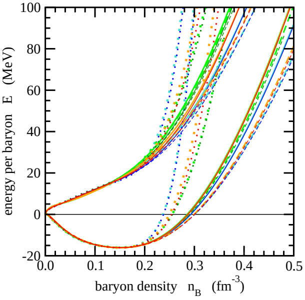
Equations of State of Relativistic Mean-Field Models with Different Parametrisations of Density Dependent Couplings

Stefan Typel

TECHNISCHE UNIVERSITÄT DARMSTADT

7th International Symposium on Nuclear Symmetry Energy
GANIL, Caen, France
September 4 – 7, 2017



DFG Deutsche Forschungsgemeinschaft
SFB 1245
HELMHOLTZ GEMEINSCHAFT
GSI

Outline

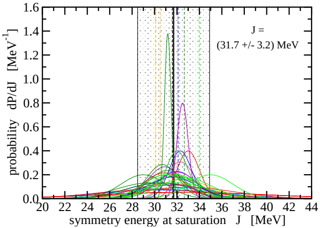
TECHNISCHE UNIVERSITÄT DARMSTADT

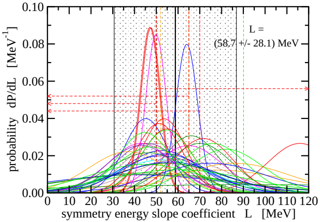
- **Motivation**
- **Relativistic Mean-Field Models**
  - Medium Dependence of Effective Interaction
  - Parametrisation of Couplings
  - Determination of Parameters
  - Choice of Functionals
- **Results**
  - Coupling Functions
  - Equations of State (Symmetric and Neutron Matter)
  - Density Dependence of Symmetry Energy
  - Nuclear Matter Parameters
- **Effects of Rearrangement Contributions**
- **Summary and Outlook**

Motivation

TECHNISCHE UNIVERSITÄT DARMSTADT

- **symmetry energy of nuclear matter**
  - density dependence
    - densities below saturation:
      convergence of different theoretical approach,
      consistency with experimental constraints
    - densities above satuation:
      large uncertainties
  - characteristic parameters at saturation
    - symmetry energy at saturation *J*
      rather well constrained
    - slope parameter *L* still has large uncertainty
- **theoretical description**
  - choice of energy density functional ?
  - effects on extrapolation ?





(M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)

Relativistic Mean-Field Models

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- **field theoretical approach**
  - energy density functional derived from Lagrangian density
  - phenomenological description
- **various versions**
  - interaction: exchange of scalar and vector mesons ($\sigma, \omega, \rho, \ldots$)
    - minimal coupling of mesons to nucleons
    - with nonlinear self-interactions
    - with density dependent couplings
  - without explicit meson fields
    - point-coupling models
- **applications**
  - description of finite nuclei and excitations
  - nuclear matter and equation of state

$\Rightarrow$ many parametrizations for different purposes

(M. Dutra et al., Phys. Rev. C 90 (2014) 055203)

Medium Dependence of Effective Interaction

TECHNISCHE UNIVERSITÄT DARMSTADT

- **interaction contribution to Lagrangian**
  - nonlinear (NL) RMF models with meson self-interactions

$$\mathcal{L}_{\text{int}} = \bar{\psi} g_\sigma \sigma \psi - \frac{A}{3}\sigma^3 - \frac{B}{4}\sigma^4 - \bar{\psi} g_\omega \omega_\mu \gamma^\mu \psi + \frac{C}{4}\left(\omega_\mu \omega^\mu\right)^2 - \bar{\psi} g_\rho \vec{\rho}_\mu \cdot \vec{\tau} \gamma^\mu \psi$$

with constants g_σ, g_ω, g_ρ, A, B, C, ...

