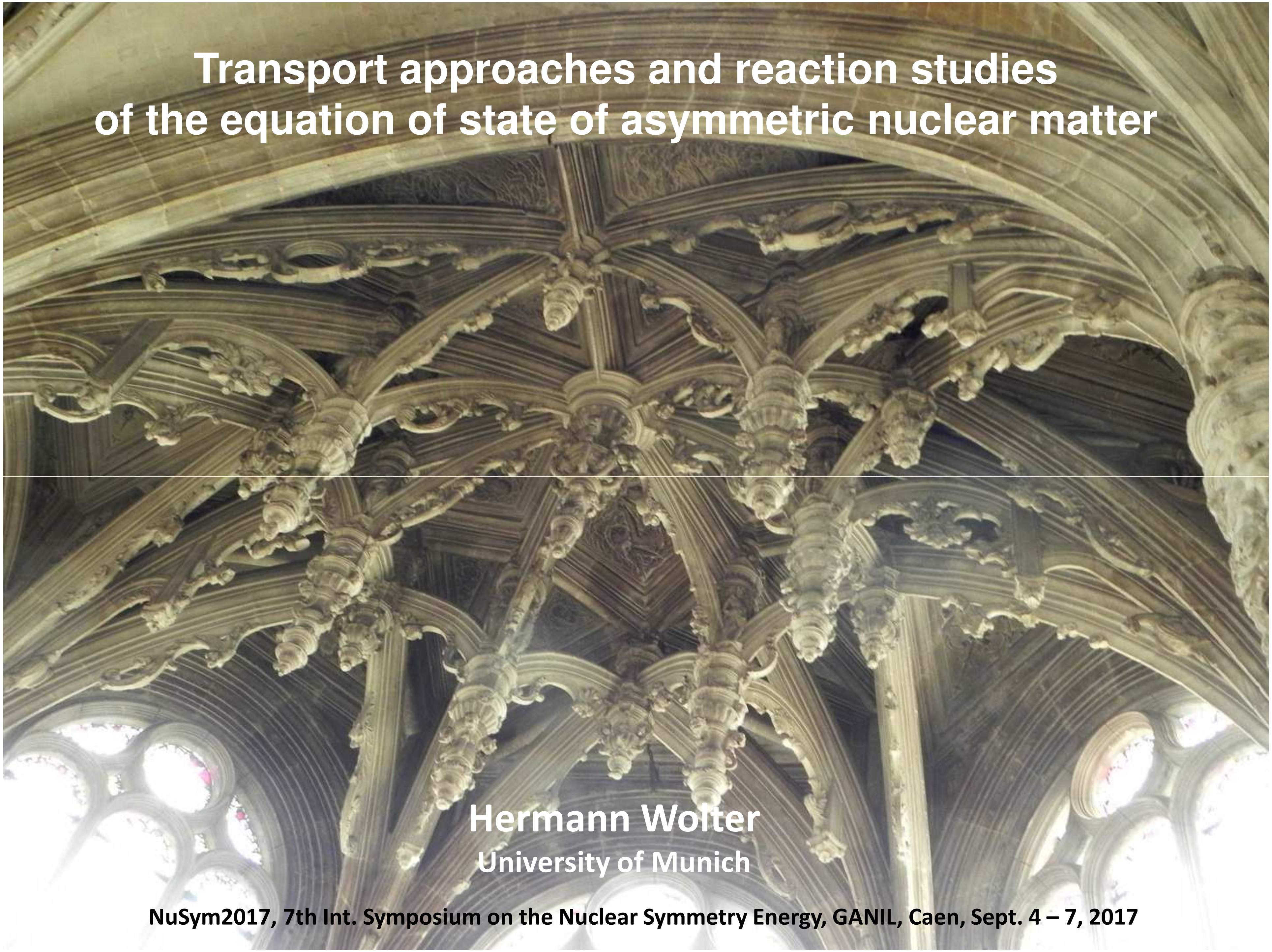
Transport approaches and reaction studies of the equation of state of asymmetric nuclear matter



Hermann Wolter
University of Munich

NuSym2017, 7th Int. Symposium on the Nuclear Symmetry Energy, GANIL, Caen, Sept. 4 – 7, 2017

A bit of history:

NuSYM2010 - RIKEN, Tokio, Japan

NuSYM2015 - IFJ/PAN, Krakow, Poland

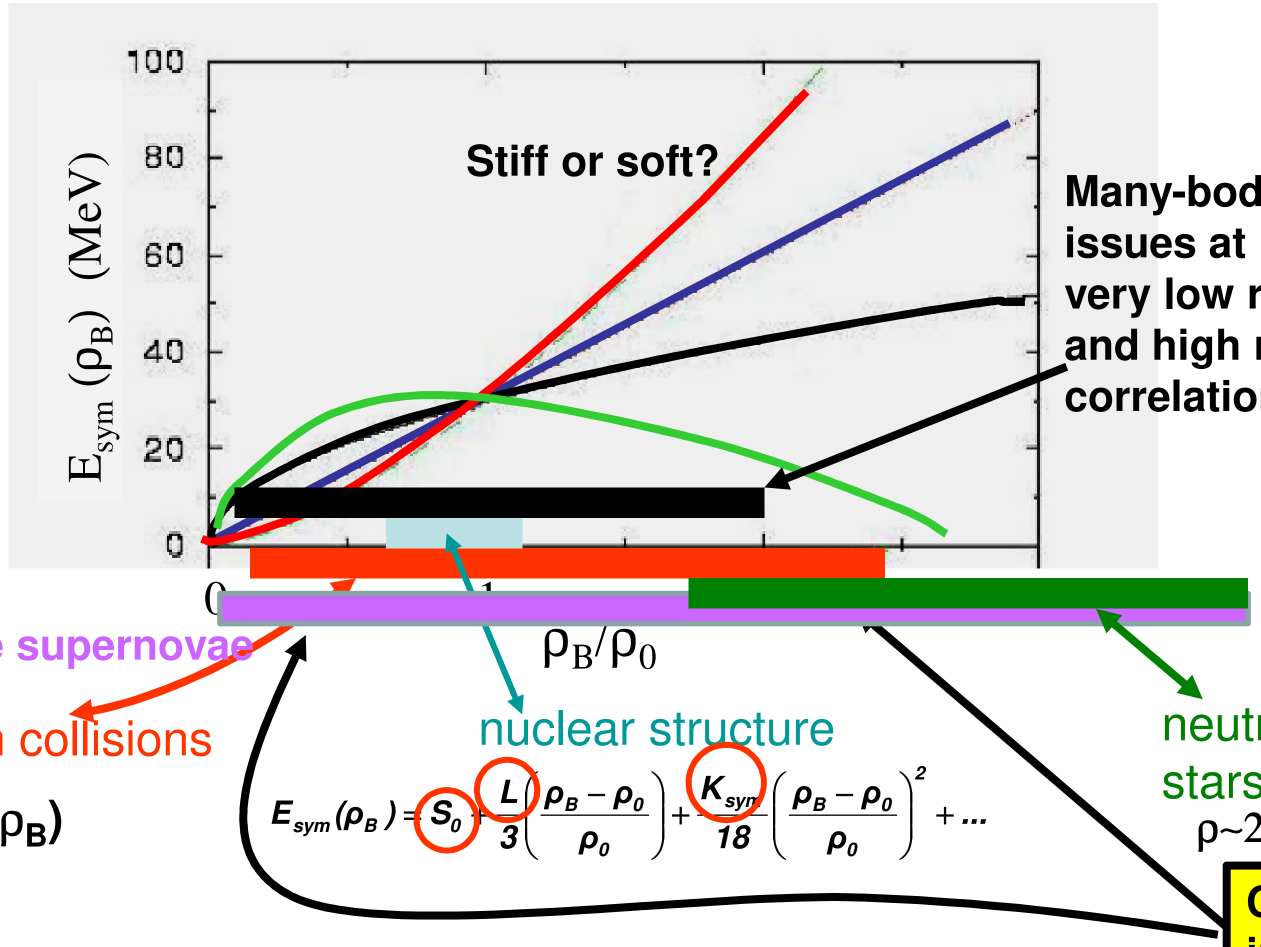
NuSYM2011 - Smith College, Northampton, MASS, USA
NuSYM2013 - NSCL/FRIB, East Lansing, MI, USA
NuSYM2014 - Univ. of Liverpool, Liverpool, UK
NuSYM2015 - IFJ/PAN, Krakow, Poland
NuSYM2016 - Tsinghua University, Beijing, China
NuSYM2017 - GANIL, Caen, France

