The nuclear structure and the pressure of asymmetric nuclear matter
— Everything as a function of (K_0, L) —

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in collaboration with

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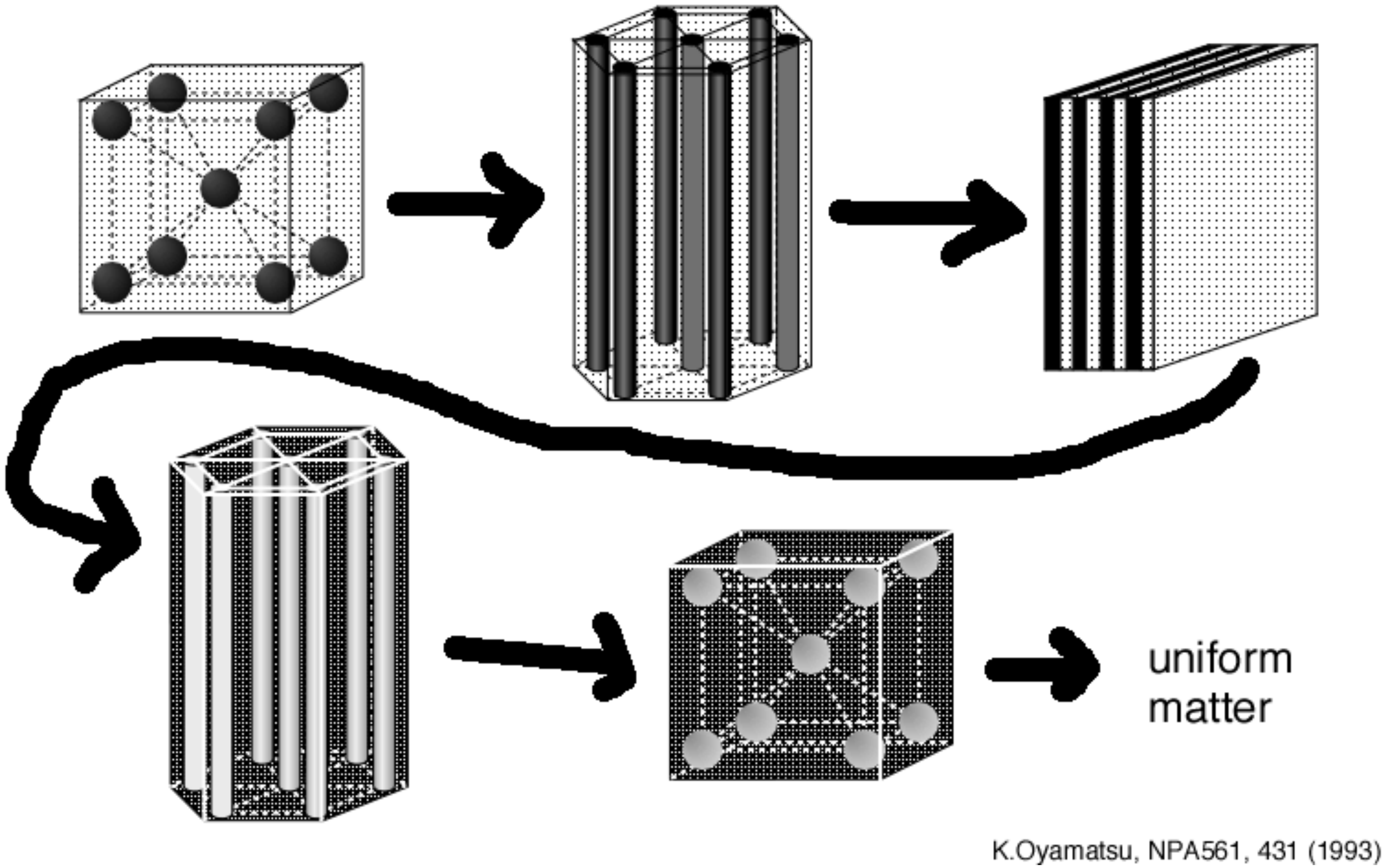
1. Oyamatsu-Iida (OI) unified EOS family
2. atomic masses (AME2003 and 2006)
3. pasta structure in neutron star crusts
4. Sotani's formula for pasta layer thickness

NUSYM17, 5 September, 2017

spherical nuclei and pasta nuclei in neutron star crusts (1993)

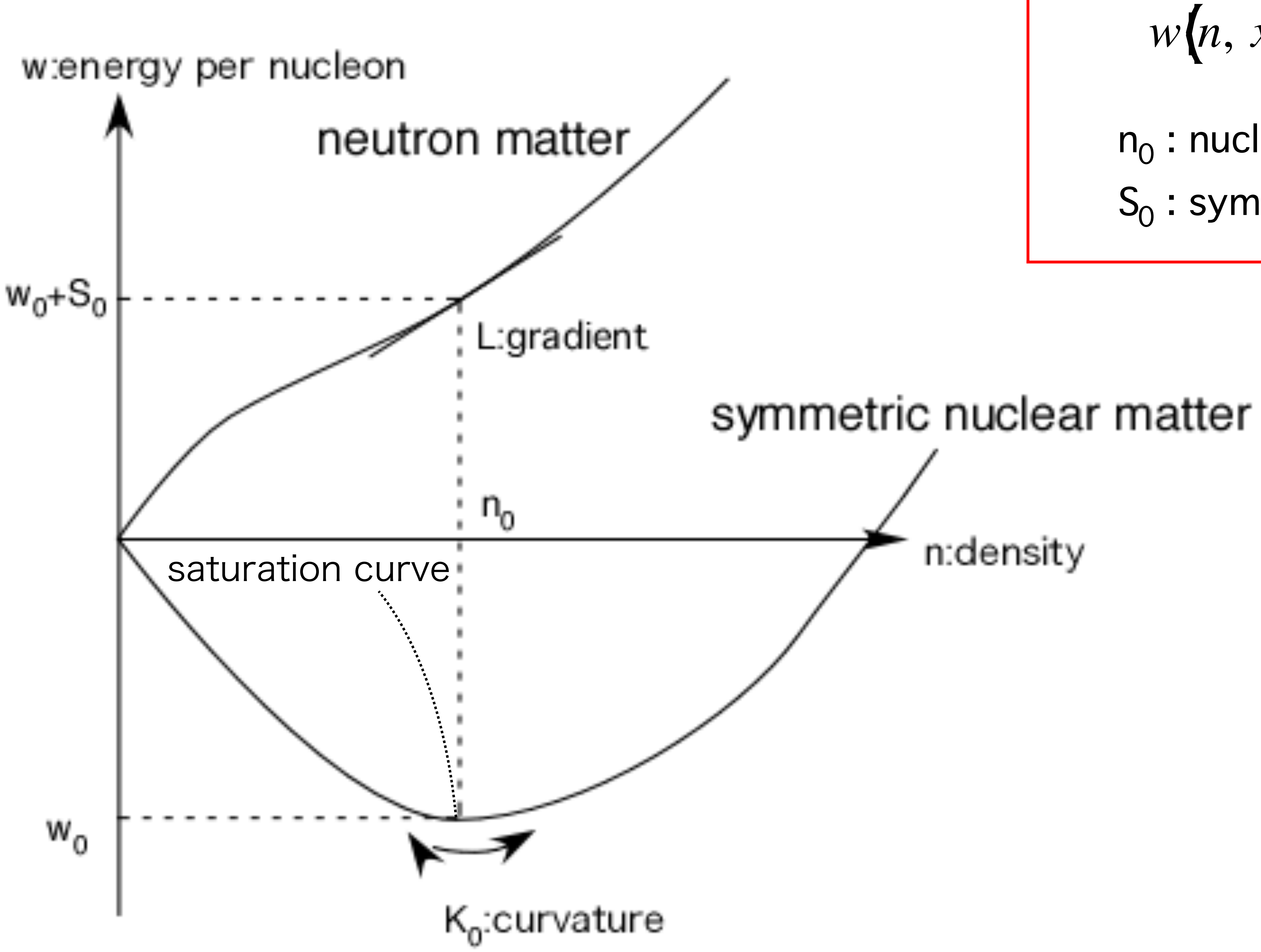


Dark domains means nuclei (proton clusters).
At low densities in neutron-star crusts, we have nuclei which are more or less spherical.
In the core we have uniform matter. Pasta nuclei could exists in between.

Existence of pasta nuclei depends on the EOS.

Lorenz C. P., Ravenhall D. G., Pethick C. J., 1993, Phys. Rev. Lett., 70, 379.

Saturation parameters and auxiliary empirical constraint



Energy per nucleon of nearly symmetric nuclear matter

$$w(n, x) \approx w_0 + \frac{K_0}{18 n_0^2}(n - n_0)^2 + (1 - 2x)^2\left[S_0 + \frac{L}{3n_0}(n - n_0)\right]$$

n_0 : nuclear density, w_0 :saturation energy, K_0 : incompressibility

S_0 : symmetry energy at $n=n_0$, L: its density derivative coefficient

Symmetry energy S_0 and its density gradient L

$$S_0 = S(n_0) \qquad L = 3n_0 \left.\frac{dS(n)}{dn}\right|_{n=n_0}$$

Saturation point of asymmetric matter $(x \approx 0.5)$

$$n_s = n_0 - \frac{3n_0 L}{K_0}\alpha^2 \qquad w_s = w_0 + S_0\alpha^2 \qquad \alpha = 1 - 2x$$

slope of saturation curve

$$y = -\frac{S_0 K_0}{3n_0 L}$$

Auxiliary empirical constraint

$$-1800 \leq y \leq -200$$

Macroscopic nuclear model

Oyamatsu K., Iida K., 2003, Prog. Theor. Phys., 109, 631.

