Crust-core transitions in neutron stars revisited

X. Viñas[a], C. González-Boquera[a], B.K. Sharma[a,b]
M. Centelles[a]

[a]Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos, Universitat de Barcelona, Barcelona, Spain
[b]Department of Sciences, Amarita Vishwa Vidyapeetham Ettimadai, 641112 Coimbatore, India

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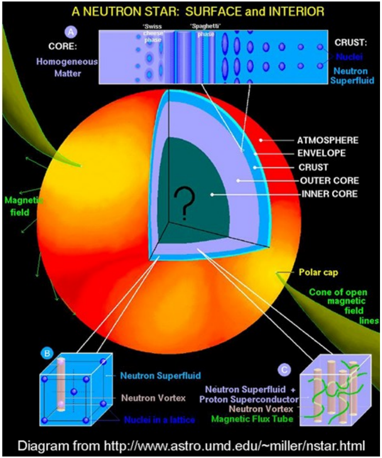
The structure of a Neutron Star



Core $\rho > 1.5 \times 10^{14}$ g/cm^3

Inner crust $\rho > 4 \times 10^{11}$ g/cm^3 Outer crust $\rho > 10^4$ g/cm^3

The inner crust. Introduction

- The structure of the inner crust consists of a Coulomb lattice of nuclear clusters permeated by the gases of free neutrons and free electrons.
- At the bottom layers of the inner crust the equilibrium nuclear shape may change from sphere, to cylinder, slab, tube (cylindrical hole) and bubble (spherical hole). These shapes are generically known as "nuclear pasta".
- Full quantal Hatree-Fock calculations of the nuclear structures in the inner crust are complicated by the treatment of the neutron gas and the eventual need to deal with different geometries.
- The formation and the properties of nuclear pasta have been investigated in three dimensional Hartree-Fock calculations in cubic boxes that avoid assumptions of the geometry of the system. However, these calculations are highly time consuming and a detailed EOS fo the whole systemn is not yet available.
- Large scale calculations of the inner crust and nuclear pasta have been performed very often with semiclassical methods such as the Compressible Liquid Drop Model (CLDM) and the Thomas-Fermi (TF) approximation.

The Inner Crust, Self-consistent Thomas-Fermi approach

The total energy of an esemble of neutrons, protons and electrons in a Wigner-Seitz cell of volume V_c is given by:

$$E = \int dV \left[\mathcal{H}\left(\rho_n, \rho_p\right) + \mathcal{E}_{elec} + \mathcal{E}_{coul} - \frac{3}{4}\left(\frac{3}{\pi}\right)^{1/3} e^2 \left({\rho_p}^{4/3} + {\rho_e}^{4/3}\right) + m_n\rho_n + m_p\rho_p \right]$$

where the nuclear energy density in the TF approach reads:

$$\mathcal{H}\left(\rho_n, \rho_p\right) = \frac{\hbar^2}{2m_n}\frac{3}{5}\left(3\pi^2\right)^{2/3}\rho_n^{5/3}(\mathbf{r}) + \frac{\hbar^2}{2m_p}\frac{3}{5}\left(3\pi^2\right)^{2/3}\rho_p^{5/3}(\mathbf{r}) + \mathcal{V}\left(\rho_n(\mathbf{r}), \rho_\mathbf{p}(\mathbf{r})\right)$$

The Coulomb energy density coming from the direct part of the proton-proton and electron-electron plus the proton-electron interaction is given by

$$\mathcal{E}_{coul} = \frac{1}{2}\left(\rho_p(\mathbf{r}) - \rho_e\right)\left(V_p(\mathbf{r}) - V_e(\mathbf{r})\right) = \frac{1}{2}\left(\rho_p(\mathbf{r}) - \rho_e\right)\int \frac{e^2}{|\mathbf{r} - \mathbf{r}'|}\left(\rho_p(\mathbf{r}') - \rho_e\right) d\mathbf{r}'.$$

assuming that the electrons are uniformly distributed in the cell

The Inner Crust, Variational equations

We perform a fully variational calculation of the energy in the WS cell under the constraints of given average density ρ_B in the WS cell of size R_c and charge neutrality in the cell. Taking functional derivatives respect to the neutron, proton and electron densities one finds

$$\frac{\delta \mathcal{H}(\rho_n, \rho_p)}{\delta \rho_n} + m_n - \mu_n = 0,$$

$$\frac{\delta \mathcal{H}(\rho_n, \rho_p)}{\delta \rho_p} + V_p(\mathbf{r}) - V_e(\mathbf{r}) - \left(\frac{3}{\pi}\right)^{1/3} e^2 \rho_p^{1/3}(\mathbf{r}) + m_p - \mu_p = 0,$$