The 7th year: the critical time,
but I am sure that this meeting will show
the big progress achieved since the beginning

The Search for the Nuclear Symmetry Energy

$$E(\rho_B,\delta)/A = E_{nm}(\rho_B) + E_{sym}(\rho_B)\delta^2 + O(\delta^4) + \dots$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$



ρ_B/ρ_0

Many-body calculations issues at very low r_B(clustering) and high r_B (short range correlations

Core collapse supernovae

heavy ion collisions

nuclear structure

neutron stars
ρ~2–4 ρ_0

$$E_{sym}(\rho_B) = \boxed{S_0} + \frac{\boxed{L}}{3}\left(\frac{\rho_B-\rho_0}{\rho_0}\right) + \frac{\boxed{K_{sym}}}{18}\left(\frac{\rho_B-\rho_0}{\rho_0}\right)^2 + \dots$$

Cosmic matter in the laboratory

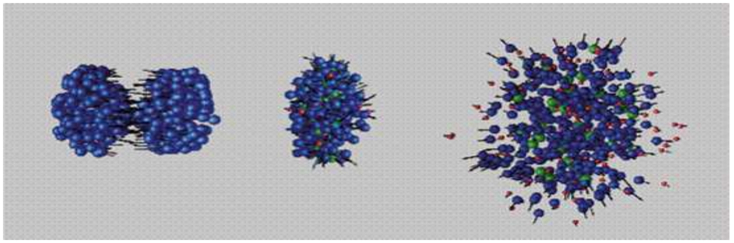
$E_{sym}(\rho_B)$

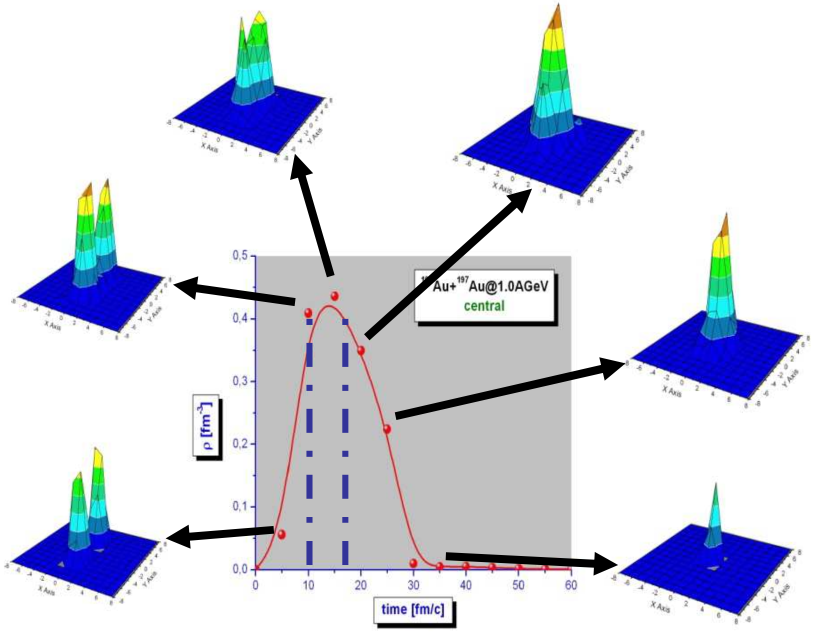
↑

→ Introductory talk about transport calculations to learn about the symmetry energy
→Issues in application of transport theories

Heavy ion collisions as non-equilibrium processes

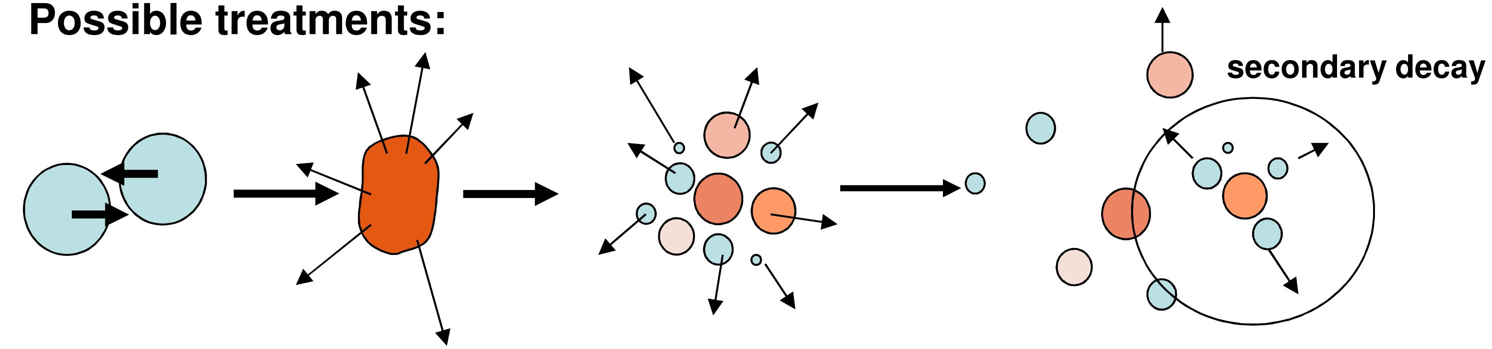
evolution in coordinate space





evolution in momentum space: non-sphericity

Possible treatments:



transport
U(ρ,p), σ
quasi-particle picture
full non-equilibrium

hydro-dynamics
EoS ε(P)
local equilibrium

statistical decay
$V_{freeze-out}$, E_{exc},A,Z
global equilibrium

used in the hadronic regime

Transport theory based on a chain of approximations

Martin-Schwinger hierarchy in many-body densities, real time formalism
truncation, introduction of self energies (1-body quantities),
irreversability

Quantum transport theory: Kadanoff-Baym theory

Semiclassical approximation :
Wigner transform, treat as phase space probabilities
Gradient approximation (separation of short and long scales)

Quasi-particle approximation
Spectral function→ delta function with effective momenta and masses
neglect off-shell effects (or treat approximately)

→ kinetic equation

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \left(\vec{\nabla}^{(r)}\boxed{U(r,p)}\,\vec{\nabla}^{(p)} + \vec{\nabla}^{(p)}\boxed{U(r,p)}\,\vec{\nabla}^{(r)}\right) f(\vec{r},\vec{p};t) =$$

$$\int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'}\, v_{21}\, \boxed{\sigma_{12}^{in-med}(\Omega)}\, (2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} \boxed{\bar{f}_1 \bar{f}_2} - f_1 f_2 \boxed{\bar{f}_{1'} \bar{f}_{2'}} \right]$$

Pauli blocking factors,
main quantum ingredienet $\bar{f}_i := (1 - f_i)$

Mean field evolution (Vlasov) + uncorr. 2-body collisions (Boltzmann)
+ Pauli-blocking of final states (Uehling-Uhlenb)

physical input:
mf potential U(r,p), momentum dependent
$\sigma^{in\text{-}med}$ in-medium cross sections

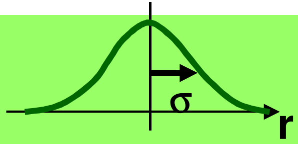
Two main transport approaches

Boltzmann-Vlasov-like (BUU/BL/BLOB)

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}-\vec{\nabla}U(r)\,\vec{\nabla}^{(p)}\right)f(\vec{r},\vec{p};t) = I_{coll}[\sigma^{in-med},f_i]$$

Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = A\prod_{i=1}^{A}\varphi(r;r_i,p_i)|0\rangle$$



$$\dot{r}_i=\{r_i,H\};\quad \dot{p}_i=\{p_i,H\};\quad H=\sum_i t_i+\sum_{i,j}V(r_i-r_j)$$

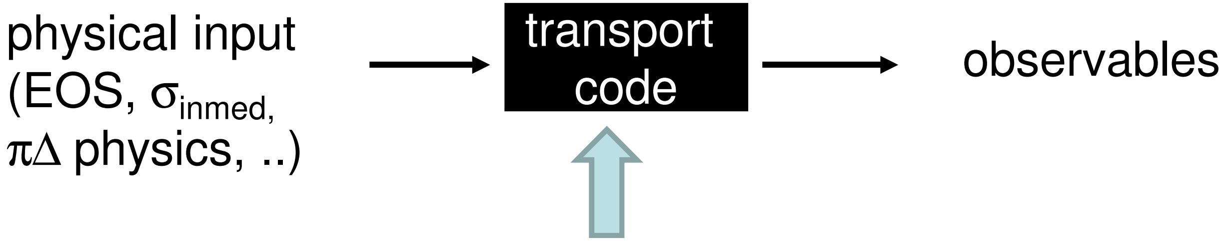
Dynamics of the 1-body phase space distribution function f with 2-body dissipation

TD-Hartree(-Fock (AMD))
(or classical molecular dynamics with extended particles)
plus stochastic NN collisions

6-dim integro-differential, non-linear eq.

6A-dim many body problem

→ very complex, simulate solutions
introduces many technical details



physical input
(EOS, σ_{inmed},
$\pi\Delta$ physics, ..)

transport code

observables

→ unique? code independent?

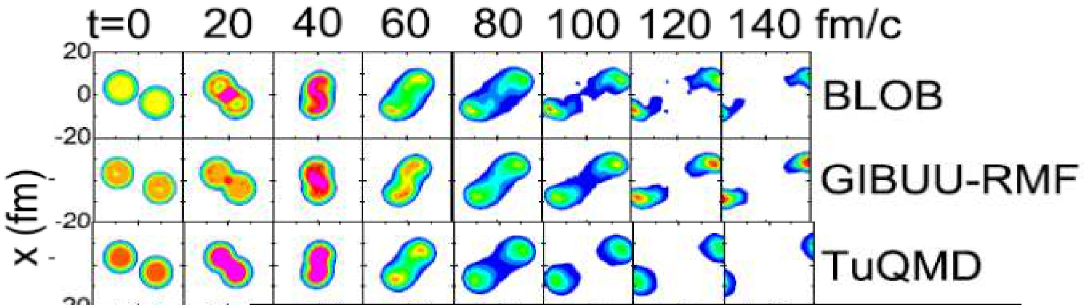
→Transport Code Evaluation (Comparison) Project

Code Comparison Project (1st stage):

check consistency of transport codes in calculations with same system (Au+Au), E=100,400 AMeV, identical physical input (mean field (EOS) and cross sections, BUU and QMD codes

idea: establish sort of theoretical systematic error of transport calculations
(and hopefully to reduce it)
published: J. Xu, et al. (31 authors); PRC 93, 044609 (2016)

HIC at b=7m (midcentral)
selected contour plots;
different evolution apparent
→compare collision numbers,
blocking, and observables

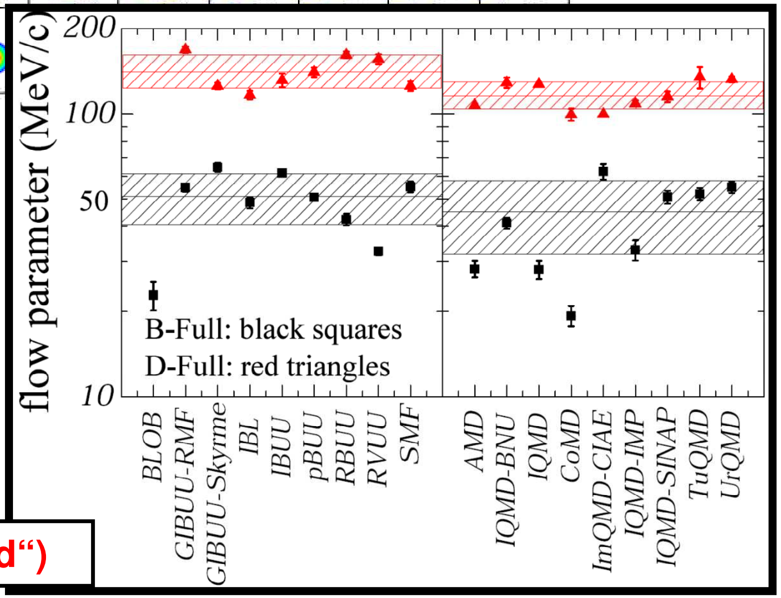


quantify spread of simulations by value of „flow“=slope of transverse momentum at midrapidity

BUU and QMD approx. consistent

uncertainity

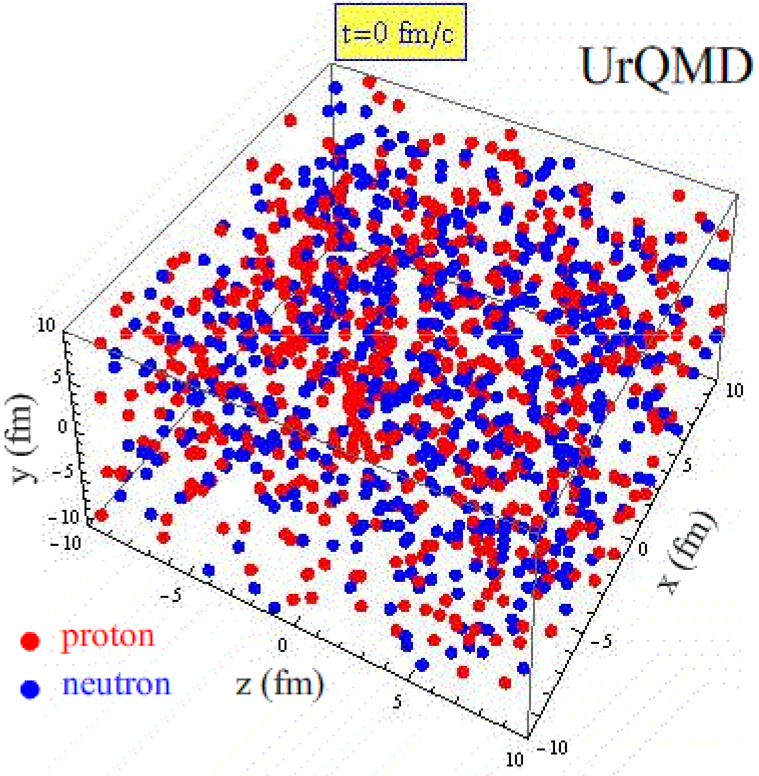
100 AMeV: ~30%
400 AMeV: ~13%



Understand differences better (effects „entangled“)

2nd stage: Box calcualation comparison

simulation of the static system of infinite nuclear matter,
→ solve transport equation in a periodic box



Useful for many reasons:

- check consistency of calculation
  e.g. EoS energy dens ε vs. pressure P
- check consistency of simulation:
  collision numbers, blocking
  (exact limits from kinetic theory)
- check aspects of simulation separately
  Cascade: only collisions
  without/with blocking
  Vlasov: only mean field propagation
- check ingredients of particle production
  e.g. pion production

→Code comparison in box calculations
→ session tomorrow morning
(Jun Xu, YX Zhang, Maria Colonna, Akira Ono)
→ Workshop on Friday