Energy per cell (or mass of a nucleus)

electron (relativistic)

Coulomb (lattice)

$$W=\int d^3r\left[\underline{\epsilon_0\left(n_n(r),n_p(r)\right)}+\underline{F_0|\nabla n(r)|^2}+m_n n_n(r)+m_p n_p(r)\right]+W_e+W_C$$

n_n (n_p) : neutron (proton) density, $n=n_n+n_p$, $x=n_p/n$,

$\boxed{F_0\ :\ \text{surface energy (interaction range effect)}}$

$\underline{\epsilon_0\left(n_n(r),n_p(r)\right)\ :\ \text{local energy density (EOS)}}$

$$\epsilon_0\left(n_n(r),n_p(r)\right)=\frac{3}{5}(3\pi^2)^{2/3}\left(\frac{\hbar^2}{2m_n}n_n^{5/3}+\frac{\hbar^2}{2m_p}n_p^{5/3}\right)+\underline{(1-(1-2x)^2)v_s(n)+(1-2x)^2v_n(n)}$$

free kinetic energy density

potential energy density

potential energy density

$$\underline{v_s(n)=a_1n^2+\frac{a_2n^3}{1+a_3n}},\quad \underline{v_n(n)=b_1n^2+\frac{b_2n^3}{1+b_3n}}$$

symmetric matter

neutron matter

a_1,b_1 : 2 body energy

a_2,b_2 : 3 body energy

a_3,b_3 : small tuning

$(0<a_3,b_3\lesssim 5)$

For given (K_0, L), values of $\boxed{a_1\text{-}a_3,\ b_1,\ b_2}$ and $\boxed{F_0}$ are fitted masses and radii of stable nuclei (b_3=1.5863 fm^3)

Simplified Thomas-Fermi calculation

energy minimization with respect to parameters of $n_n(r)$ and $n_p(r)$ (and lattice constant)

neutron (proton) density distribution n_n (n_p)

$$n_i(r) = \begin{cases} \left(n_i^{in} - n_i^{out}\right)\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out} & r < R_i \\ n_i^{out} & r < R_i \end{cases}$$

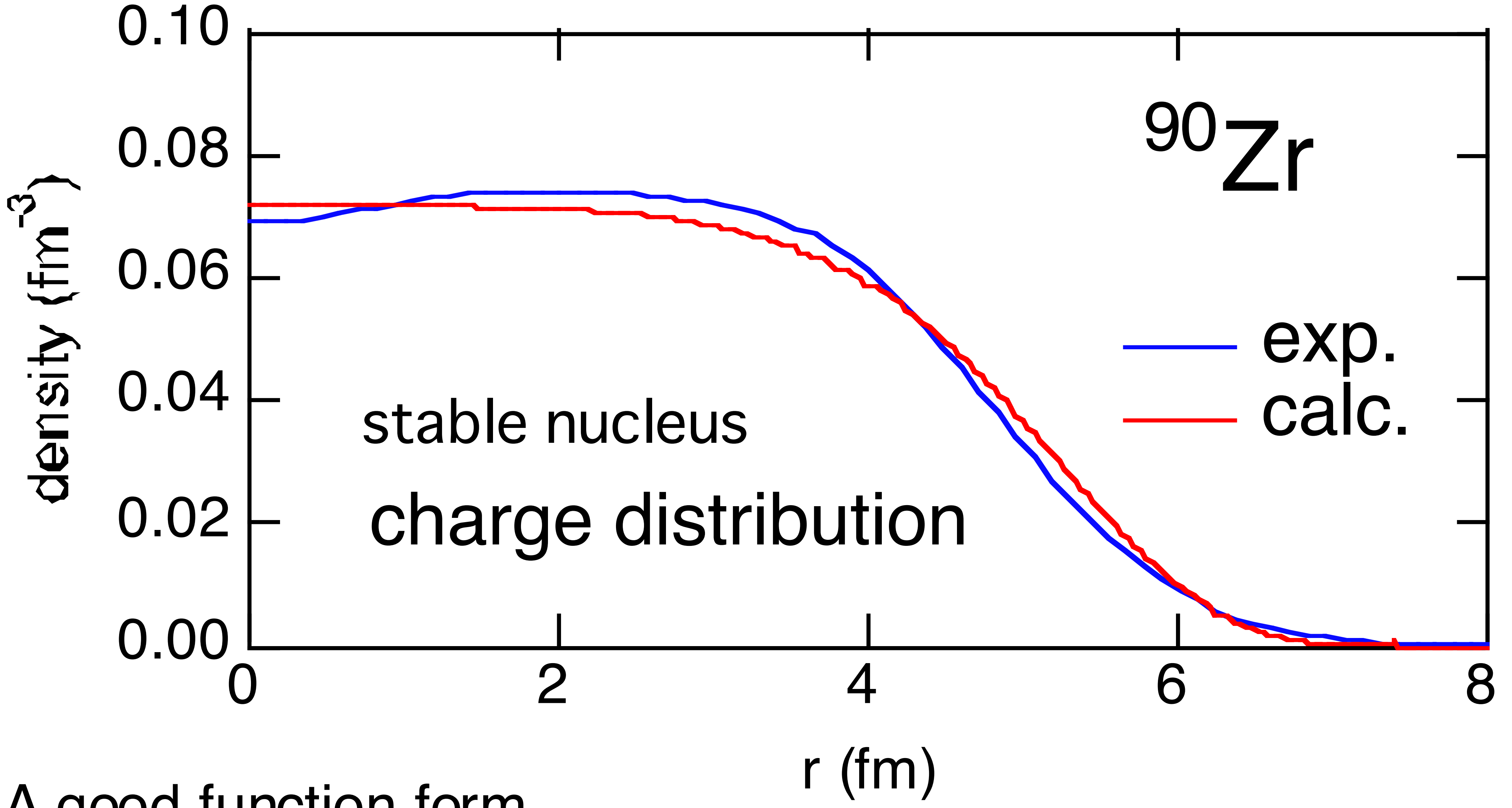
R_n (R_p) : neutron (proton) radius parameter

t_n (t_p) : neutron (proton) surface thickness parameter

density

n_i^{in} : central desnity

n_n^{out} : neutron gas density (n_p^{out}=0)



A good function form

The n and p distributions are independent.

=> neutron skin

The empirical information is limited: radius and thickness.

The gradient term in Euler Eq. is continuous.

The density is zero beyond the classical turning point.

The values of parameters $a_1 \sim b_3$(EOS) and F_0 are determined

to fit masses and radii of stable nuclei.

EOS parameters and liquid drop mass formula

	OI EOS	**saturation parameters**	**liquid drop mass formula**
symmetric matter	a_1, a_2, a_3	n_0, w_0, K_0	a_v (volume) => w_0 a_c (Coulomb) => n_0
Symmetry energy or neutron matter	b_1, b_2, b_3(=1.59)	S_0, L w_{n0}, L	a_i (symmetry)
surface	F_0	(F_0)	a_s (surafce)

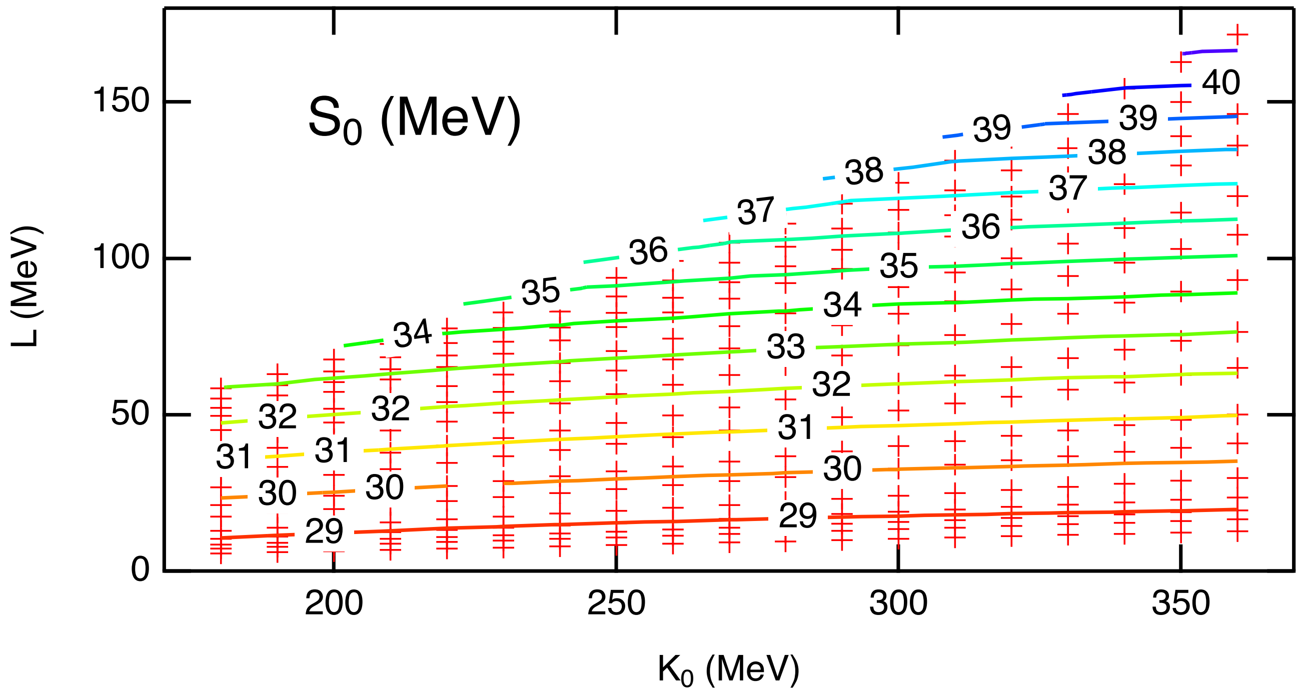
Each OI EOS is labeled with (K_0, L).

