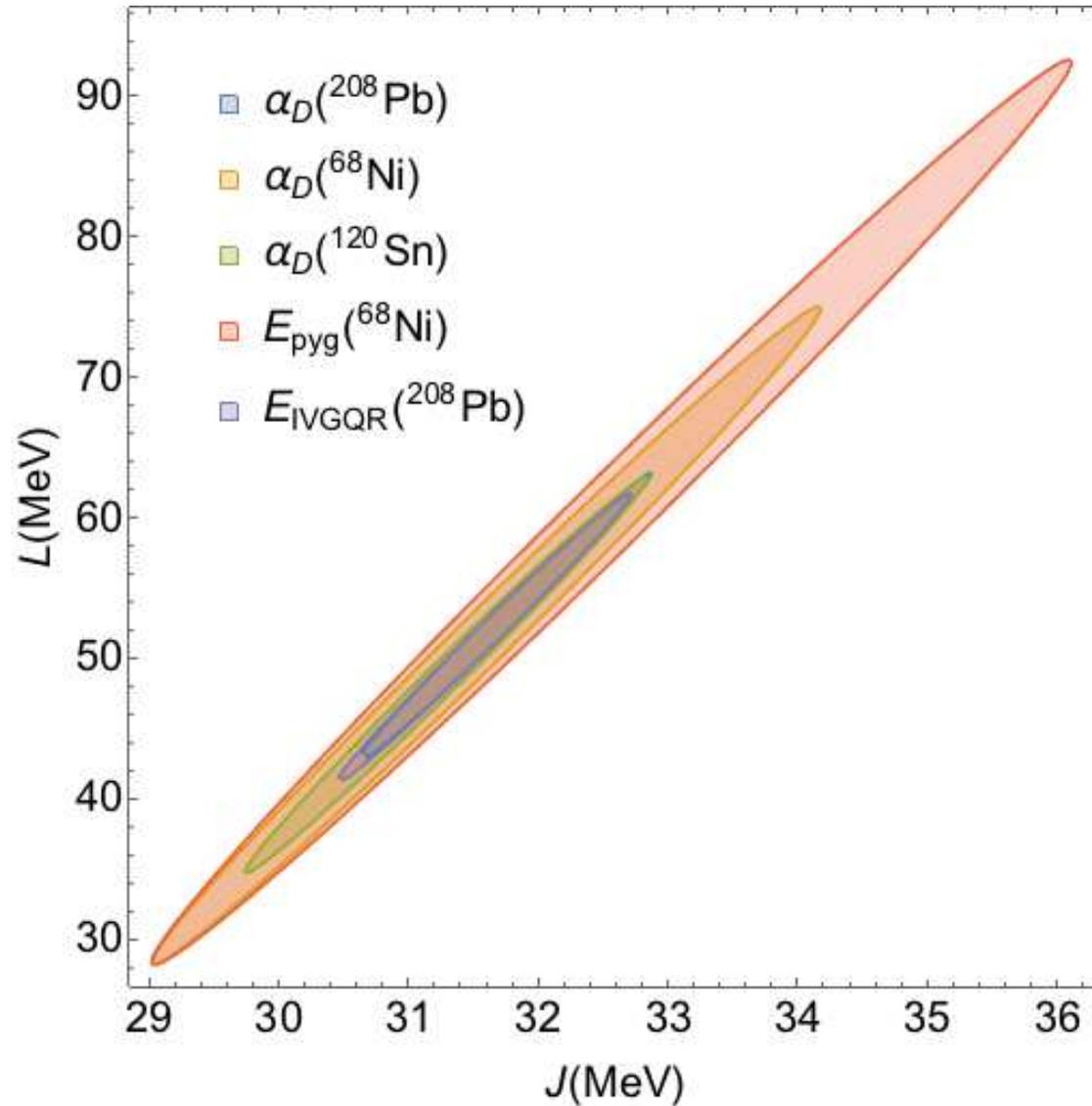
- Precise determination of isovector giant quadrupole resonances available and could be used to constrain the theory

S.S. Henshaw, M.W. Ahmed, G. Feldman et al,
PRL 107, 222501 (2011)