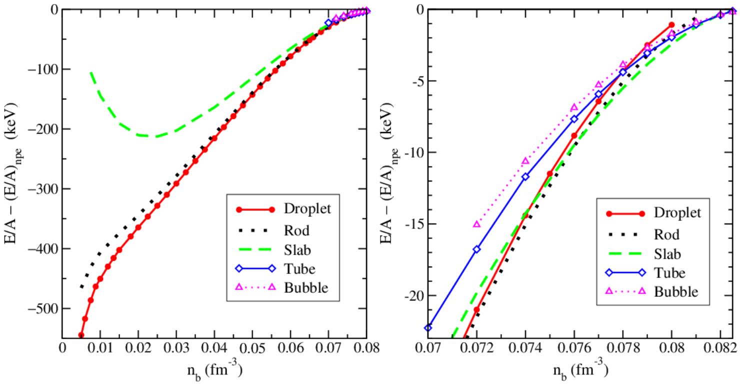
$$\sqrt{k_F^2 + m_e^2} - V_p(\mathbf{r}) + V_e(\mathbf{r}) - \left(\frac{3}{\pi}\right)^{1/3} e^2 \rho_e^{1/3} = \mu_e,$$

together with the β-equilibrium condition

$$\mu_e = m_n - m_p + \mu_n - \mu_p,$$

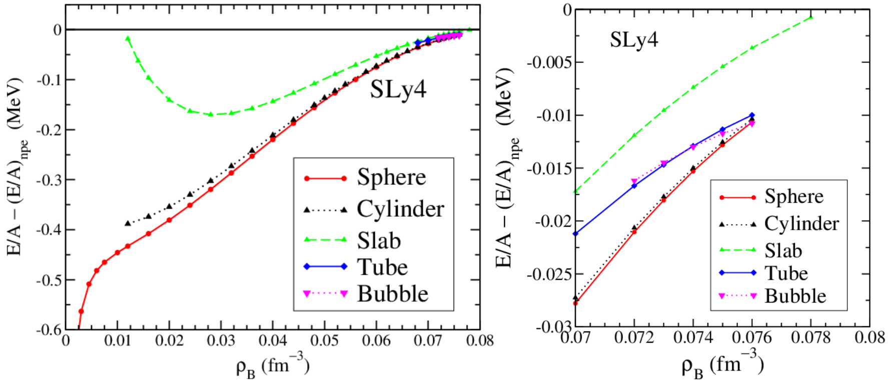
imposed by the the aforementioned constraints.

The Inner Crust, Energy per particle

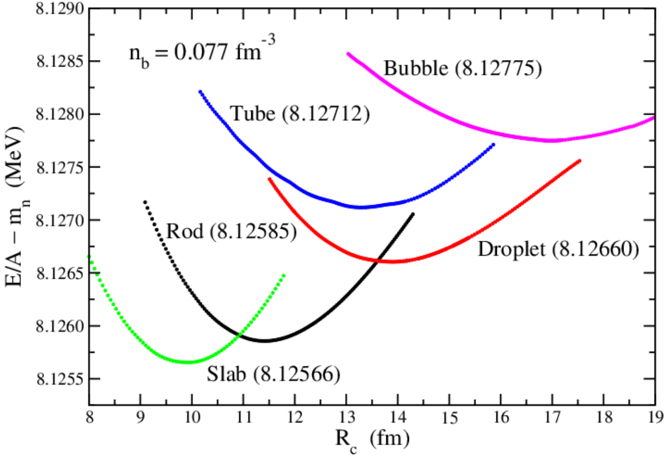


From B.K. Sharma et al A&A **584**, A103 (2015)