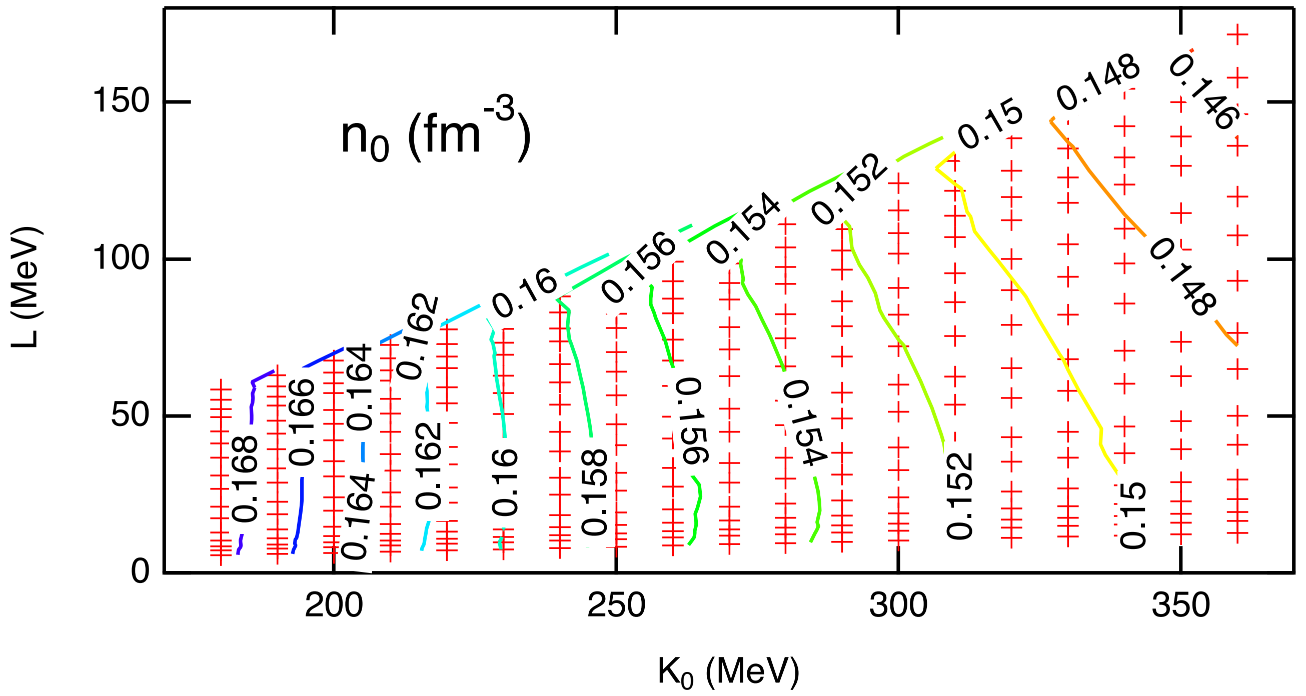
Determination of potential parameters and F_0

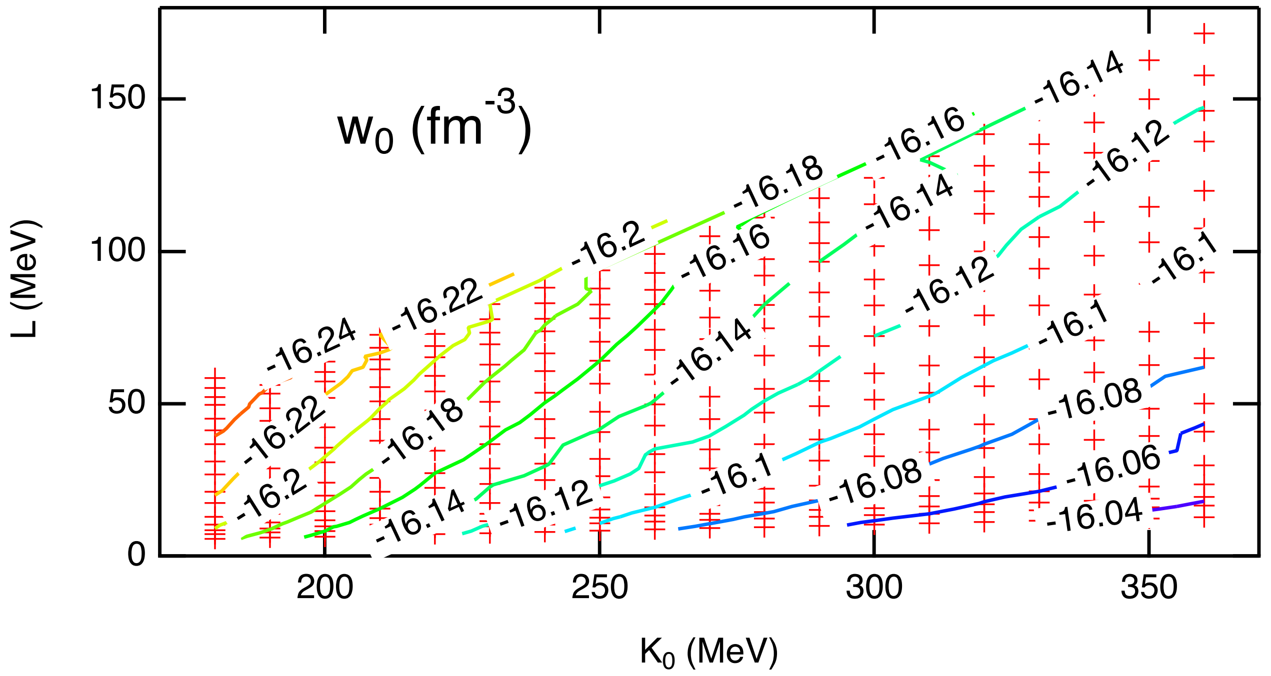
- Practically, for each of 304 sets of (y, K_0) values, a_1-a_3 and b_1, b_2 (b_3=1.59) and F_0 are fitted to masses and radii of stable nuclei.
- We have one-to-one correspondence between (y, K_0) and (K_0, L).
- We obtain 304 EOS's (interaction parameter sets) labeled with (K_0, L).
- With these EOS's, we calculate nuclei in nuclear chart and in neutron star crusts, neutron star matter EOS, and neutron star structure. We obtain them as functions of (K_0, L).