- The IVGQR energy is strongly correlated with the symmetry energy properties

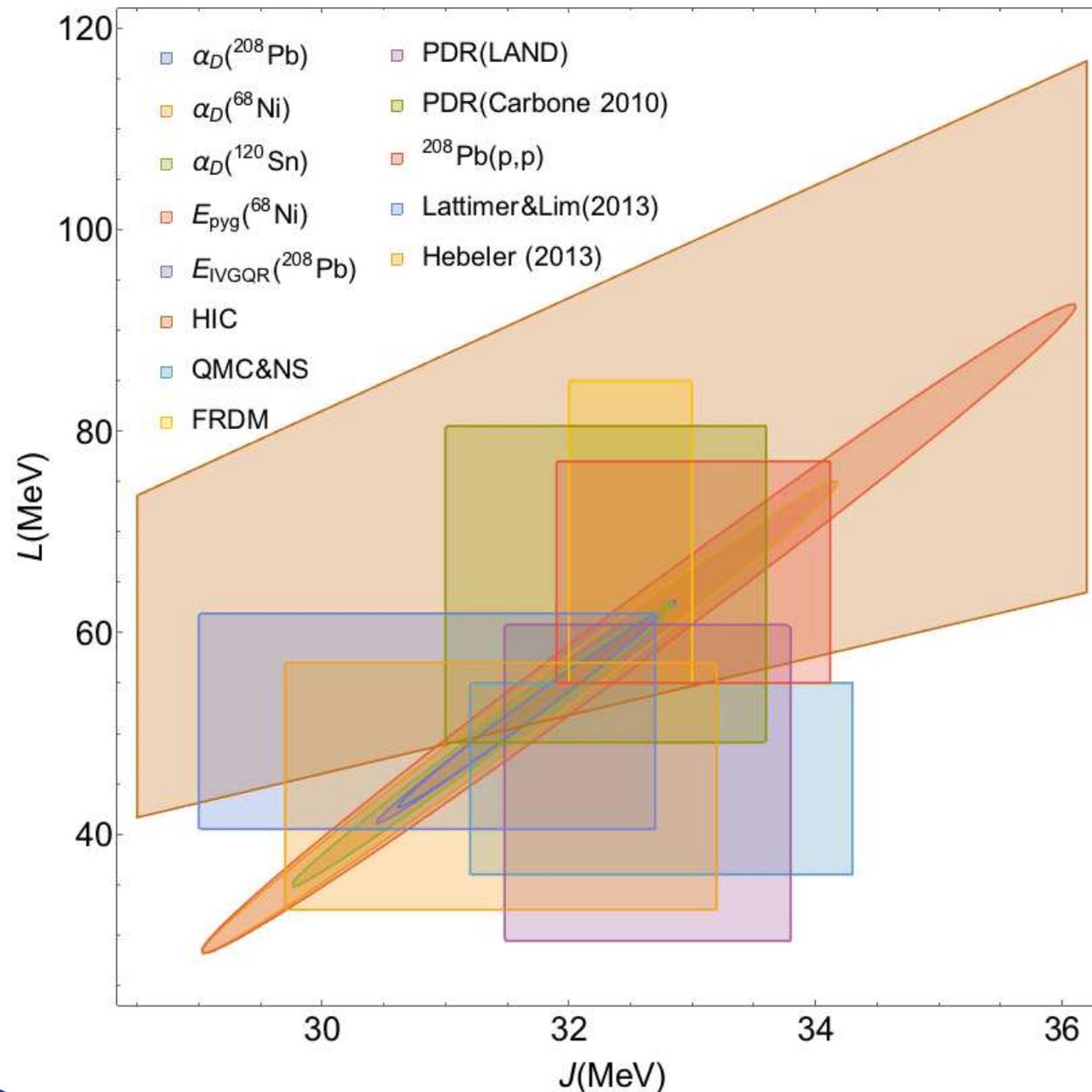


CONSTRAINING THE SYMMETRY ENERGY (J-L)



- Correlation between J & L taken into account
- The overall result is centered around $J=31.6$ MeV and $L=50.6$ MeV