(ususally scalar and vector contributions not coupled, cross terms added later)

  - density dependent (DD) RMF models

$$\mathcal{L}_{\text{int}} = \bar{\psi} \Gamma_\sigma \sigma \psi - \bar{\psi} \Gamma_\omega \omega_\mu \gamma^\mu \psi - \bar{\psi} \Gamma_\rho \vec{\rho}_\mu \cdot \vec{\tau} \gamma^\mu \psi$$

with functionals Γ_σ, Γ_ω, Γ_ρ, ... depending on Lorentz scalars
constructed from $\bar{\psi}$, ψ

(more flexible than NL models)

Parametrisation of Couplings

TECHNISCHE UNIVERSITÄT DARMSTADT

- **dependence of Γ_i on**
  - **vector density** $\varrho^{(v)} = \sqrt{j^\mu j_\mu}$ with current $j^\mu = \bar{\psi}\gamma^\mu\psi$
    $\Rightarrow$ standard choice
  - **scalar density** $\varrho^{(s)} = \bar{\psi}\psi$
    $\Rightarrow$ not really explored so far
  - ...
- **form of dependence** (introduced in S. Typel and H.H. Wolter, Nucl. Phys. A 565 (1999) 331)
  - **general parametrisation:**
    $\Gamma_i(\varrho) = \Gamma_i(\varrho_{\text{ref}}) f_i(x)$ with $x = \varrho/\varrho_{\text{ref}}$ and reference density ϱ_{ref}
  - **rational function:**

    $$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \quad \text{with additional constraints on } f_i$$

  - **exponential:**

    $$f_i(x) = \exp[-a_i(x - 1)]$$

  - other

Determination of Parameters

TECHNISCHE UNIVERSITÄT DARMSTADT

- **fit to properties of nuclei**
  - minimisation of function $\chi^2(\{p_k\}) = \sum_{n=1}^{N_{\text{data}}} \left[\frac{O_n^{(\text{exp})} - O_n^{(\text{model})}(\{p_k\})}{\Delta O_n} \right]^2$ with parameters $\{p_k\}$
  - set of 12 nuclei:
    - ^{16}O, ^{24}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{68}Ni, ^{90}Zr, ^{100}Sn, ^{114}Sn, ^{132}Sn, ^{140}Ce, ^{208}Pb
  - observables O_n with assumed errors ΔO_n:
    - binding energy (0.1 MeV, 12 data)
    - charge radius (0.01 fm, 8 data)
    - diffraction radius (0.01 fm, 5 data)
    - surface thickness (0.005 fm, 5 data)
    - spin-orbit splitting (0.1 MeV, 14 data)

  $\Rightarrow$ 44 data in total
- no fit to nuclear matter parameters (derived quantities)

Choice of Functionals

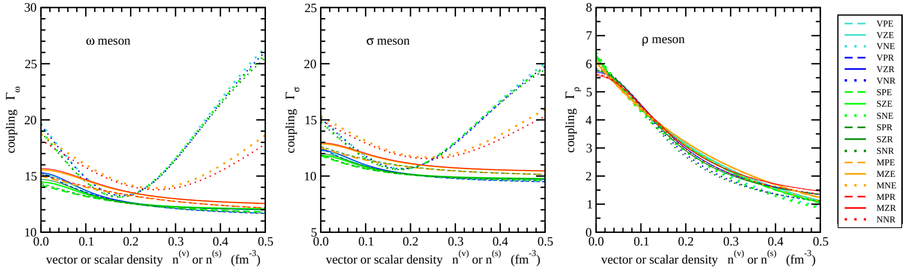
TECHNISCHE UNIVERSITÄT DARMSTADT

- **density dependence**
  - **'V':** dependence of Γ_ω, Γ_σ, Γ_ρ on vector density $\varrho^{(v)}$
  - **'S':** dependence of Γ_ω, Γ_σ, Γ_ρ on scalar density $\varrho^{(s)}$
  - **'M':** dependence of Γ_ω, Γ_ρ (Γ_σ) on vector (scalar) density $\varrho^{(v)}$ ($\varrho^{(s)}$)
- **functional form for ω and σ mesons**
  - rational function $f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}$ with conditions $f_i(1) = 1$ and
  - **'P':** $f_i''(0) = 0$, $d_i > 0$ (positive)
  - **'Z':** $f_i'(0) = 0$, $d_i = 0$ (zero)
  - **'N':** $f_i''(0) = 0$, $d_i < 0$ (negative)
- **functional form for ρ meson**
  - **'E':** exponential function $f_i(x) = \exp[-a_i(x-1)]$
  - **'R':** rational function $f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}$ with conditions $f_i(1) = 1$, $f_i'(0) = 0$, $d_i = 0$, $f_i'(1)/f_i(1) = f_i''(1)/f_i'(1)$