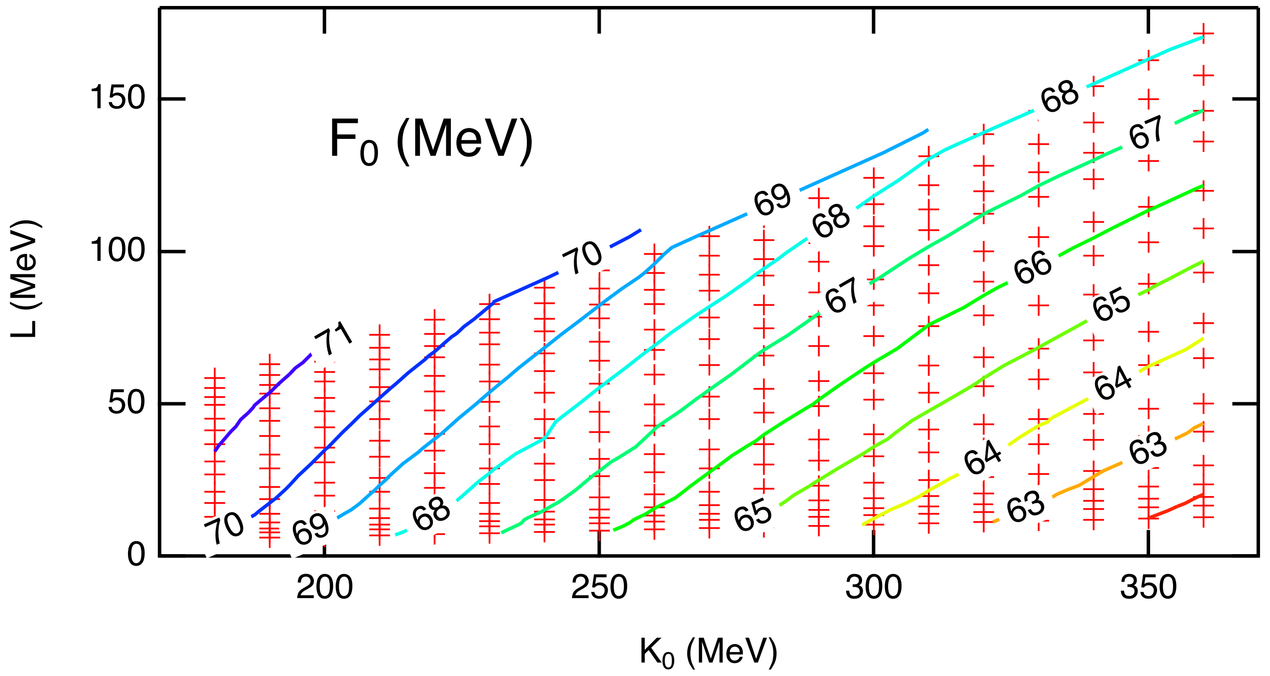
Saturation parameters

contour plots in K$_0$-L plane

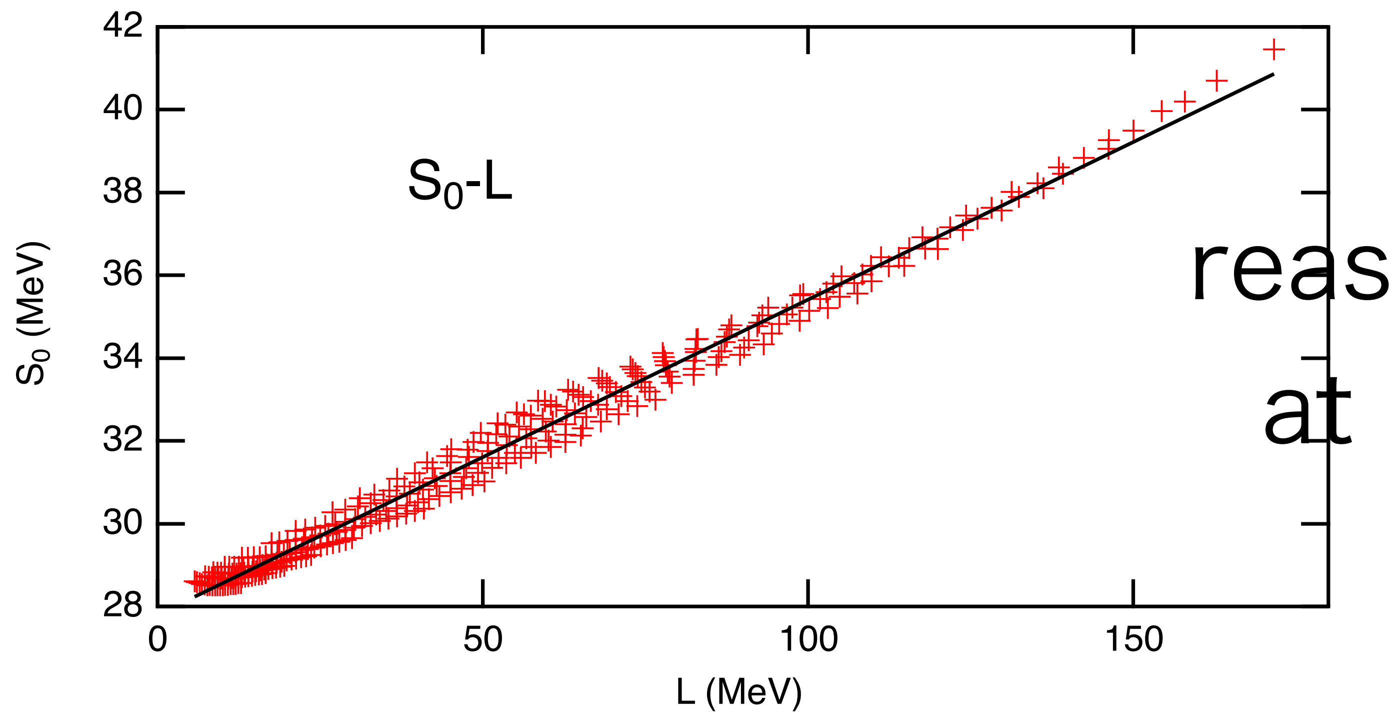




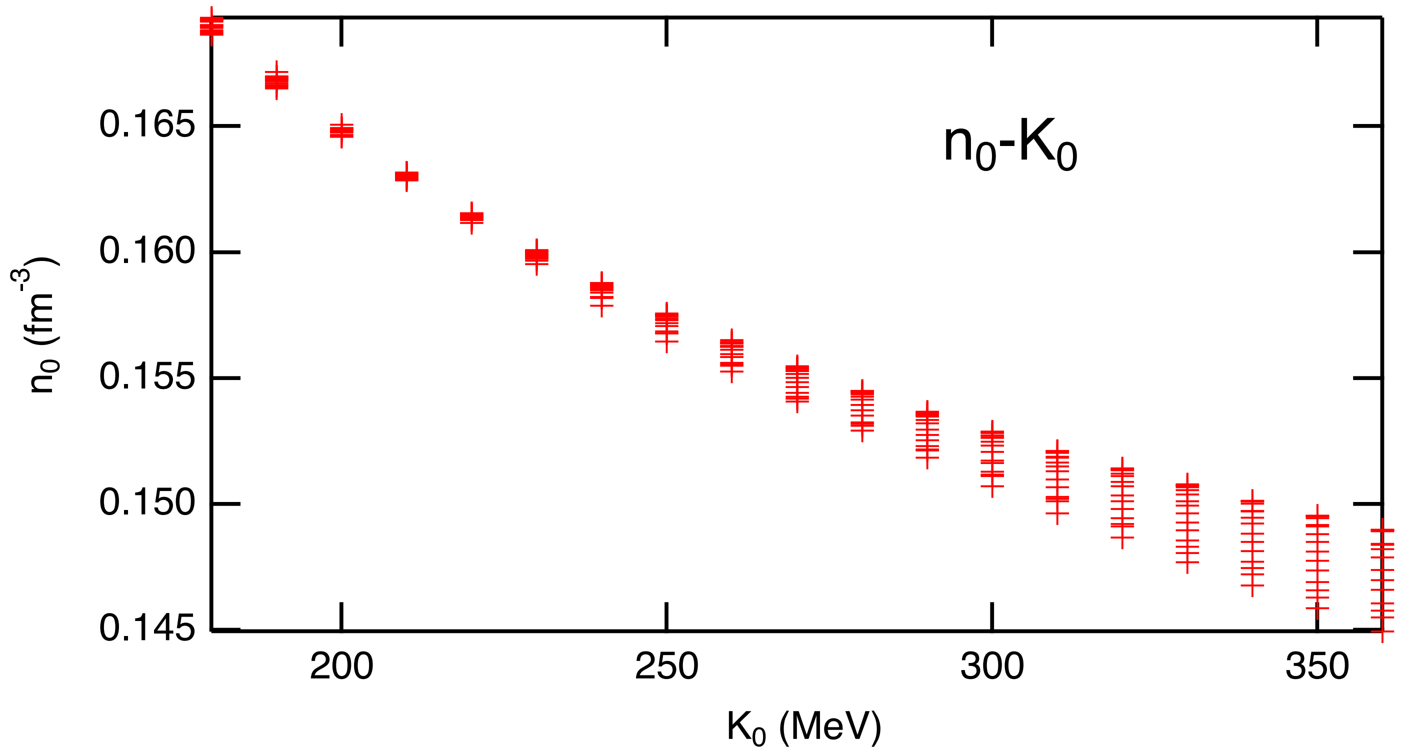




Correlations

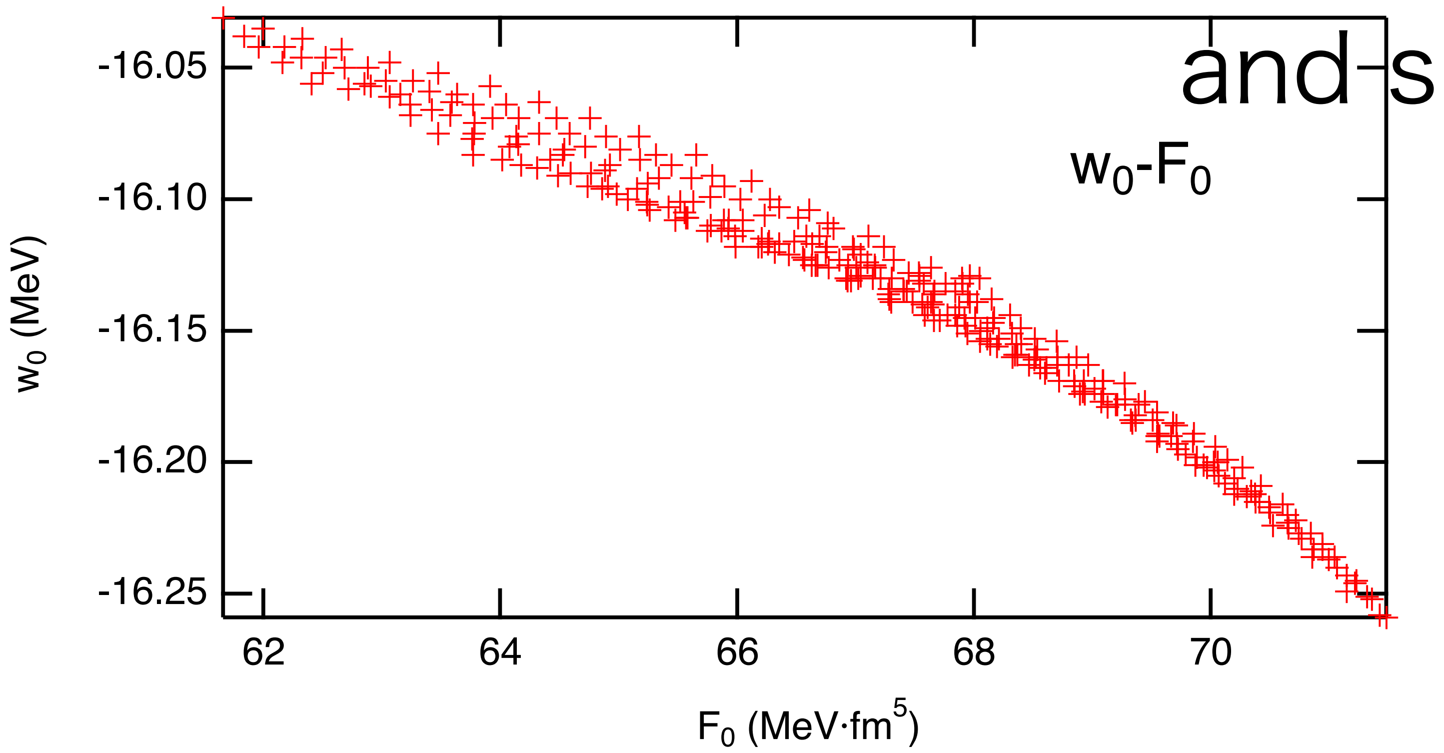


reasonable symmetry energy at nuclear surface density



radii of stable nuclei

balance between volume and surface energies