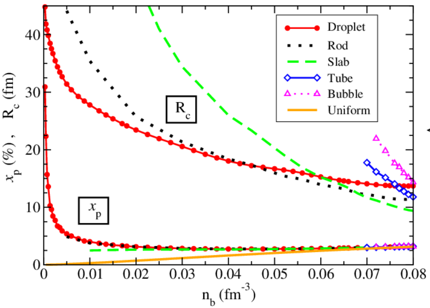
The Inner Crust, Energy per particle

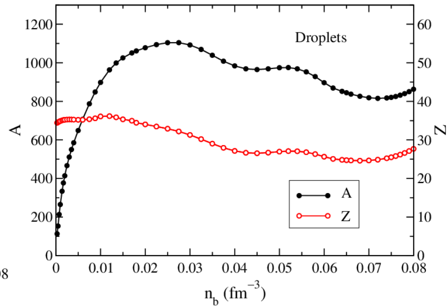


The Inner Crust, Energy per particle

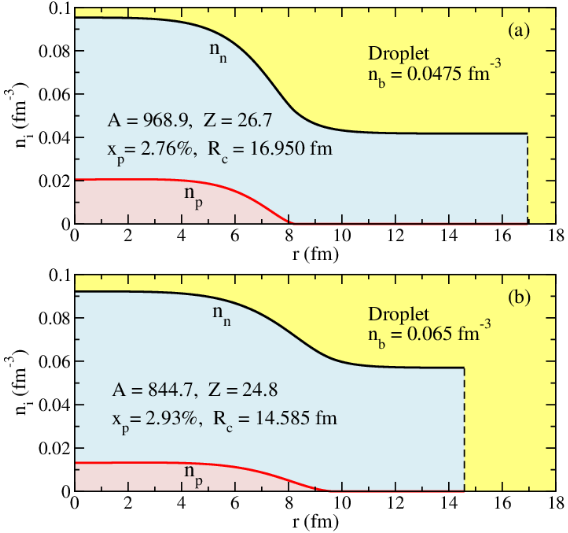


The Inner Crust, Size and Composition





The Inner Crust, Density Profiles



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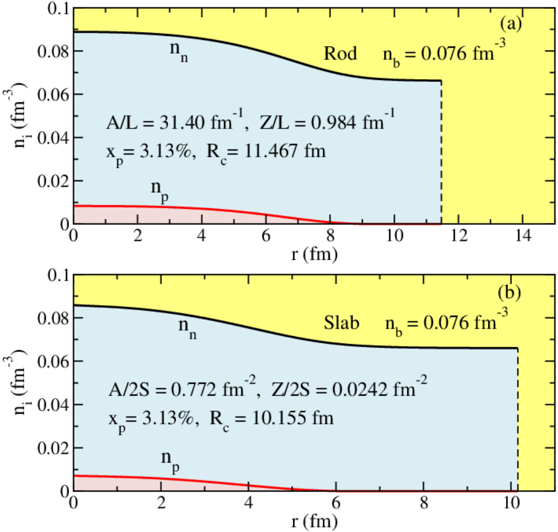
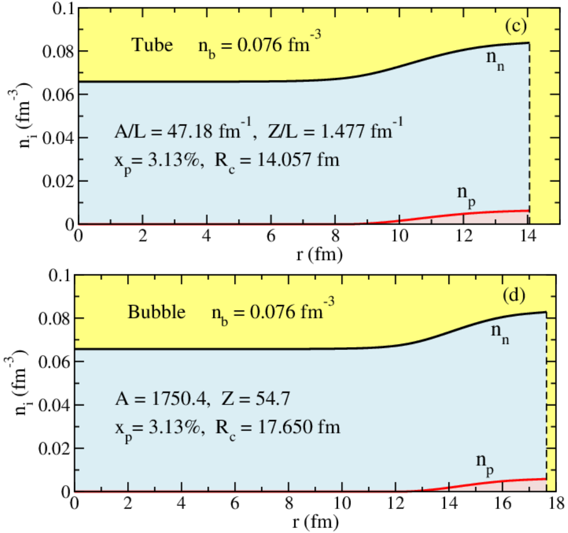
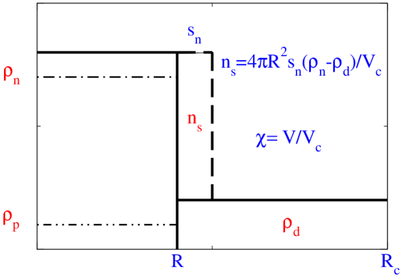


The Inner Crust, Density Profiles



The Compressible Liquid Drop Model (I)

CLDM was introduced by Baym, Bethe and Pethick, NPA **175** 225 (1971). See also Douchin and Haensel A&A **380** 151 (2001).