$\Rightarrow$ 18 models with 9 parameters (including ϱ_{ref} and m_σ), similar quality in describing nuclei

TECHNISCHE UNIVERSITÄT DARMSTADT

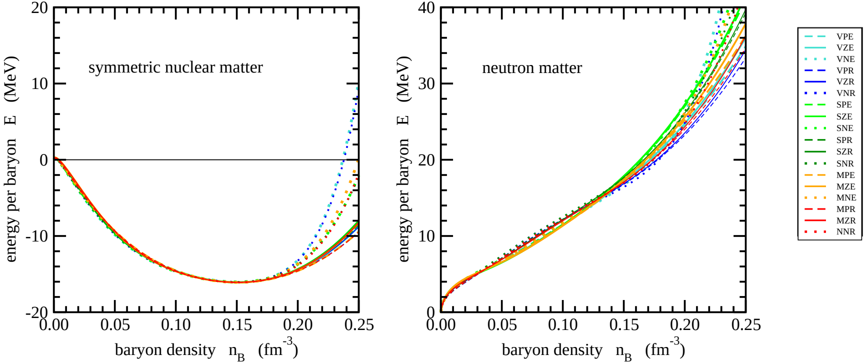
Results (Preliminary)
Coupling Functions



- similar smooth functions for 'P' and 'Z' parametrisations
- minimum in functions for 'N' parametrisations (ω and σ mesons)
- only small differences between 'E' and 'R' parametrisations (ρ meson)

Results (Preliminary)
Equations of State

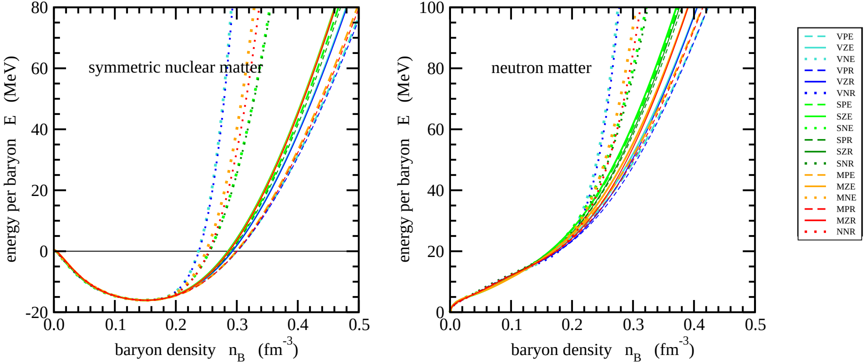
TECHNISCHE UNIVERSITÄT DARMSTADT



- very similar below saturation density
- divergence above saturation density
- strong stiffening for 'N' parametrisations

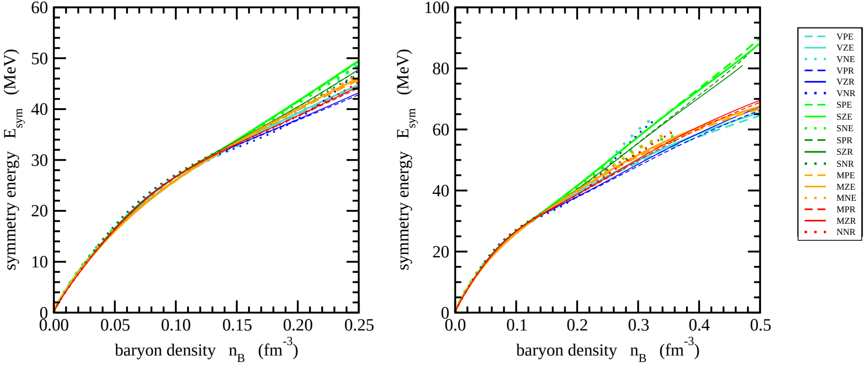
Results (Preliminary)
Equations of State