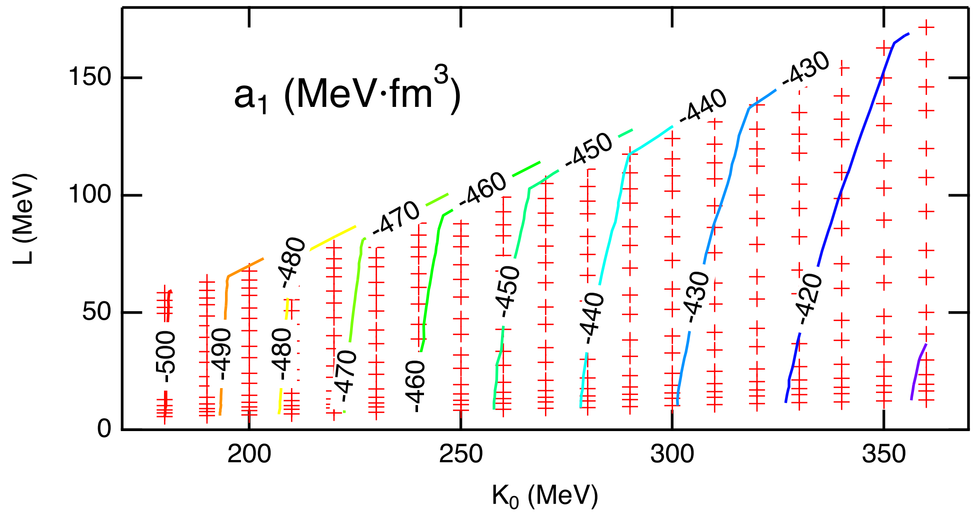


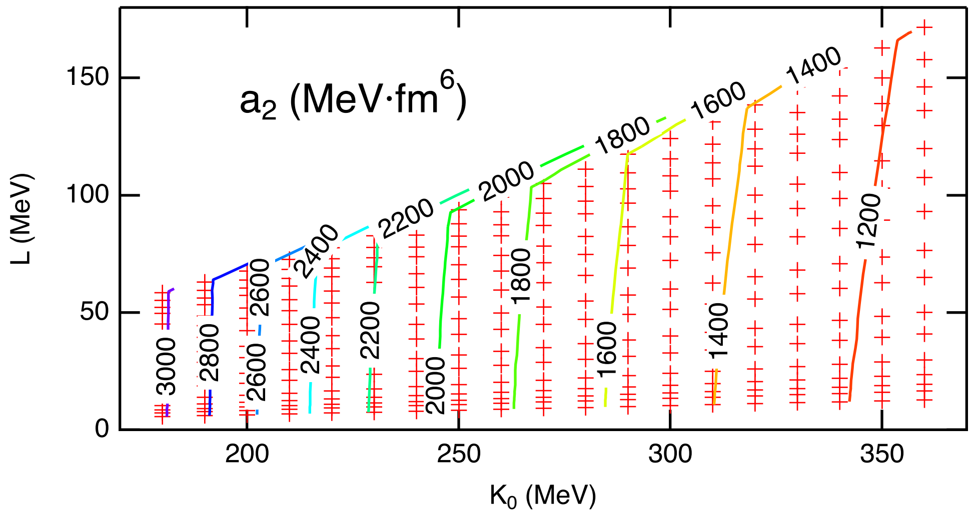
Potential energy coefficients

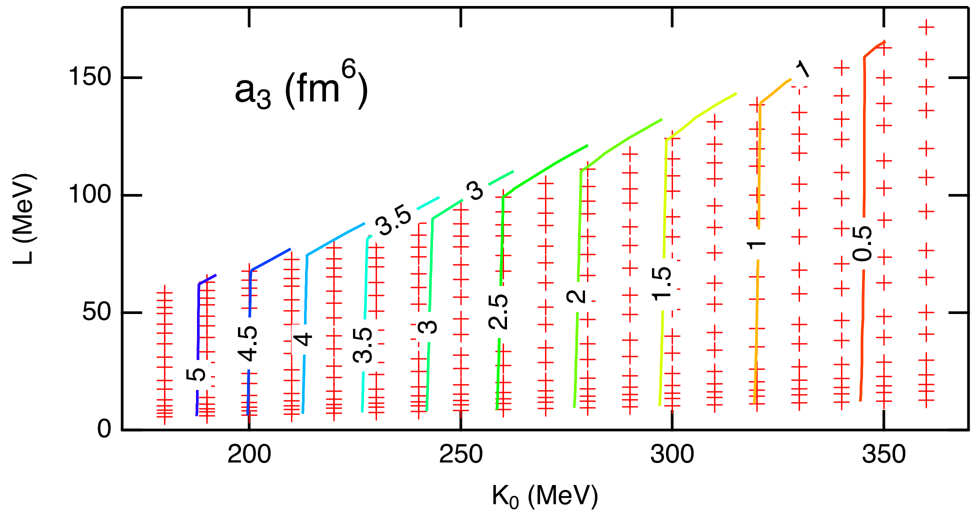
symmetric matter

two body

three body





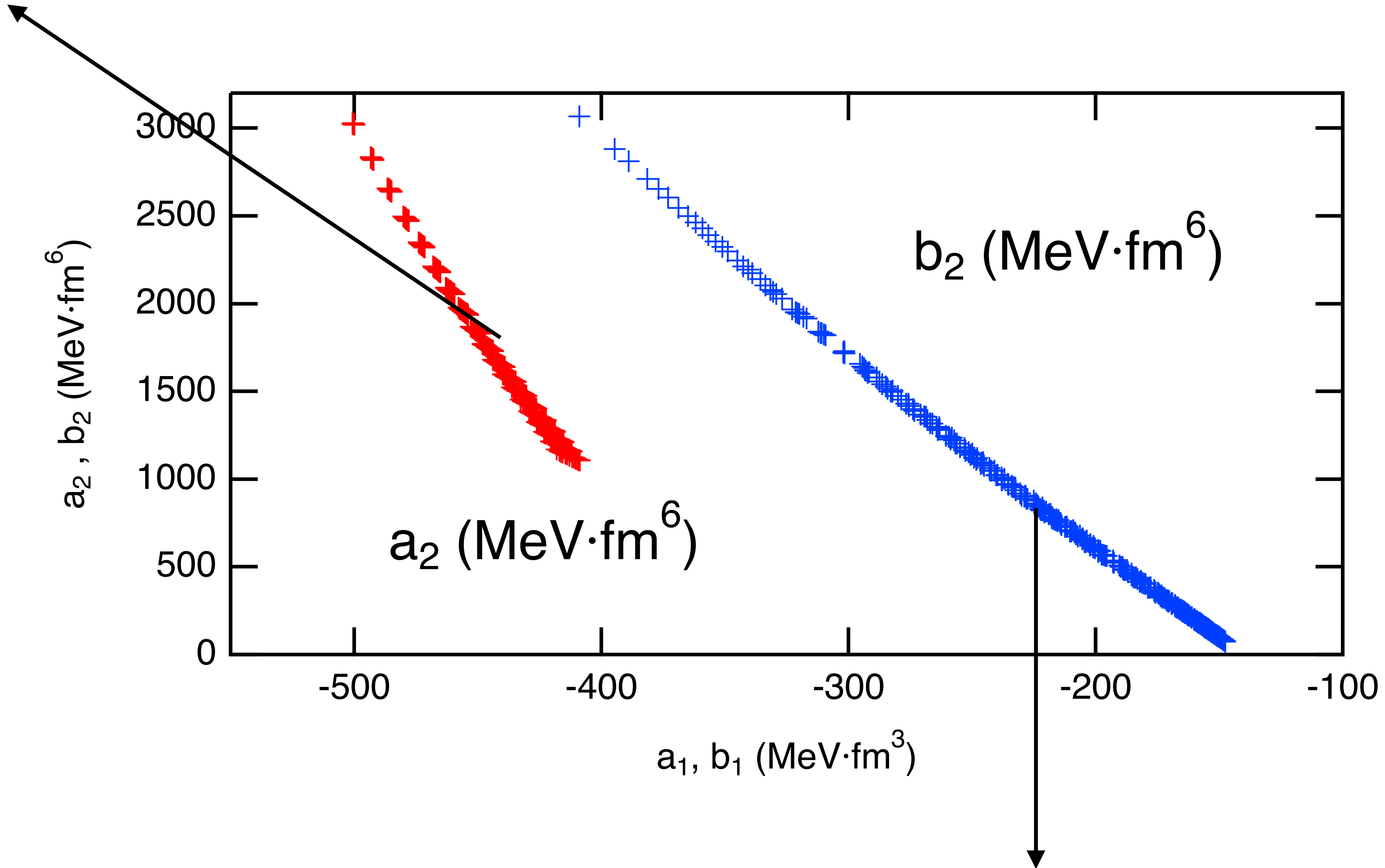


OI EOSF covers wide range of three-body energy.

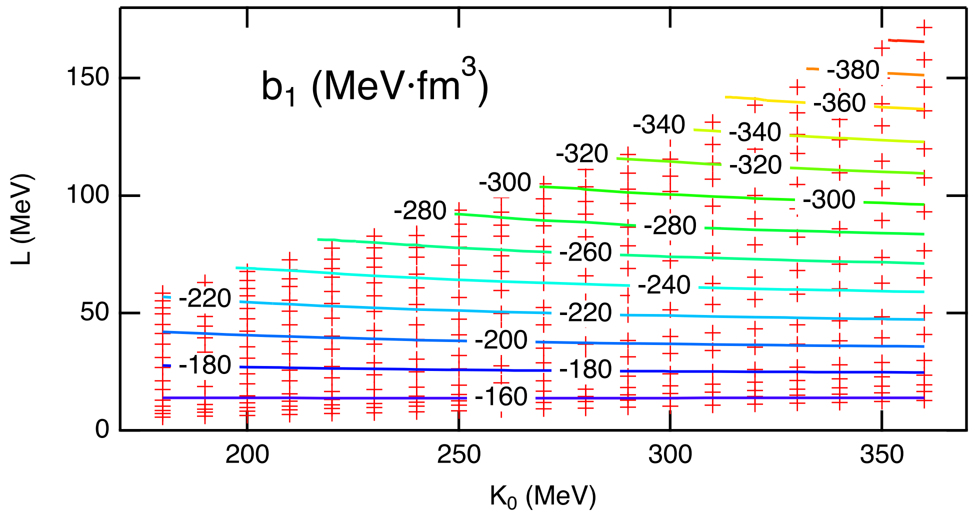
b_1 and b_2 are more uncertain than a_1 and a_2