I will use some results to illustrate some points

Transport and Symmetry Energy

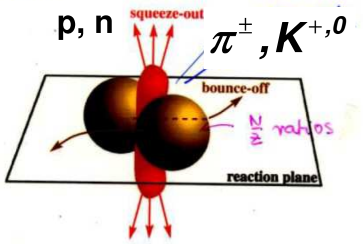
Coupled transport eqs. for neutrons and protons, τ=n,p

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}-\vec{\nabla}^{(r)}U(r,p;\delta,\tau)\vec{\nabla}^{(p)}-\vec{\nabla}^{(p)}U(r,p;\delta,\tau)\,\vec{\nabla}^{(r)}\right)f_\tau(\vec{r},\vec{p})$$
$$=\sum_{\tau'}I_{coll}[\sigma_{\tau\tau'},f_\tau,f_{\tau'}];\quad \tau=n,p$$

Momentum dependence of symmetry potential (isoscalar and isovector effective mass)

$$U(r,p;\delta,\tau)=U_0(r,p)+\underbrace{U_{sym}(r,p)(\delta\tau)+\ldots}_{\text{small (\textasciitilde10\%) relative to isoscalar}} \qquad \frac{m^*_\tau}{m}=\left(1+\frac{m}{p}\frac{\partial U_\tau}{\partial p}\right)^{-1}$$

„Primary" observables



E.g. „flow" (Fourier decomp. of azimuthal momentum distrib.)

$$\boxed{N_\tau(\Theta;y,p_t)=N_0(1+v_1^\tau(y,p_t)\cos\Theta+v_2^\tau(y,p_t)\cos2\Theta+\ldots}$$

v_1: directed flow **v_2: elliptic flow**

discuss p/n ratios or differences of observables
→ sometimes influenced by final state effects

$$v_i^n/v_i^p;\quad v_i^n-v_i^p$$

„Secondary" observables
particle production, e.g. pions, strangeness

$$\frac{\rho_n}{\rho_p}\downarrow\Rightarrow\frac{Y(\Delta^{0,-})}{Y(\Delta^{+,++})}\downarrow\Rightarrow\frac{\pi^-}{\pi^+}\downarrow$$

$$\begin{tikzcd}
NN \arrow[r] & N\Delta \arrow[dr, leftrightarrow] \arrow[ddrrr] & & & \\
 & & N\pi \arrow[r] & \Lambda K & \\
 & & & & N\Lambda K
\end{tikzcd}$$

Can be more sensitive to high density region

The Symmetry Energy at High Density

Au+Au @ 400 AMeV
new experiment ASY-EOS
(Russotto,NuSYM 2015, Krakow; submitted PRC)

$$N(\Theta; y, p_t) = N_0(1 + v_1 \cos\Theta + v_2 \cos 2\Theta + \ldots)$$

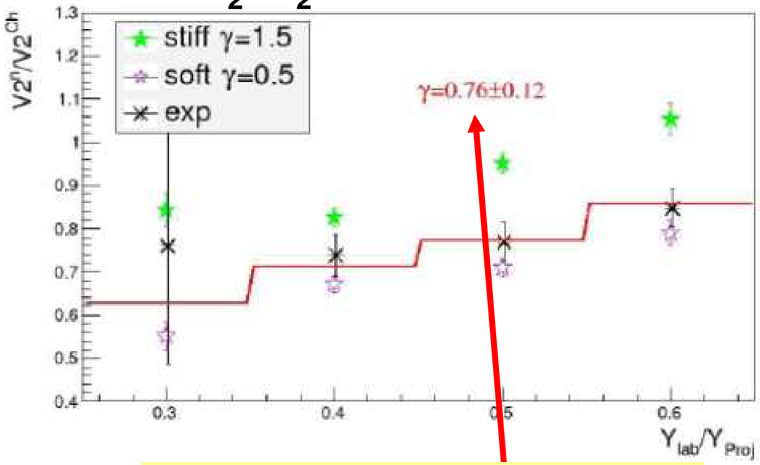
ratio of neutron to hydrogen flow
- Elliptic flow v_2 in this energy region good probe of high density

analysis of density dependence of SE in terms of power law exponent γ

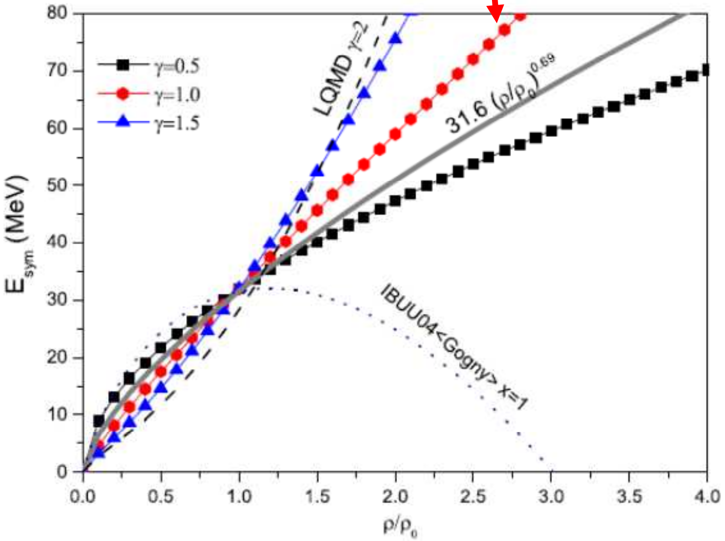
$$E_{sym}(\rho) = \tfrac{1}{3}\varepsilon_F\left(\frac{\rho}{\rho_0}\right)^{2/3} + C\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Big step forward in constraining the high-density symmetry energy
→ See talk by Paolo Russotto

ratio $v_2^n/v_2^{charged}$



not very precise (yet) but indicates rather stiff SE, γ~1



Beyond dissipative mean field dynamics: Fluctuations

Boltzmann-Vlasov-like (BUU/BL/BLOB)

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}-\vec{\nabla}U(r)\vec{\nabla}^{(p)}\right)f(\vec{r},\vec{p};t)$$
$$=I_{coll}[\sigma^{in-med}]+\delta I_{fluct}$$

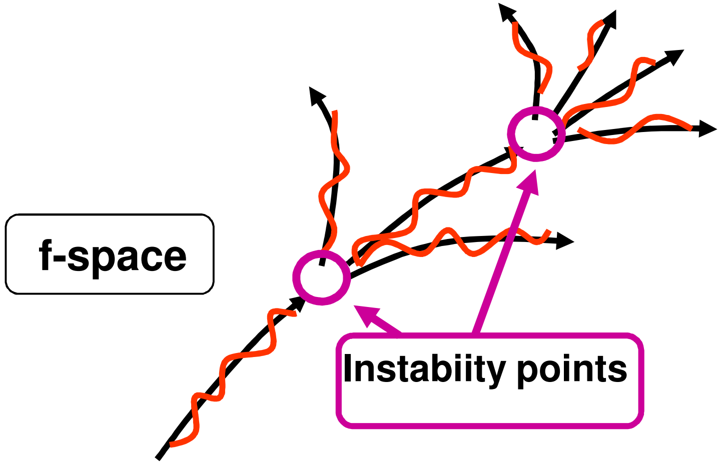
Dynamics of the 1-body phase space distribution function f with 2-body dissipation

fluctuations around diss. solution

$$f(r,p,t)=\bar{f}(r,p,t)+\delta f(r,p,t)$$

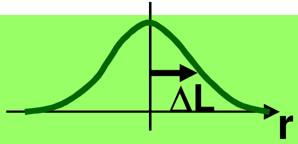
$$\boxed{\frac{df}{dt}=I_{coll}+I_{fluc}}$$

Boltzmann-Langevin eq.



Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = A\prod_{i=1}^{A}\varphi(r;r_i,p_i)|0\rangle$$



$$\dot{r}_i=\{r_i,H\};\quad \dot{p}_i=\{p_i,H\};\quad H=\sum_i t_i+\sum_{i,j}V(r_i-r_j)$$

TD-Hartree(-Fock)
(or classical molecular dynamics with extended particles)
plus stochastic NN collisions

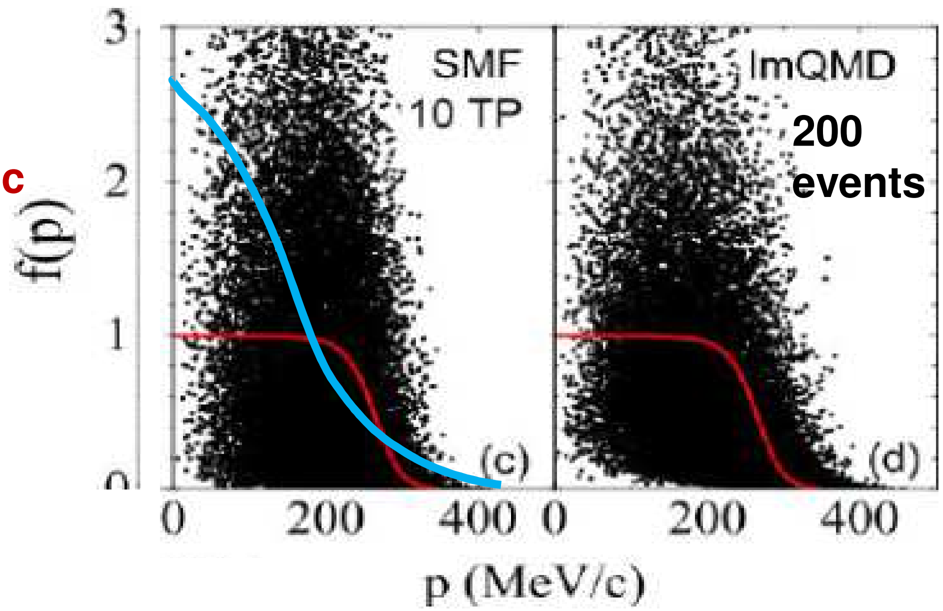
No quantum fluctuations,
but classical N-body fluctuations,
damped by the smoothing.

However, more fluctuations than BUU, since dof are nucleons and not test particles:
→ more fluctuations in representation of phase space distribution
→ more fluctuation gained from collision term
→ amount controlled by width of single particle packet ΔL

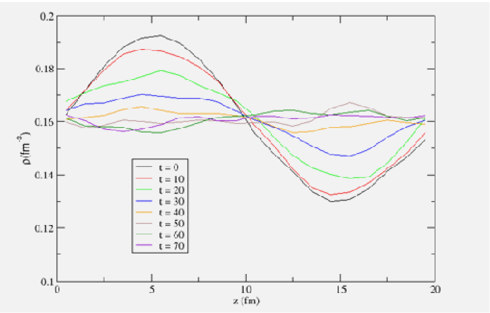
Examples of fluctuation from box calculations:

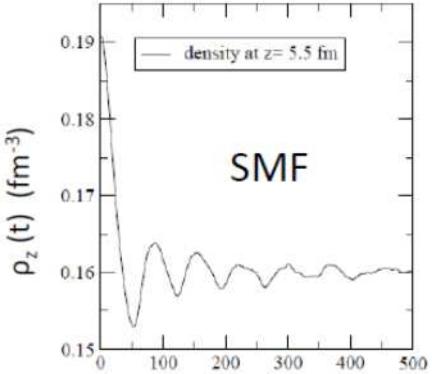
(1) calculation of the occupation prob.
for an initialized system at T=5 MeV
large fluctuations arounf Fermi-Dirac

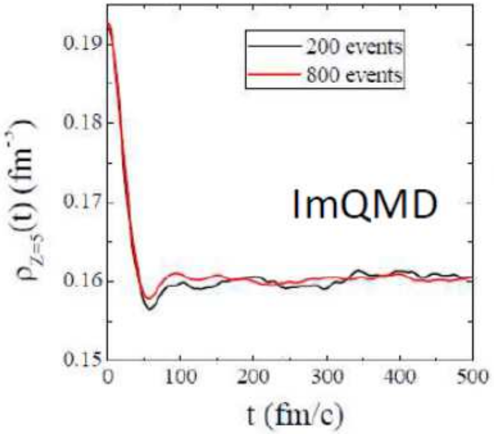
Consequence:
instability of fermionic system,
Fermi-Dirac→Maxwell-Boltzmann
→ talk by Yingxun Zhang tomorrow



(2) Evolution of a standing wave in time







$$\rho(z,t=t_0)= \rho_0 + a_\rho \sin(k_i z)$$

$$k=2\pi/L$$

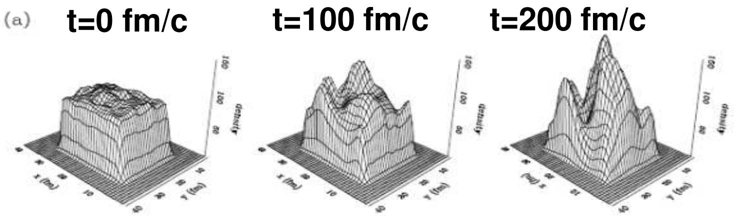
→ talk by Maria Colonna tomorrow

Importance of fluctuations

(1) Formation of fragments:
intermediate mass fragments (IMF) (5≥A≥~30)
develop from fluctuation as seeds
which are amplified by the mean field

issue: correct amplitude and spectrum of fluctuations

BUU calculation in a box with initial conditions inside the instability region: $\rho=\rho_0/3$, T=5 MeV, $\delta=0$
(V.Baran, et al., Phys.Rep.410,335(05))



BUU: fluctuations around dissipative solution, approx. to BL

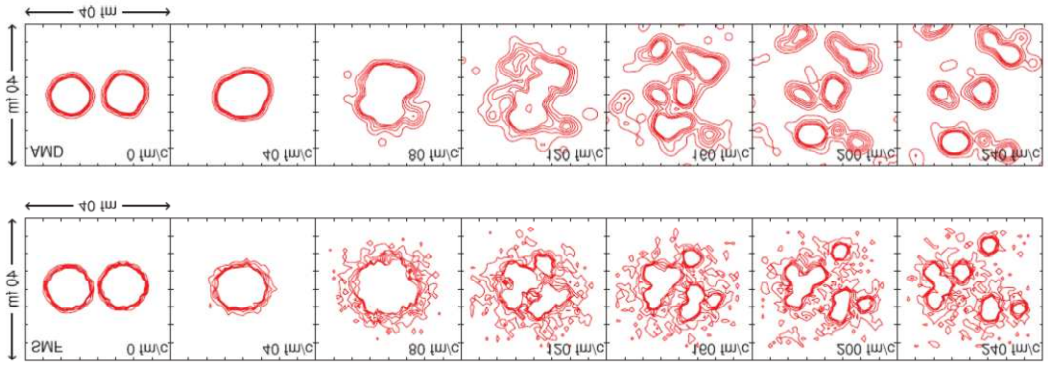
$$f(r,p,t) = \bar{f}(r,p,t) + \delta f(r,p,t)$$

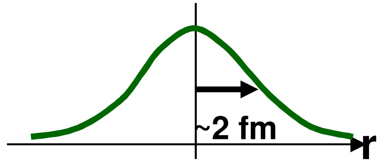
$$\boxed{\frac{df}{dt} = I_{coll} + I_{fluc}}$$

δf taken from statistical fluctuation in a Fermi system $\sigma_f^2(r,p) = \bar{f}(r,p)(1-\bar{f}(r,p))$

SMF (stochastic mean field): project on density fluctuations
BLOB (Boltzmann-Langevin One-Body dynamics) Move N_{TP} testparticles simultaneously (in p-space) to simulate fluctuation connected to NN collisions (→talk by Napolitani)

QMD: fluctuations controlled by wave packet width ΔL:

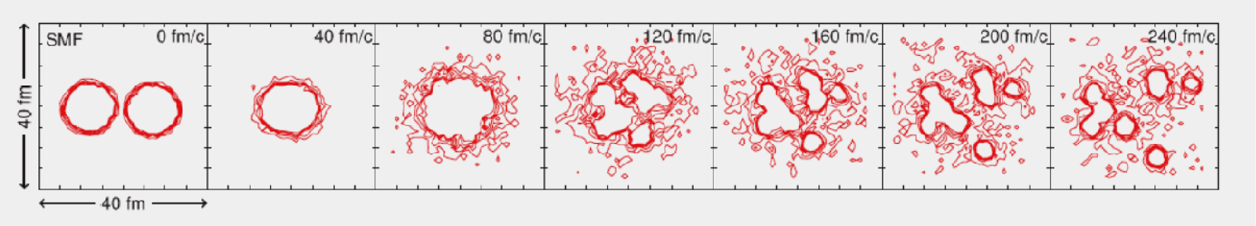




Comparison of simulations:
BUU(SMF)-AMD:
(Rizzo, Colonna,Ono, PRC82 (2010))

(2) Influence dynamics (e.g. via blocking (see box calc.) or via classical correlations)
→ less well understood in HIC

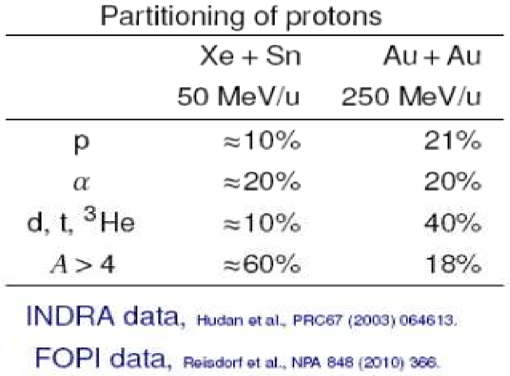
Correlations: Light clusters (LC, A≤4) in transport



Clusters (A≤4) (correlation stabilized)

Fragments (IMF, A>4) (mean field stabilized)

large fractions of particles in clusters, e.g.



Partitioning of protons

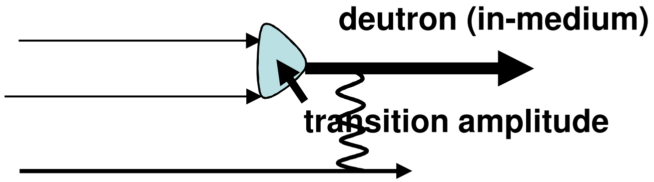
	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	≈10%	21%
α	≈20%	20%
d, t, ^{3}He	≈10%	40%
$A>4$	≈60%	18%

INDRA data, Hudan et al., PRC67 (2003) 064613.

FOPI data, Reisdorf et al., NPA 848 (2010) 366.

LC: correlation dominated

(common density functionals are not sophisticated enough to describe LC properly)
→ introduce explicitely, LC medium modified !

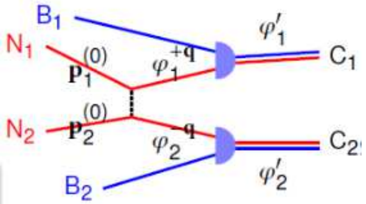


deutron (in-medium)

transition amplitude

BUU: Introduce LCs as explicit degrees of freedom formed in 3-body colisions

(P. Danielewicz and Q. Pan, PRC 46 (1992)) (d,t,3He, but no α!) → see talk of Pawel

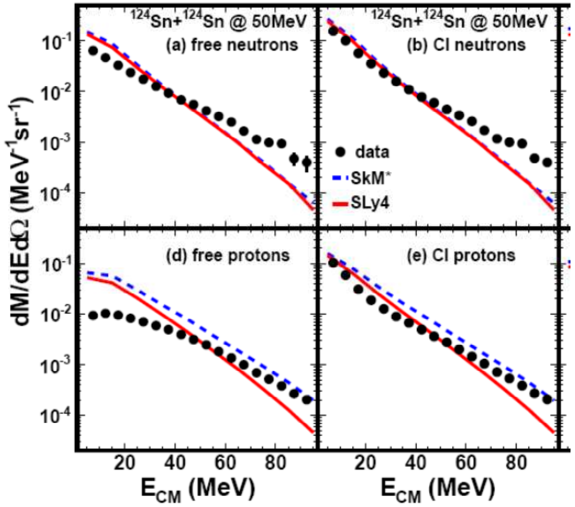


QMD: classical correlations (if 2-body interaction employed)

AMD: in collision determine overlap with cluster wave function
rearrange nucleons into cluser, but propagate as nucleons
cluser-cluster collisions for heavier clusters → see talk by Akira

Light cluster description in transport approaches: Comparison to data

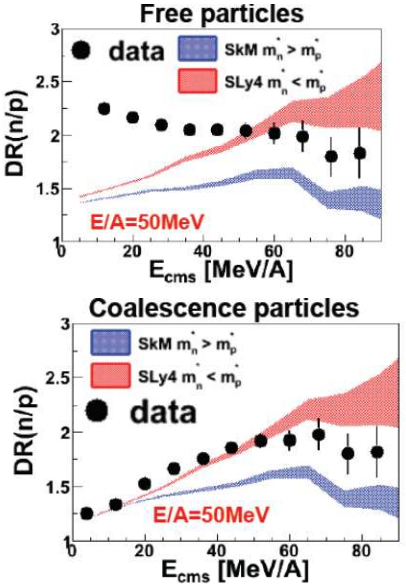
p,n spectra: free coalescence invariant (CI)



Double Ratios

$$\frac{^{124}Sn + {}^{124}Sn}{^{112}Sn + {}^{112}Sn}$$

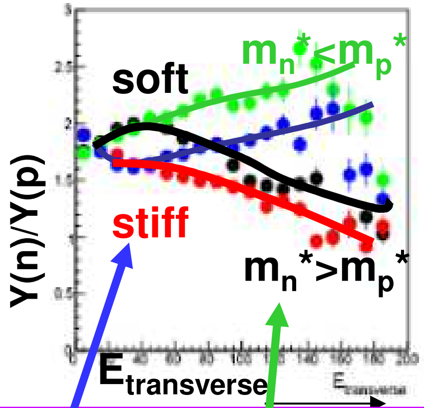
agree only for CI spectra



Y.X. Zhang, M.B.Tsang,et al., PLB 732, 186 (2014)
D.D.S.Coupland, arXiv 1406.4546

Role of effective mass in Light Cluster production

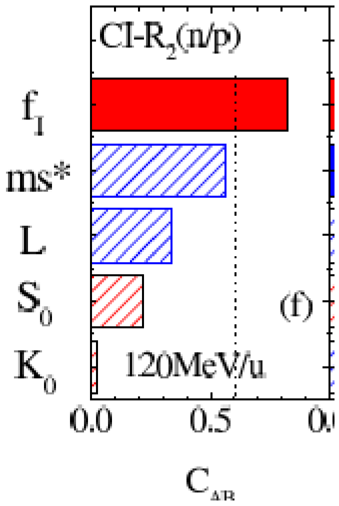
136Xe+124Sn, 150 MeV



H. H. Wolter, et. al,
EPJ WoC, 03097 (2014)

density dep. dominates for slow particles;
mom.dep. (effective mass) for fast particles,

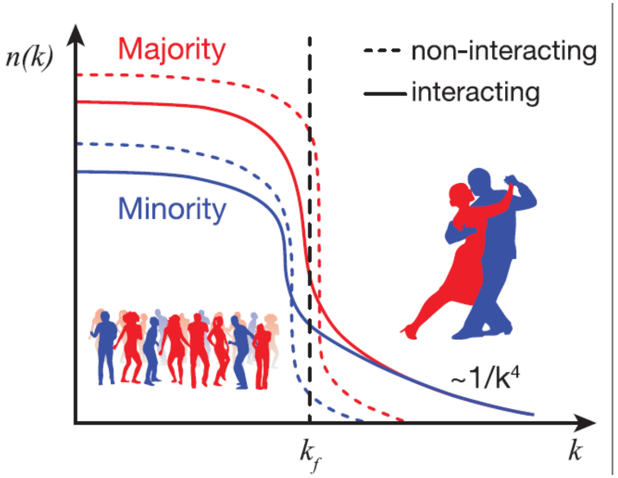
Correlation analysis of five parameters S_0, L, K_0, m_s^*, $f_I \sim m_V^*$ to fit of coalescence-invariant single ot double n/p ratios (Sn+Sn, 120 AMeV)
→ dominantly isovector effective mass



Y. Zhang et al. / Physics Letters B 749 (2015) 262–266

Role of Short-Range-Correlations

Well known from many-body calculations from infinite nuclear matter, that momentum distribution has high energy tail due to short range correlations.Recently confirmed by experiment from Jlab (O.Hen, et al.,Science 346, 6209 (14)). In asymmetric nuclear matter, this is different for neutrons and protons, for $k<k_F$, but similar for $k>k_F$.