TECHNISCHE UNIVERSITÄT DARMSTADT



- very similar below saturation density
- divergence above saturation density
- strong stiffening for ’N’ parametrisations

Results (Preliminary)
Density Dependence of Symmetry Energy



- very similar below saturation density
- divergence above saturation density, stiffest for 'S' paremetrisations
- convergence problems for some parametrisations

Nuclear Matter Parameters I

TECHNISCHE UNIVERSITÄT DARMSTADT

- **energy per nucleon**

$$E(n_B, \delta) = E_0(n_B) + E_{\text{sym}}(n_B)\delta^2 + \mathcal{O}(\delta^4)$$

with baryon density $n_B = n_n + n_p$ and isospin asymmetry $\delta = (n_n - n_p)/n_B$

- **energy per nucleon in symmetric nuclear matter**

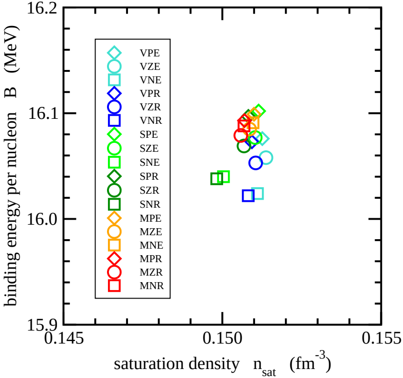
$$E_0(n_B) = m_{\text{nuc}} - B_{\text{sat}} + \frac{1}{2}Kx^2 + \frac{1}{6}Qx^3 + ... \qquad \text{with} \qquad x = \frac{n_B - n_{\text{sat}}}{3n_{\text{sat}}}$$

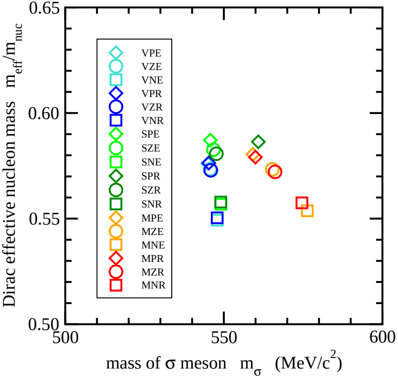
- **symmetry energy**

$$E_{\text{sym}}(n_B) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + ...$$

- **parameters** $n_{\text{sat}}, B_{\text{sat}}, K, Q, J, L, K_{\text{sym}}, \ldots$

Results (Preliminary)
Nuclear Matter Parameters II

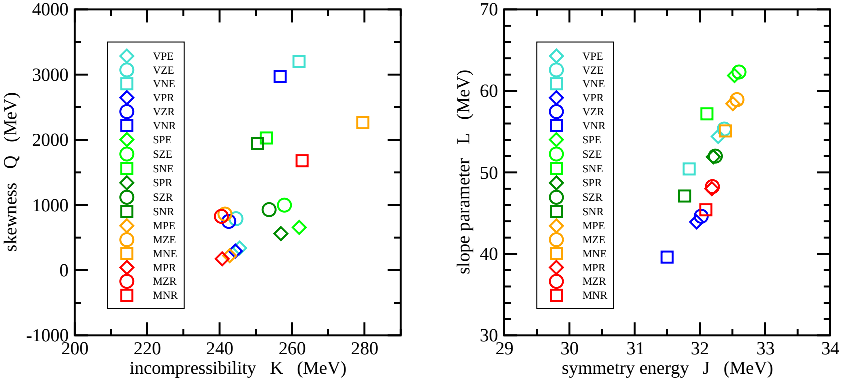




- small variation in n_{sat} and B, some spread in m_{eff}, a few outliers for m_σ

Results (Preliminary)
Nuclear Matter Parameters III

TECHNISCHE UNIVERSITÄT DARMSTADT



- large spread in K, Q, and L
- systematics of Q with K and correlation of L with J