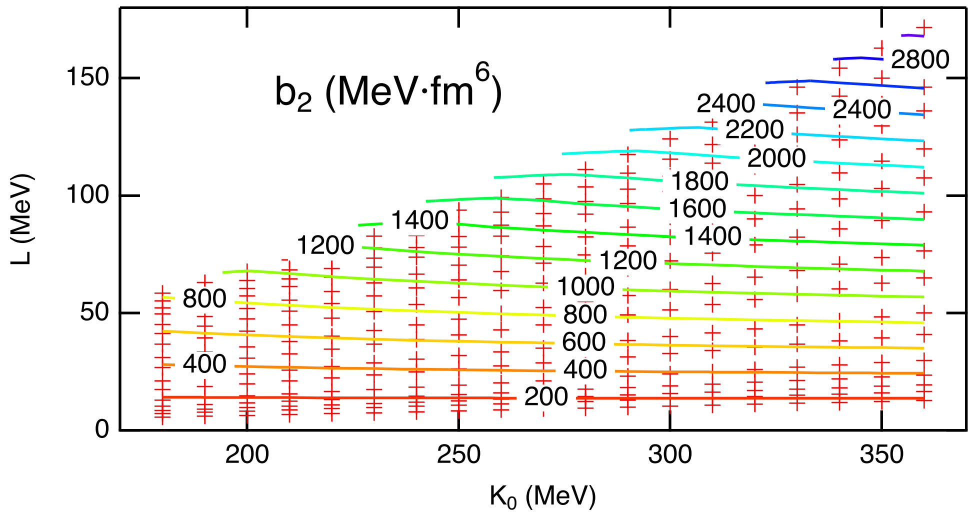
n_0-K_0 correlation

a_1, a_2 and a_3: mainly dependent on K_0



neutron matter





two body

three body

S_0-L correlation

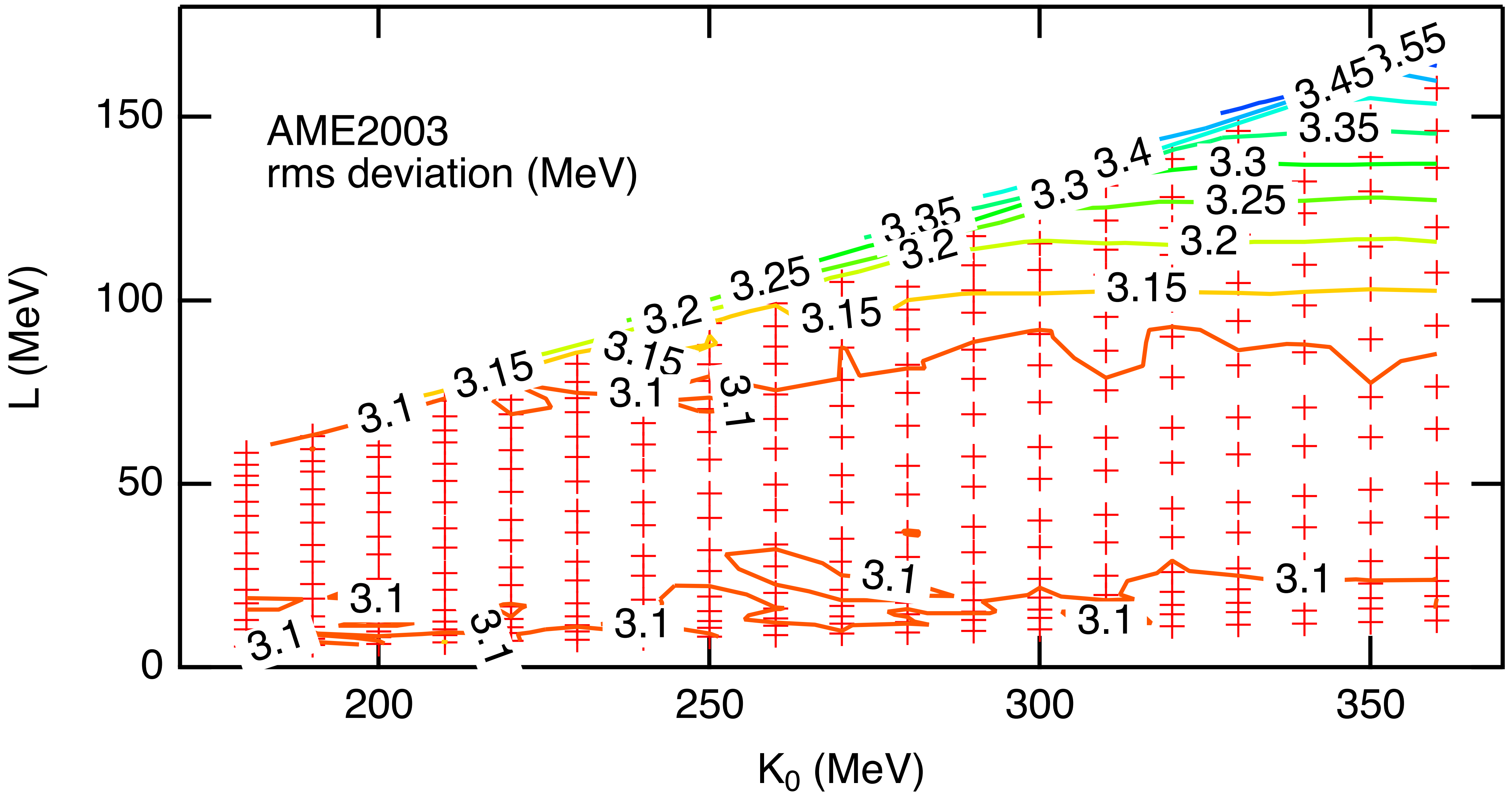
b_1 and b_2 : mainly dependent on L

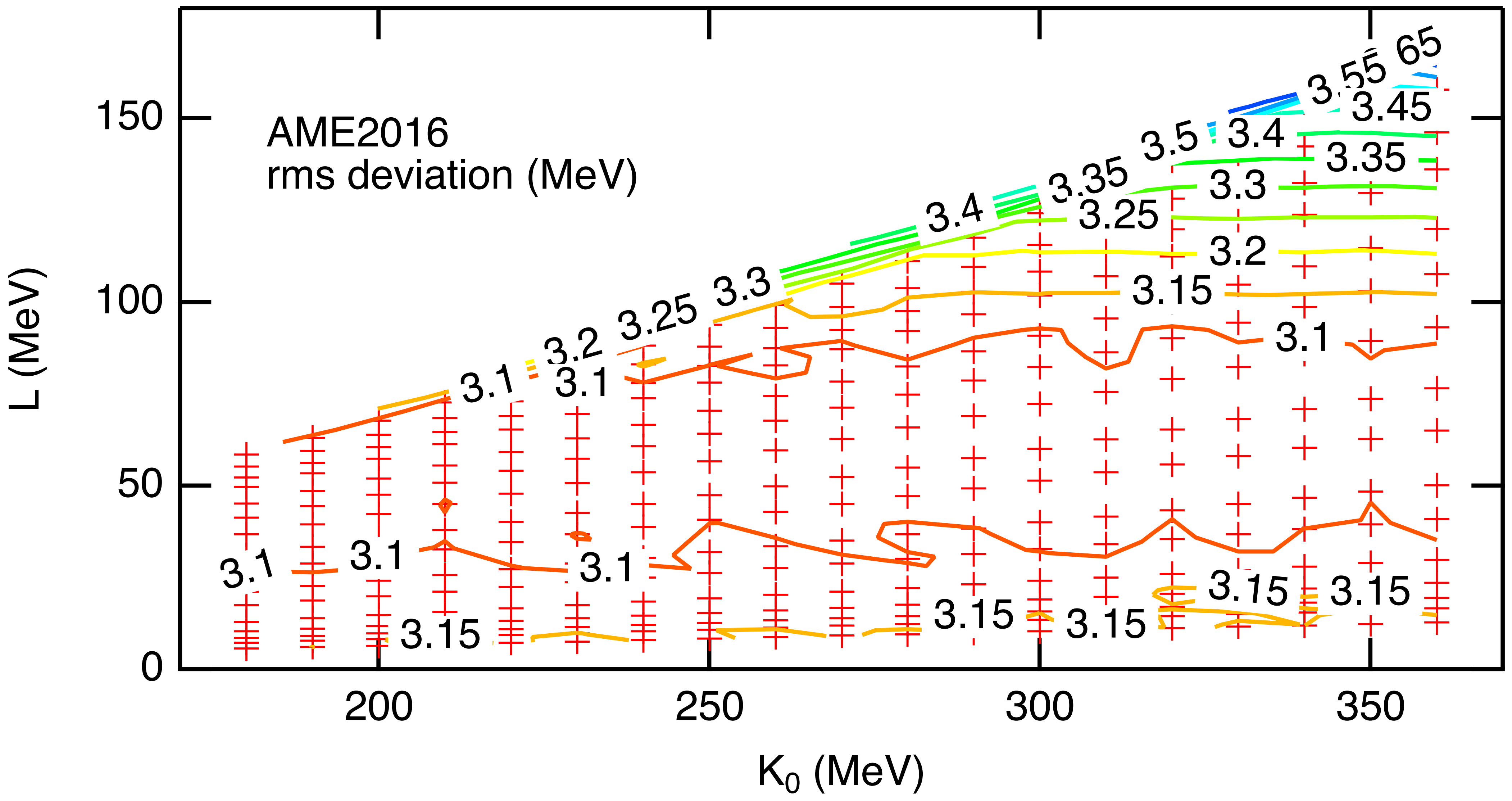
b_3 =1.599 to reasonable fit FP EOS

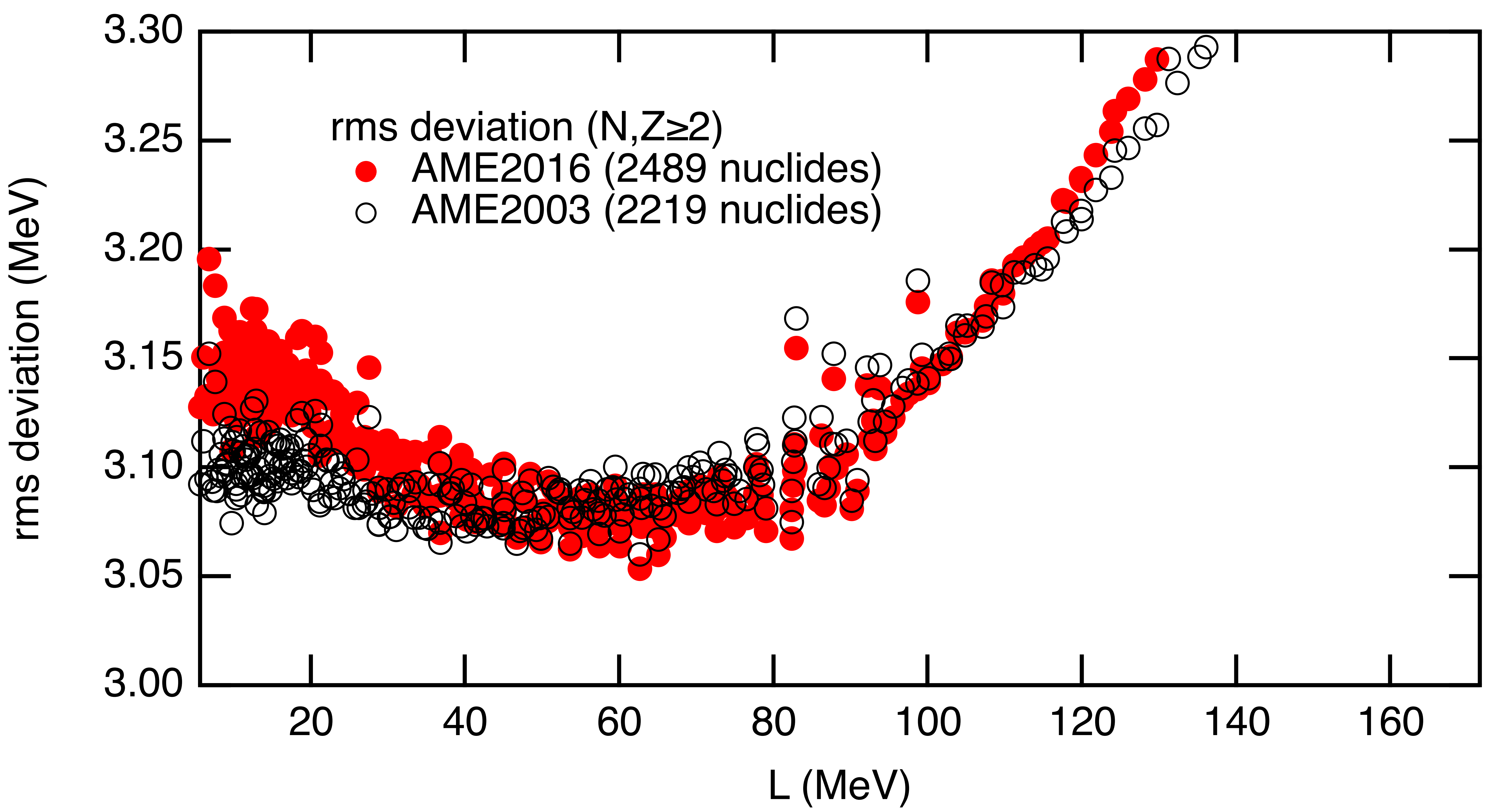