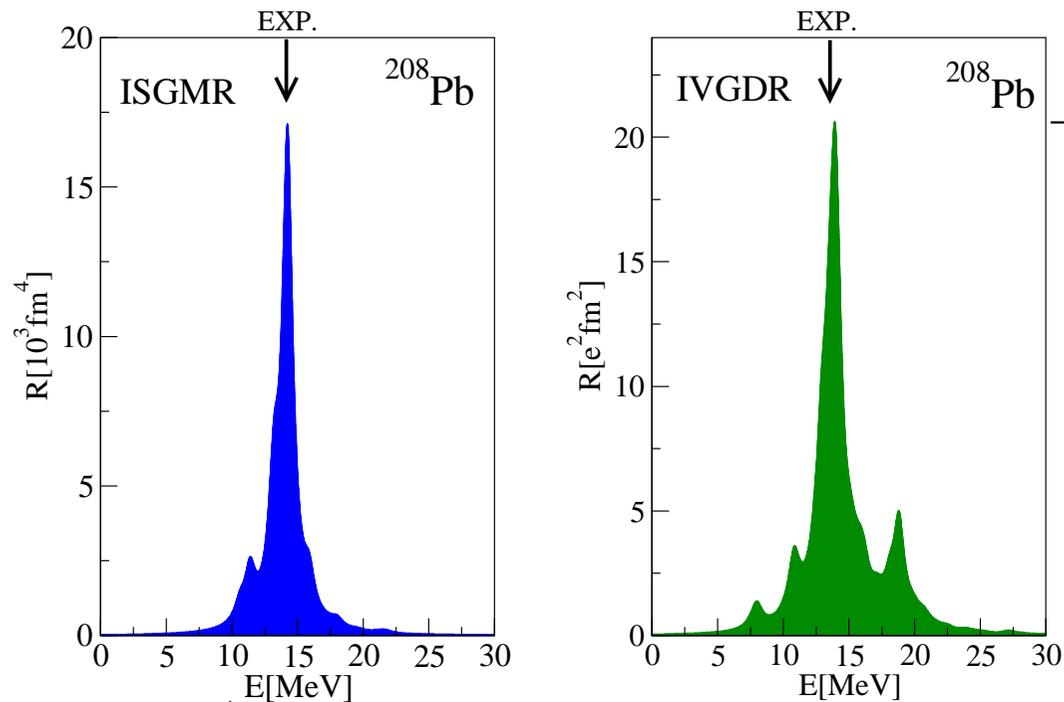
CONSTRAINING THE SYMMETRY ENERGY (J-L)



- Lattimer & Lim, ApJ. 771, 51 (2013)
– compilation from various approaches
- K. Hebeler et al., AJ 773, 11 (2013)
– nuclear interactions derived from chiral EFT
- $^{208}\text{Pb}(p,p)$ – J. Zenihiro et al., PRC 82, 044611 (2010)
- A. Carbone, G. Colo, A. Bracco, et al., PRC 81, 041301 (2010)
– PDR
- A. Klimkiewicz et al. (LAND), PRC 76,051603(R) (2007)
- P. Moller, et al., PRL 108, 052501 (2012).
- A. W. Steiner and S. Gandolfi, PRL 108, 081102 (2012)
– QMC (Av8')+ neutron stars
- H. S. Xu et al., PRL 85, 716 (2000)
- Z. Y. Sun et al., PRC 82, 051603(R) (2010)
- M. B. Tsang et al., PRC 86, 015803 (2012)

CONSTRAINING THE SYMMETRY ENERGY: 2nd approach

- use the experimental data on collective excitations to constrain the symmetry energy within the fitting protocol to determine the parameters of the functional



Dipole polarizability:
 $\alpha_D = (19.68 \pm 0.21) \text{ fm}^3$

Exp.
 $\alpha_D = (19.6 \pm 0.6) \text{ fm}^3$

A. Tamii et al., PRL 107, 062502 (2011). + update (2015).

- IVGDR – α_D determine the symmetry energy for the PC interaction

J = 31.89 MeV
L = 51.48 MeV

- ISGMR energy determines the nuclear matter incompressibility: **$K_{nm} = 232.4 \text{ MeV}$**