Inside the WS cell we impose charge neutrality $\chi\rho_p = \rho_e = V/V_c$ and fixed average density ρ_B. The surface tension σ and neutron skin s_n are taken form a self-consistent ETF calculation in semi-infinite nuclear matter Centelles et al NPA **635** 193 (1998).

The Compressible Liquid Drop Model (II)

$$\mathcal{E}_{cell} = \frac{E}{V_c} = \chi \mathcal{E}\left(n_n, n_p\right) + (1-\chi)\,\mathcal{E}\left(n_d, 0\right) + \frac{3\chi\sigma_{surf}}{R_p} + n_{surf}\,\mu_s$$
$$+\frac{3}{4}\left(3\pi^2\right)^{1/3}\chi^{4/3} n_p^{4/3} + \frac{4\pi}{5} e^2 R_p^2 n_p^2 \chi \left(1 - \frac{3}{2}\chi^{1/3} + \frac{1}{2}\chi\right)$$

$$E_{surf} = \mathcal{A}\sigma_{surf} + N_{surf}\,\mu_s,$$

where $\mathcal{A}$ is the area of the reference surface, σ_{surf} the surface tension, and N_{surf} and μ_s are the number and the chemical potential of the neutrons adsorbed onto the reference surface.

$$4\pi R_p^2 \sigma_{surf} = 4\pi R_p^2 \left[\sigma_s + \frac{2}{R_p}\sigma_c\right]$$

and

$$N_{surf} = 4\pi R_p^2 \left[s_n + \frac{2}{R_p}\frac{b_n^2 - b_p^2 + s_n^2}{2}\right](n_n - n_d)$$

where s_n is the skin thickness and b_n and b_p the neutron and proton surface widths.

The Compressible Liquid Drop Model (III)

Performing variations respect to n_n, n_p, n_d, n_{surf}, χ and R_p with the constraints of fix average density $n_B = \chi(n_n + n_p) + (1-\chi)n_d + n_{surf}$ and charge neutrality $n_e = \chi n_p$ one obtains the equilibrium conditions

$$\mu_n - \mu_p - \mu_e = \frac{8\pi}{5} e^2 R_p^2 n_p \left(1 - \frac{3}{2}\chi^{1/3} + \frac{1}{2}\chi\right) \quad (1)$$

$$P_i - P_o = \frac{2}{R_p}\left(\sigma_s + \frac{\sigma_c}{R_p}\right) - \frac{4\pi}{15} e^2 R_p^2 n_p^2 (1-\chi) \quad (2)$$

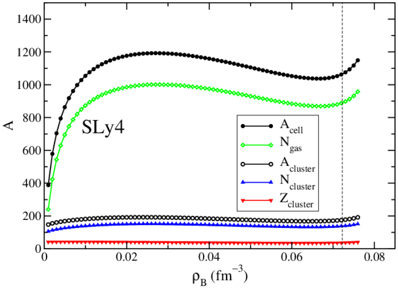
$$4\pi R_p^2 \left(\sigma_s + \frac{4\sigma_c}{R_p}\right) = \frac{32\pi^2}{15} e^2 R_p^5 n_p^2 \left(1 - \frac{3}{2}\chi^{1/3} + \frac{1}{2}\chi\right), \quad (3)$$

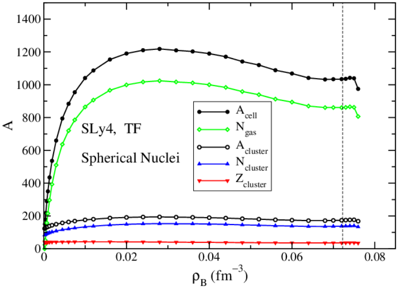
where $\mu_n = \partial\mathcal{E}(n_n, n_p)/\partial n_n$, $\mu_p = \partial\mathcal{E}(n_n, n_p)/\partial n_p$ and $\mu_e = \partial\mathcal{E}_{elec}(n_e)/\partial n_e$ are the neutron, proton and electron chemical potentials.