Everything is a function of (K_0, L).

Atomic mass data
from 2003 to 2016

L value from atomic masses



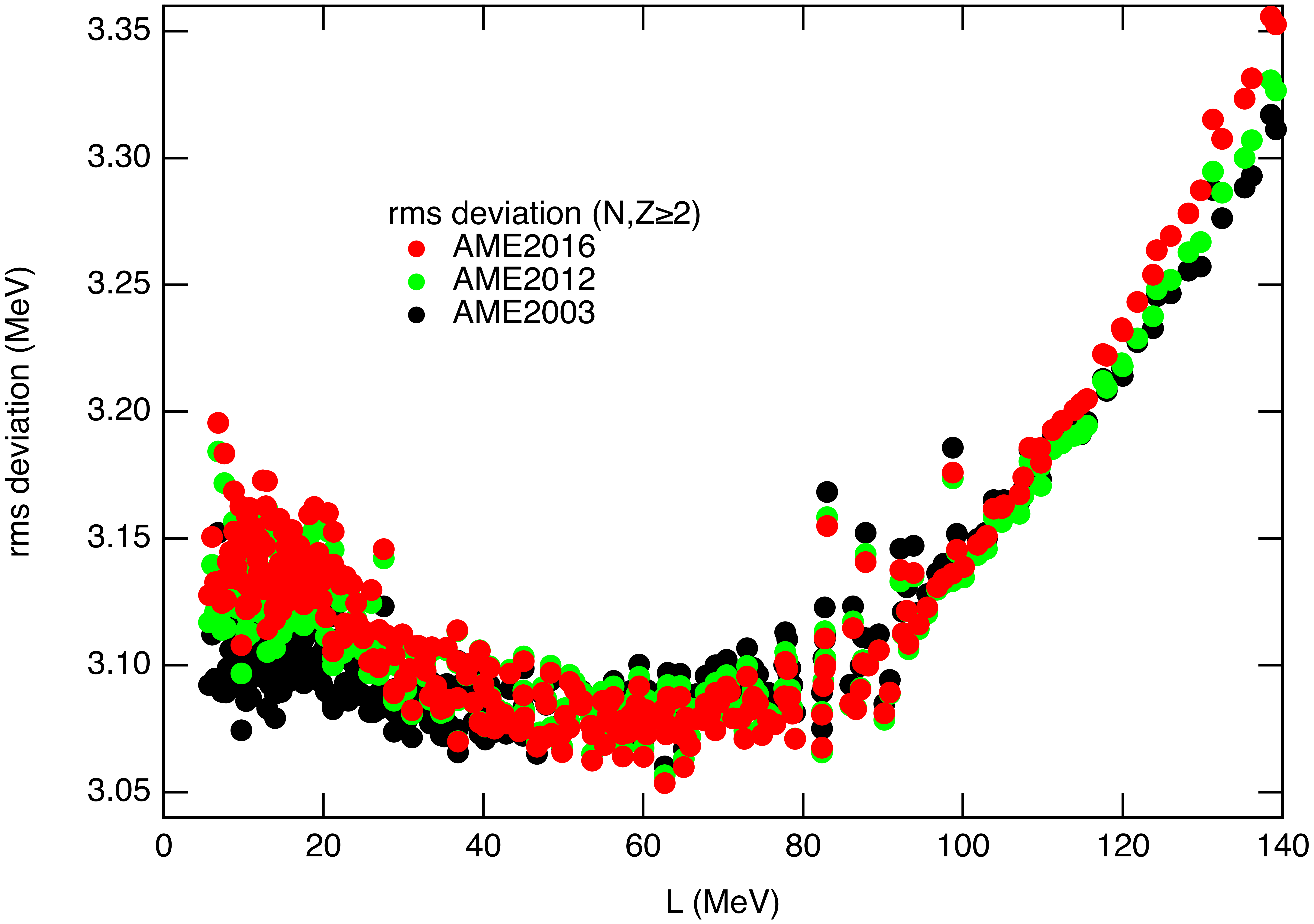




AME2016 excludes small and large L values.

The range of L from AME2016 is roughly 40 ≤ L ≤ 90 MeV.

Steady progress from 2003 to 2006



neutron star crust

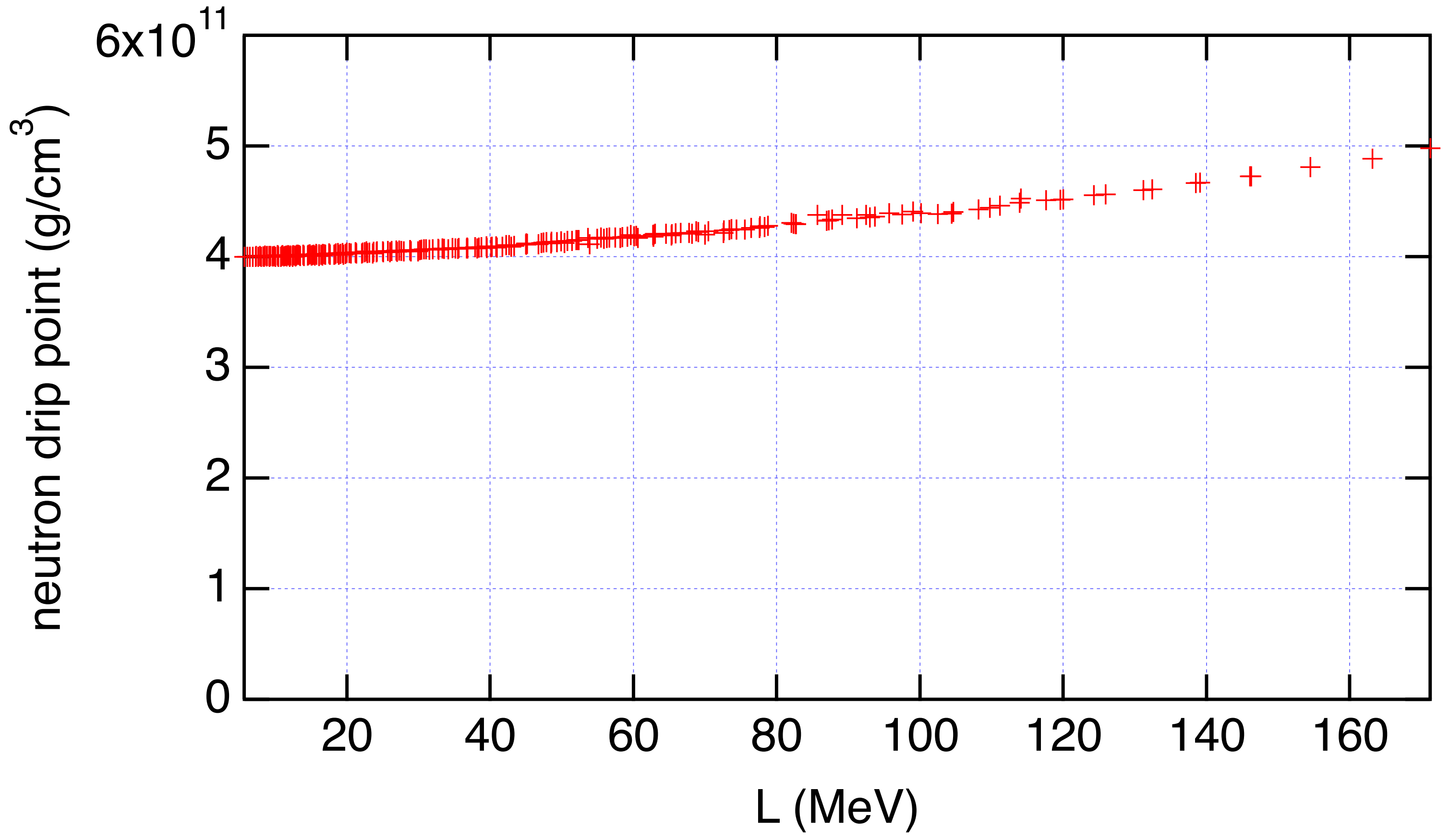
nuclei in neutron-star crusts (2007)