$E \text{ (Exp.)} = (13.91 \pm 0.11) \text{ MeV (TAMU)}$

$E \text{ (Exp.)} = (13.7 \pm 0.1) \text{ MeV (RCNP)}$

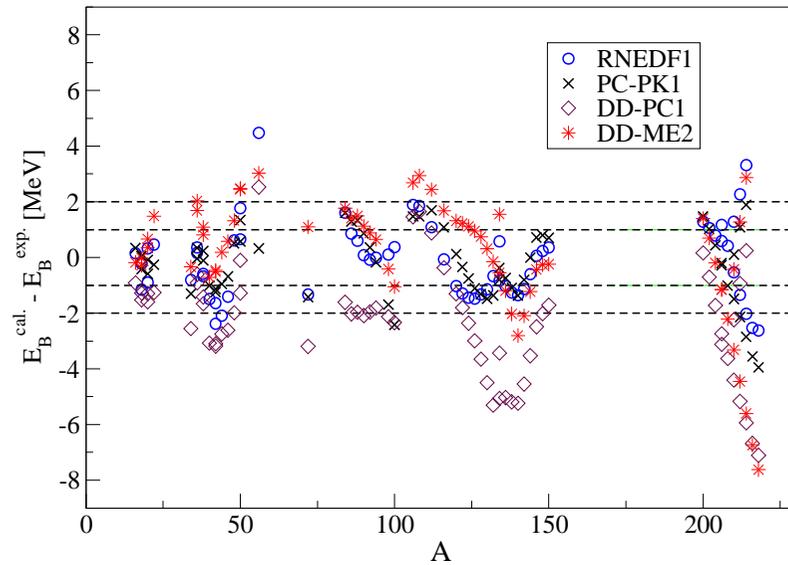
“Correct symmetry energy” is the one obtained for the interaction that reproduces the exp. data on dipole polarizability

CONCLUDING REMARKS

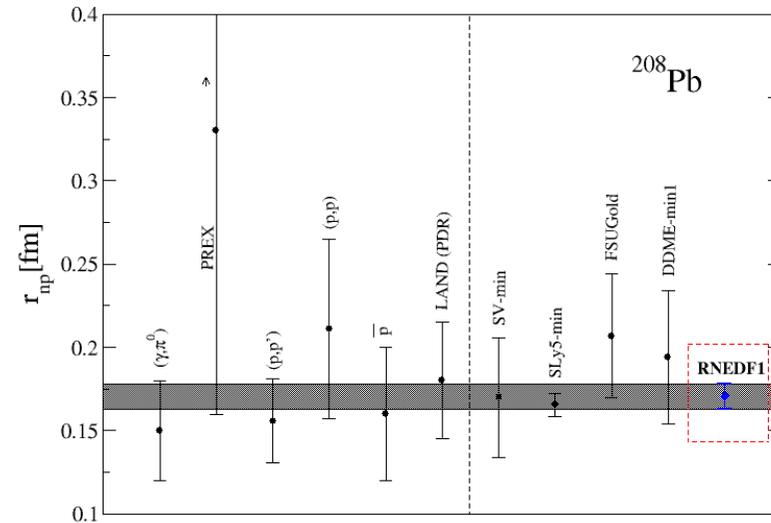
- Dipole excitations in nuclei (PDR, α_D , IVGDR, AGDR) and other modes (IVGQR,...) provide useful constraints for the nuclear matter symmetry energy J & L
- Microscopic theory frameworks are well established and (some) accurate exp. data are available
- Accurate measurements of collective excitations have important implications to reduce uncertainties in the symmetry energy
- On the other side, they allow improved constraining of the EDFs (their isovector channel)
- Prospects to include directly the properties of collective excitations as additional observables in the fitting protocols to determine the parameters of the EDF

SOME OTHER PROPERTIES...

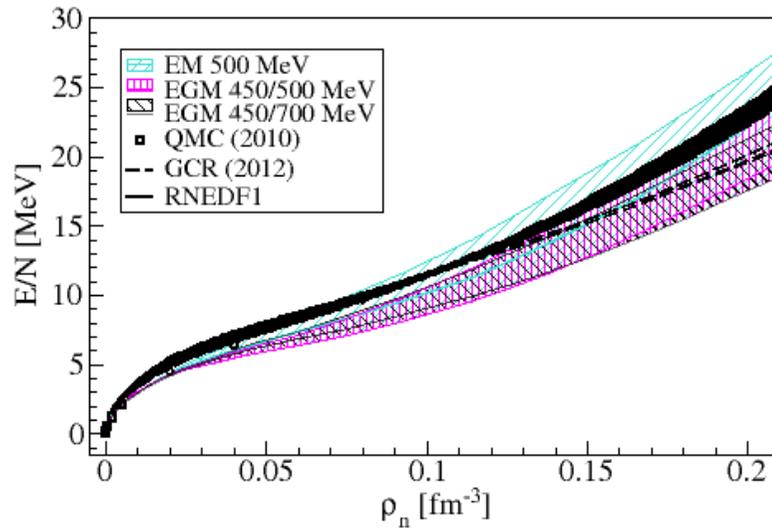
Nuclear binding energies (calc. – exp.)



Neutron skin thickness



Pure neutron matter



Neutron star mass-radius relationship