CLDM in other shapes

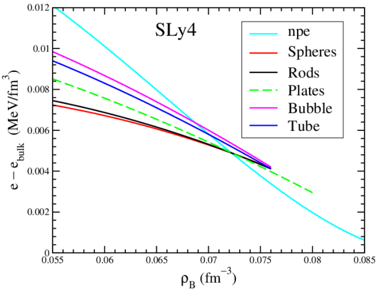
- The bulk nuclear and the electron contributions are shape independent.
- Surface and Coulomb energy densities can be written as
  $\varepsilon_{surf} = \frac{\chi d}{R}\left[(\rho_n - \rho_d)\mu_n s_n + \sigma\right]$
  $\varepsilon_{Coul} = \frac{4\pi}{5}(\rho_p e R)^2 f_d(\chi)$
  where d is the dimensionality and
  $f_d(\chi) = \frac{5}{d+2}\left[\frac{1}{d-2}(1 - \frac{1}{2}\chi^{1-2/d}) + \frac{1}{2}\chi\right] \qquad f_2 = \frac{5}{8}(ln\frac{1}{\chi} - 1 + \chi)$
- The virial theorem ($\varepsilon_{surf} = 2\varepsilon_{Coul}$) is valid for any phase.
  $P_i - P_0 = (d-1)\frac{\sigma}{R} + \frac{4\pi}{5}(\rho_p e R)^2 f_d(\chi)\left)\left[\frac{2}{d} + \chi\frac{f_d'(\chi)}{f_d(\chi)} - 1\right]$
- For holes (tubes and bubbles) one shall put a "-" sign in front of the surface term and replace χ by $1-\chi$.

CLDM versus TF

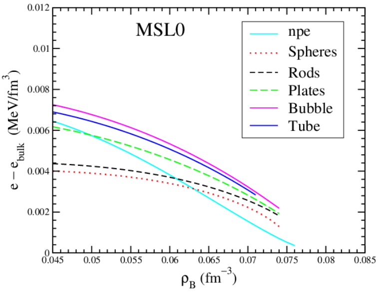




CLDM Predictions(I)

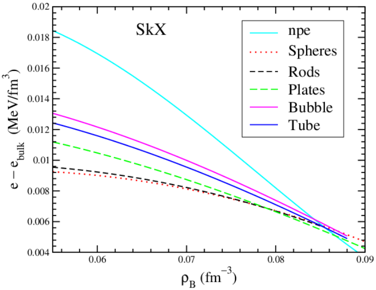


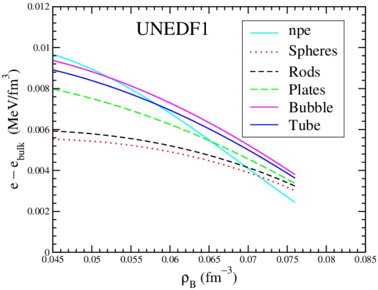
$\rho_t = 0.072$ fm^{-3}
$L = 45.96$ MeV



$\rho_t = 0.065$ fm^{-3}
$L = 60.00$ MeV

CLDM Predictions(II)





$\rho_t = 0.086$ fm^{-3}
$L = 33.19$ MeV

$\rho_t = 0.071$ fm^{-3}
$L = 40.01$ MeV

Conclusions (I)

- We have analyzed the shape phase transitions that appear in the inner crust of neutron stars using the Compressible Liquid Drop Model (CLDM) and the Thomas-Fermi (TF) approximation.
- These semiclassical methods allow to study in a rather simple way not only conventional spherical droplets but also another different geometries such as cylinders, plates, tubes (cylyndrical holes) and bubbles (spherical holes).
- The study of the different phases in the inner crust is very delicate from the numerical point of view due to the extremly small differences of the energy per baryon computed with the different geometries.
- Calculations performed with the CLDM are in good agreement with the predictions of the TF method.
- A detailed analysis of pasta phases in the inner crust allows to determine the transition density to a single liquid phase (core).
- Below a transition density $\rho_t \simeq 0.075$ fm^{-3} and slope of the symmetry energy $L \simeq 45$ MeV only droplets appear in the inner crust. Above these values of the transition density and the slope of the symmetry energy another pasta phases are also possible in the inner crust.