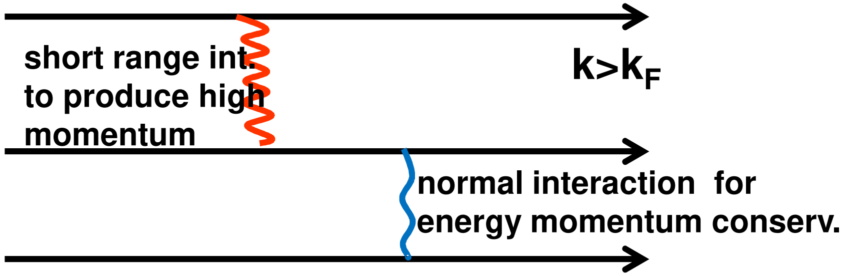
This could be important in HIC in particle production (threshold effect)
Current debate, how to take into account in transport:

1. Initialize momentum distribution with high momentum tail, e.g. GC Yong, PLB 765 (2017) 104
   Should be quickly lost due to colliusions and is not regenerated, which should be the main effect.
2. Subtract correlation energy from mean field potential, e.g. B.A.Li+, PRC 91, 044601 (2015).

   $$E_{\mathrm{sym}}(\rho)=\eta\cdot E_{\mathrm{sym}}^{\mathrm{kin}}(\mathrm{FG})(\rho)+\left[S_0-\eta\cdot E_{\mathrm{sym}}^{\mathrm{kin}}(\mathrm{FG})(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

   argument: determination of U_{sym} more realistic, since correlation energy not assumed as symmetry energy.
   but: but does not affect kinetic energy and produces no high momentum tails

3.? Treat explicitely in 3-body collision, in a sense similar to problem of LC production.



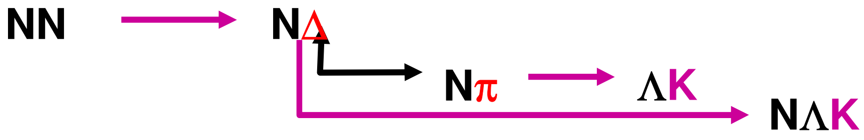
short range int. to produce high momentum

$k>k_F$

normal interaction for energy momentum conserv.

Particle Production

Inelastic collisions: Production of particles and resonances, Coupled transport equations



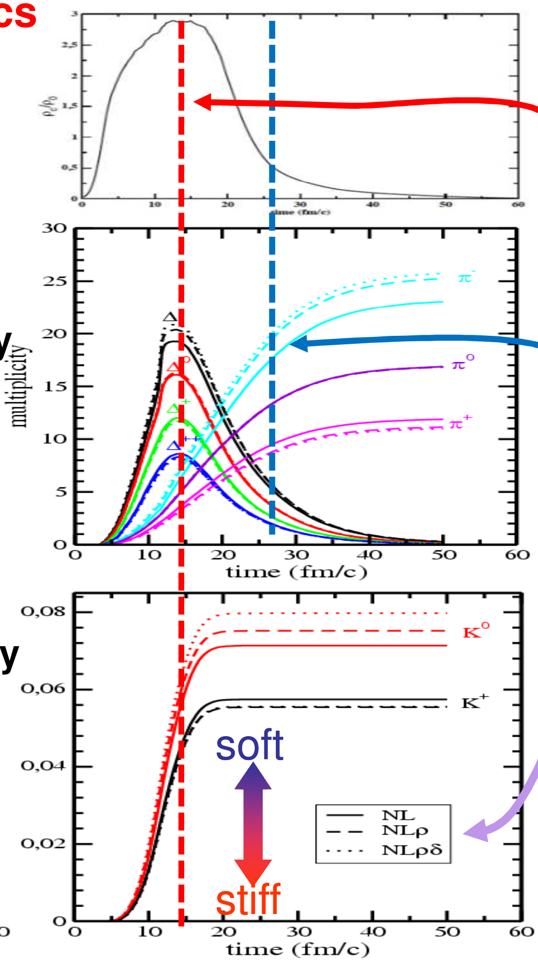
Many new potentials, elastic and inelastic cross sections needed, Δ dynamics in medium