Effects of Rearrangement Contributions I

TECHNISCHE UNIVERSITÄT DARMSTADT

- essential for thermodynamic consistency of model
- general form of potentials (in symmetric nuclear matter):

$$S = \Gamma_\sigma \sigma + S^{(R)} \qquad V = \Gamma_\omega \omega_0 + V^{(R)} \qquad \text{with}$$

$$S^{(R)} = \frac{\partial \Gamma_\sigma}{\partial \varrho^{(s)}} n_\sigma \sigma - \frac{\partial \Gamma_\omega}{\partial \varrho^{(s)}} n_\omega \omega_0 \qquad V^{(R)} = \frac{\partial \Gamma_\omega}{\partial \varrho^{(v)}} n_\omega \omega_0 - \frac{\partial \Gamma_\sigma}{\partial \varrho^{(v)}} n_\sigma \sigma$$

  with source densities n_σ and n_ω
- dependence of couplings on scalar density $\varrho^{(s)}$
  $\Rightarrow$ rearrangement contribution $S^{(R)}$ to scalar potential S
- dependence of couplings on vector density $\varrho^{(v)}$
  $\Rightarrow$ rearrangement contribution $V^{(R)}$ to scalar potential V
  (as in standard DD-RMF models)

Effects of Rearrangement Contributions II

TECHNISCHE UNIVERSITÄT DARMSTADT

- dependence of Γ_ω on scalar density $\varrho^{(s)}$ with $\frac{\partial \Gamma_\omega}{\partial \varrho^{(s)}} < 0$, zero temperature
  - $\Rightarrow$ Dirac effective mass $m_{\text{nuc}}^{(\text{eff})} = m_{\text{nuc}} - S$ can approach zero
  - $\Rightarrow$ scalar density $\varrho^{(s)} = n_\sigma \to 0 \Rightarrow \sigma \to 0$
  - $\Rightarrow$ scalar potential $S \to -\frac{\partial \Gamma_\omega}{\partial \varrho^{(s)}} \frac{\Gamma_\omega}{m_\omega^2} n_\omega^2$
  - $\Rightarrow$ limiting maximum density $n_{\max} = \sqrt{-\left(\frac{\partial \Gamma_\omega}{\partial \varrho^{(s)}}\right)^{-1} \frac{m_{\text{nuc}} m_\omega^2}{\Gamma_\omega}}$
- dependence of Γ_σ on vector density $\varrho^{(v)}$ with $\frac{\partial \Gamma_\sigma}{\partial \varrho^{(v)}} \neq 0$ at finite temperature T, baryon density $n_B = 0$
  - $\Rightarrow$ $n_\omega = 0$, $\omega_0 = 0$, $n_\sigma > 0$, $\sigma > 0$ (antiparticles!)
  - $\Rightarrow$ vector potential $V = V^{(R)} = -\frac{\partial \Gamma_\sigma}{\partial \varrho^{(v)}} \frac{\Gamma_\sigma}{m_\sigma^2} n_\sigma^2$
  - $\Rightarrow$ baryon chemical potential $\mu_B = V \neq 0$
- model should work in whole hadronic part of phase diagram
  - $\Rightarrow$ constraints on density dependent couplings
  - $\Rightarrow$ exclusion of some parametrisations

Summary and Outlook

TECHNISCHE UNIVERSITÄT DARMSTADT

- extension of RMF models with density dependent couplings
  - different dependencies on vector and scalar densities
  - model parameters from fit to properties of finite nuclei
    - similar quality in description of all 18 parametrisations
  - equations of state, symmetry energy and nuclear matter parameters
    - only small variations below saturation density
    - divergence above saturation density
    - exceptional behavior of 'N' parametrisations
    - some parameters (K, Q, L) not well constrained and model dependent
  - constraints from rearrangement contributions to potentials
    - only parametrisations VZR, SZR, and Mxx viable, others problematic
- next steps
  - include δ meson
  - include pairing effects
  - detailed analysis of uncertainties and correlations