Reaching the crust-core interface fom the core side

- To find the inner edge of the neutron stars crust corresponding to the phase transition from homogeneous matter at high density to inhomogeneous matter at low density requires large scale calculations in the crust as the ones explained before.
- A well stablished approach is to search for the density at which the uniform liquid density becomes unstable against to small-amplitude density fluctuations, indicating the onset for the formation of nuclear clusters.
- There are basically two different methods, the thermodynamical method (S. Kubis PRC **70**,065804 (2004), **76**,035801 (2007)) and the dynamical method (Baym at al Ap.J **170**,299 (1971) (See Jun Xu et al ApJ **697**, 1549 (2009) for more details)

Methods for locaiting the inner edge of neutron star crust

a) Thermodynamical method

$$-\left(\frac{\partial P}{\partial v}\right)_{\mu} \qquad -\left(\frac{\partial \mu}{\partial q_c}\right)_{v},$$

where v and q_c are the volume and charge per baryon number $\mu = \mu_n - \mu_p$ and $P = P_b + P_e$ is the total pressure of the *npe* system.

$$\rho^2 \frac{\partial^2 E_b}{\partial \rho^2} + 2\rho \frac{\partial E_b}{\partial \rho} - \left(\frac{\partial^2 E_b}{\partial x_p \partial \rho}\right)\left(\frac{\partial^2 E_b}{\partial x_p^2}\right)^{-1} > 0$$

b) Dynamical method

$$v(q) = (\mu_{pp} + D_{pp} q^2 + \frac{4\pi e^2}{q^2}) - \frac{\frac{4\pi e^2}{q^2}}{\mu_{ee} + \frac{4\pi e^2}{q^2}} - \frac{(\mu_{pn} + D_{pn} q^2)^2}{\mu_{nn} + D_{nn} q^2} > 0,$$

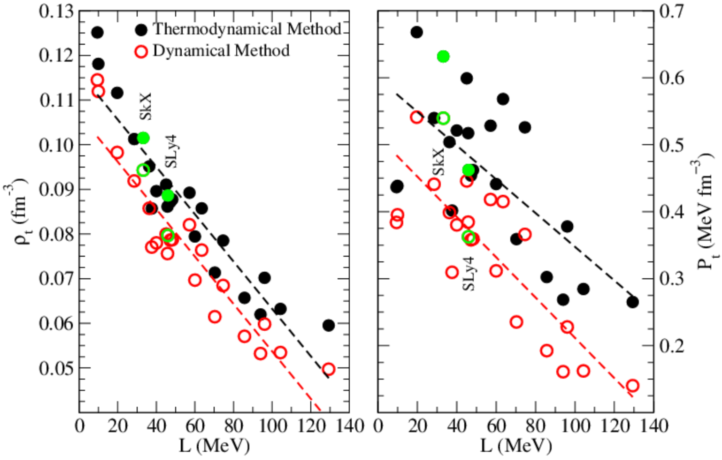
where $\mu_{xy} = \partial \mu_x / \partial \rho_y$, $D_{nn} = D_{pp} = \frac{3}{16}[t_1(1 - x_1) - t_2(1 + x_2]]$ and $D_{np} = D_{pn} = \frac{1}{16}[3t_1(2 + x_1) - t_2(2 + x_2]]$

Crust-core transition density and pressure

Force	ρ_{CLDM}	ρ_{TF}	$\rho_{thermal}$	ρ_{dynam}
SLy4	0.072 fm^{-3}	0.076 fm^{-3}	0.0886 fm^{-3}	0.0794 fm^{-3}
MSL0	0.065 fm^{-3}	—-	0.0795 fm^{-3}	0.0694 fm^{-3}
SkX	0.086 fm^{-3}	—-	0.1015 fm^{-3}	0.0942 fm^{-3}
UNEDF1	0.071 fm^{-3}	—-	0.0896 fm^{-3}	0.0771 fm^{-3}

Force	$P_{thermal}$	P_{dynam}
SLy4	0.4623 MeV fm^{-3}	0.3625 MeV fm^{-3}
MSL0	0.4412 MeV fm^{-3}	0.3117 MeV fm^{-3}
SkX	0.6318 MeV fm^{-3}	0.5396 MeV fm^{-3}
UNEDF1	0.5212 MeV fm^{-3}	0.3803 MeV fm^{-3}