Dynamics

Central density

π and Δ multiplicity

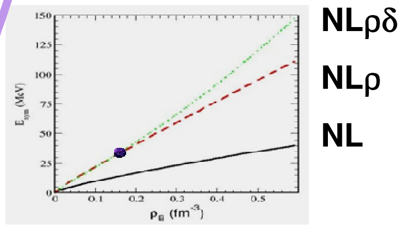
$K^{0,+}$ multiplicity



Δ and K: production in high density phase

Pions: low and high density phase

Sensitivity to asy-stiffness



G. Ferini et al., Nucl. Phys. A 762, 147 (2005)

Dependence on asy-stiffness

n/p ratio

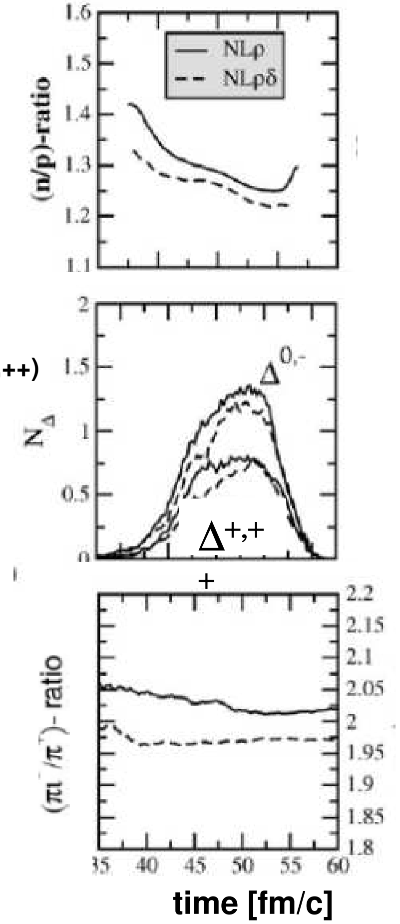
↓

$Y(\Delta^{0,-})$ vs. $Y(\Delta^{+,++})$

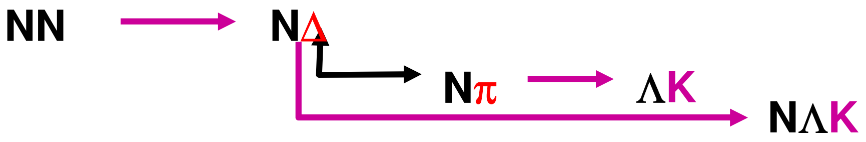
↓

π^-/π^+

Au+Au, 0.6AMeV



(π,Δ,K) dynamics: many new physical ingredients


NN → NΔ

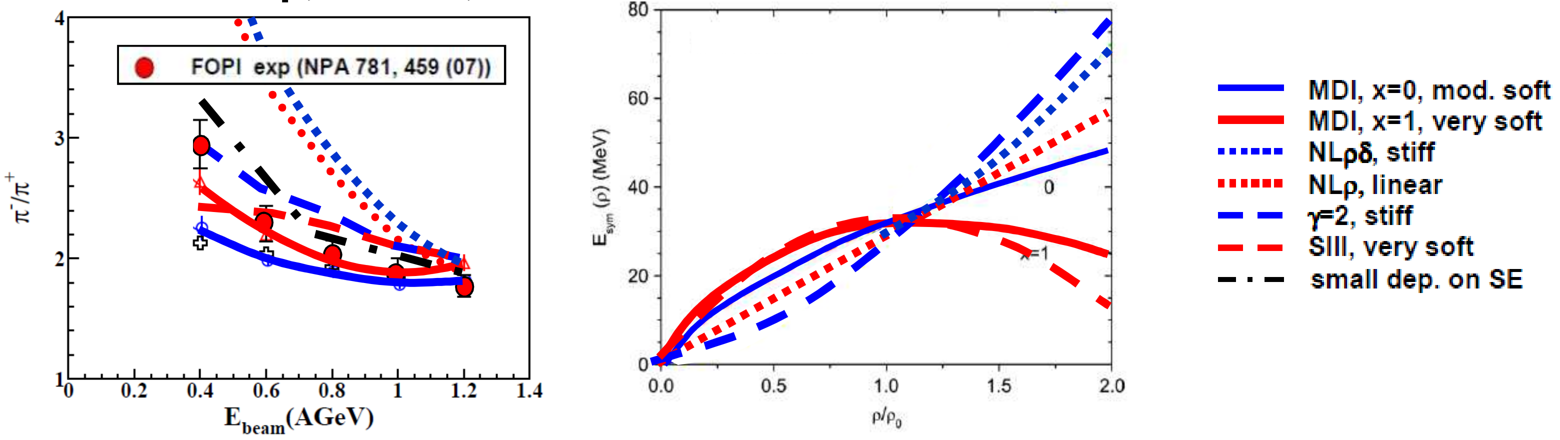
NΔ → Nπ → ΛK

NΔ → NΛK

- → potentials U_π, U_Δ, U_K → effective masses in medium, threshold effects
- → inelastic cross sections, e.g. NN→NΔ, in medium
- → Δ Resonances with decay widths, mass distributions,spectral fcts, more general: off-shell transport
- → talks by C.M. Ko, P. Danielewicz, Jun Xu, …

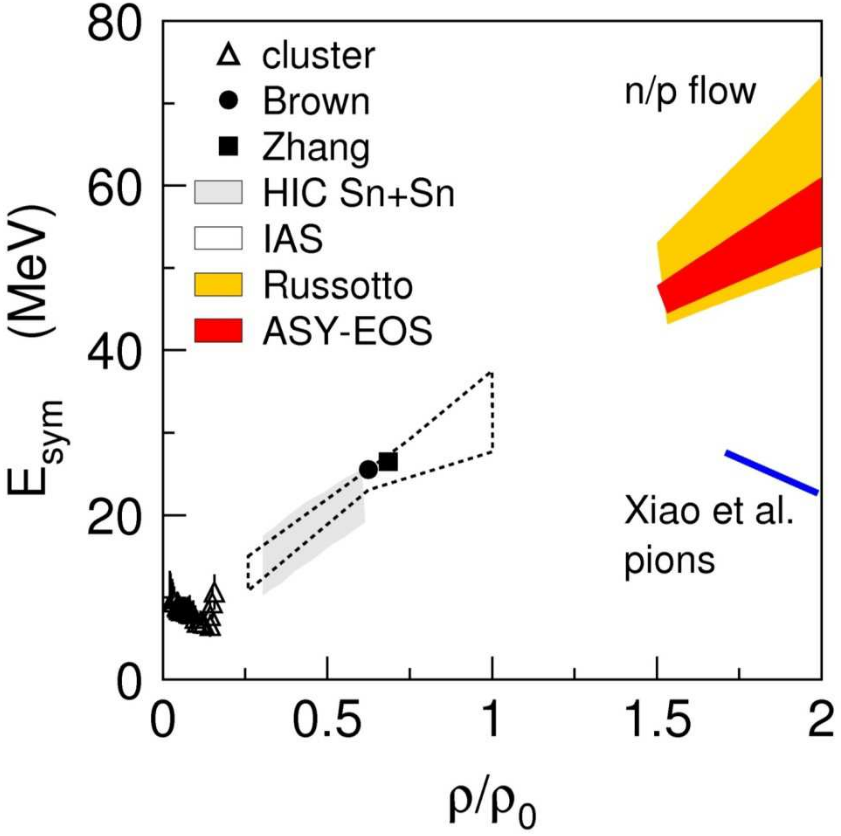
present situation unsatisfactory:

FOPI exp, Au+Au, 0.4-1.2 GeV/A



- → check of (π,Δ) physics in box calculations, talk by Akira Ono tomorrow
- → more sensitivity in spectral distributions, Sπrit experiment, → talk by Betty Tsang,etc
- → reconsider Kaons

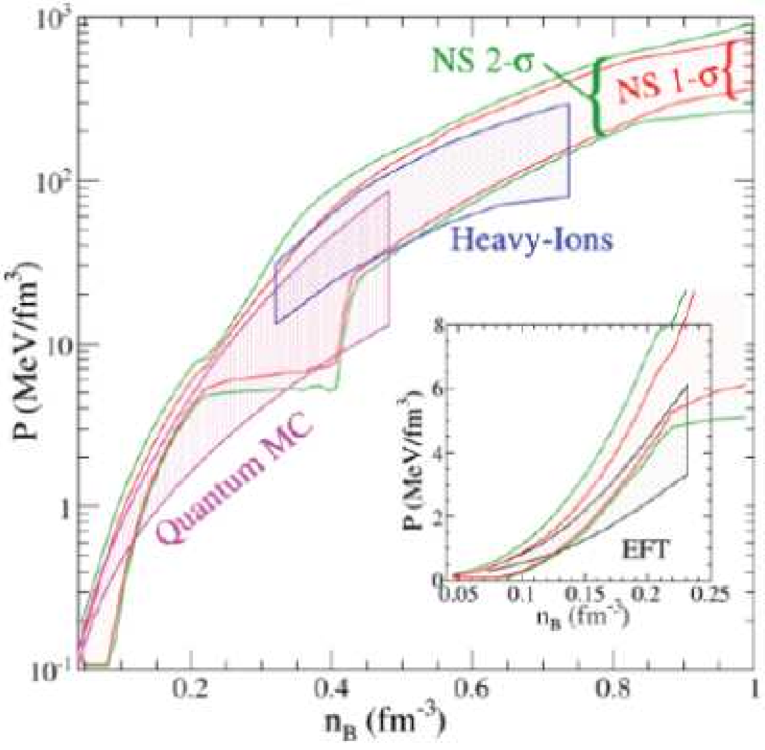
Constraints from nuclear struture and heavy ion collisions



P. Russotto et al., Phys. Rev. C 94, 034608 (2016).

Synopsis of constraints from neutron stars, HIC and microscopic calculations

(for neutron star matter, i.e. β-equilibrium)



A. Steiner, J. Lattimer, E.D. Brown, APJLett 765 (2013)

SUMMARY:

Equation-of-State (EoS) of nuclear matter of interest in itself
and important input for astrophysics:
Core Collapse Supernova, Neutron star structure, nucleosynthesis)

Investigation of EoS in the laboratory in Heavy Ion Collisions
Interpretation in complex transport models: theoretical questions
- treatment of fluctuation and correlations
to describe fragment and light cluster production
- treatment of short range correlations in transport
- check consistency of transport approaches (code comparison)
-treatment of instable particles (e.g. Δ)
Generally transport approaches are on a firmer footing today

EoS of symmetric nuclear matter ($\rho_n=\rho_p$) fairly well determined,
but symmetry energy is area of very active investigations experimentally
(new facilities) and theoretically

This overview only touched many of the questions which will be discussed
in more detail in the meeting. Hopefully useful.

Thank you for the attention!