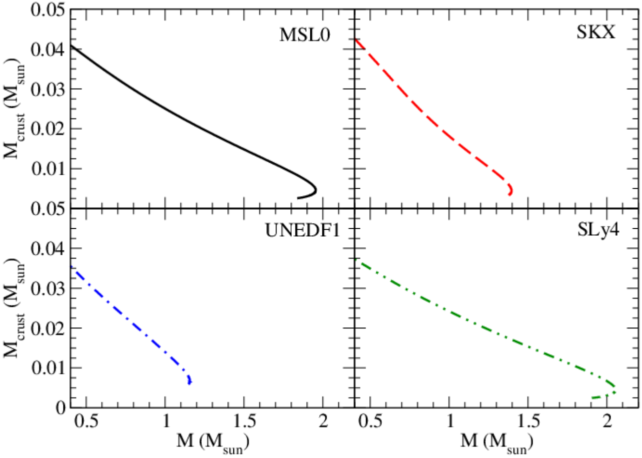
Transition density and pressure versus slope of the symmetry energy



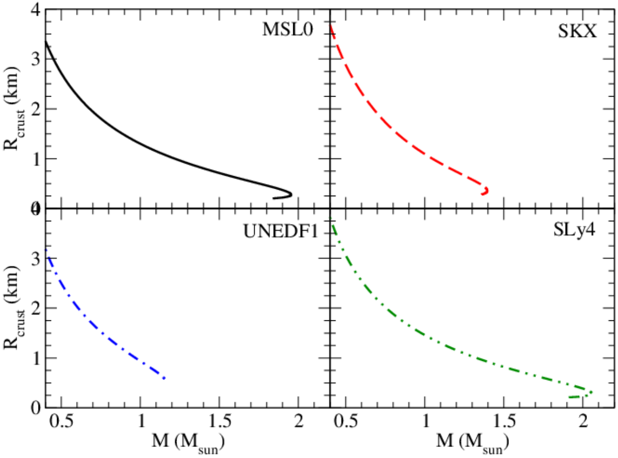
Global properties of the crust

- To determine the mass-radius relationship in a neutron star requires the knowledge of the EOS in the core but also in the crust.
- There are very few EOS computed in the crust and in the core with the same nuclear interaction.
- An approximated method, which avoids the explicit knowledge of the EOS in the crust, has been recently proposed by J.L.Zdunik, M.Fortin and P.Haensel A & A **599** A119 (2017)
- This method is based in neglecting the mass of the crust in front of the total mass of the star in the TOV equations in the region corresponding to the crust.
- Within this approximation the mass and size of the crust are given by analytical expressions that depend on the crust-core transition density and pressure as well as on the mass and radius of the core, which are obtained numerically by solving the TOV equations in this region of the neutron star.

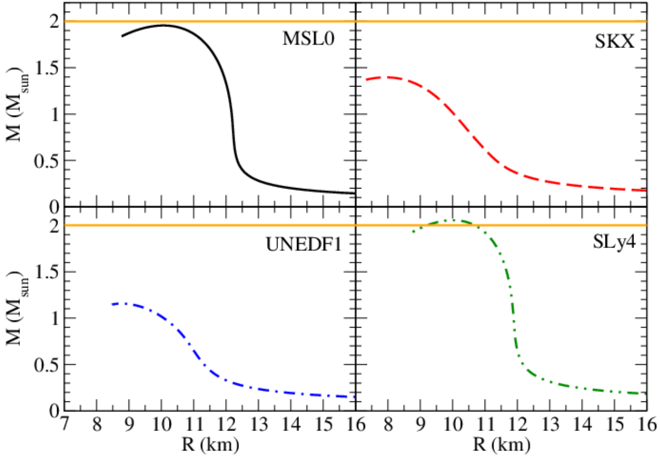
Mass of the crust



Size of the crust



Mass-Radius relationship



Conclusions (II)

- We have estimated the crust-core transition density and pressure using the so-called thermodynamical and dynamical methods.
- The crust-core transition densities estimated with the dynamical method agree within a 10% with the CLDM predictions.
- The crust-core transition densities and pressures follow a decreasing trend with increasing values of the slope of the symmetry energy. This decreasing trend is roughlly linear, which is more clear for the transition densities than for the transition pressures.
- These two properties of the crust-core transition are the input used in an approximation , recently proposed, which allows to estimate the mass and the size of the crust of the neutron stars without an explicit knowledge of the EOS in this region.
- The mass and size of the crust corresponding to a neutron star of maximum mass are roughly constant with values $\simeq$ 0.005 solar masses and $\simeq$ 300 m, respectively.