simplified calculations Oyamatsu K., Iida K., 2007, Phys. Rev. C, 75, 015801

neutron drip poit (NDP)

onset density of neutron drip

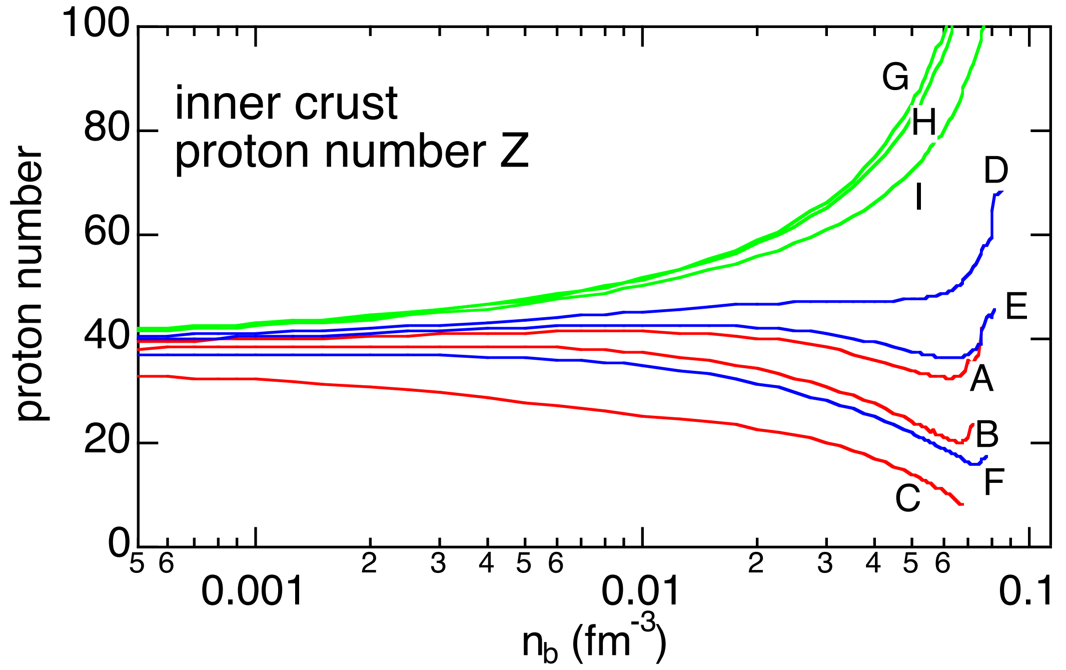
bundary of outer and inner crust

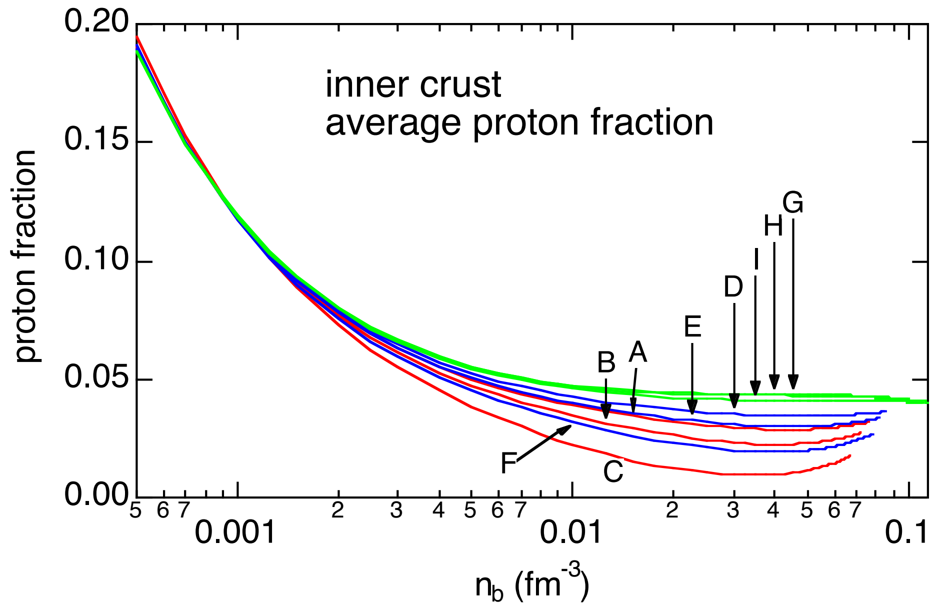


NDP slightly increases with L.

Inner crust nuclei

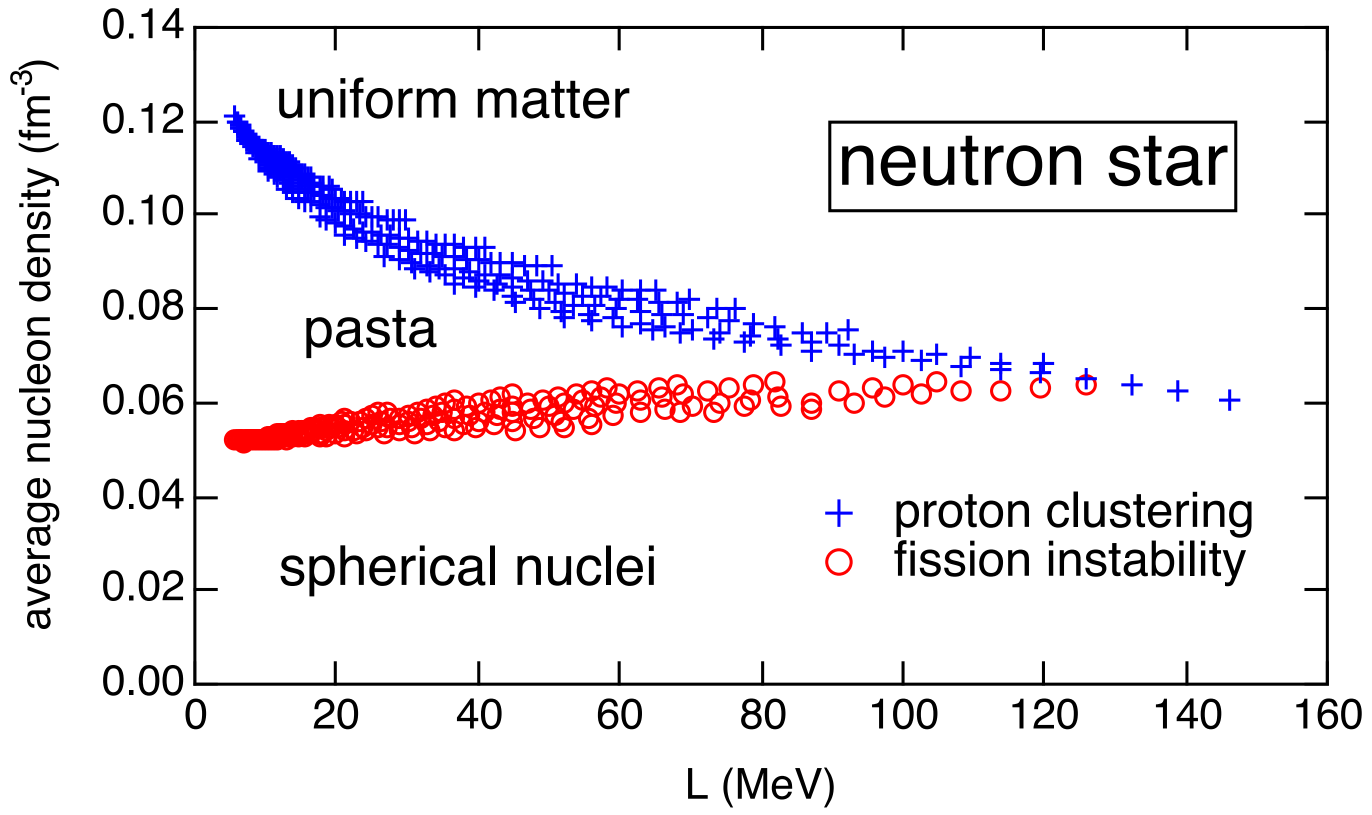
Z and Yp decrease with L.



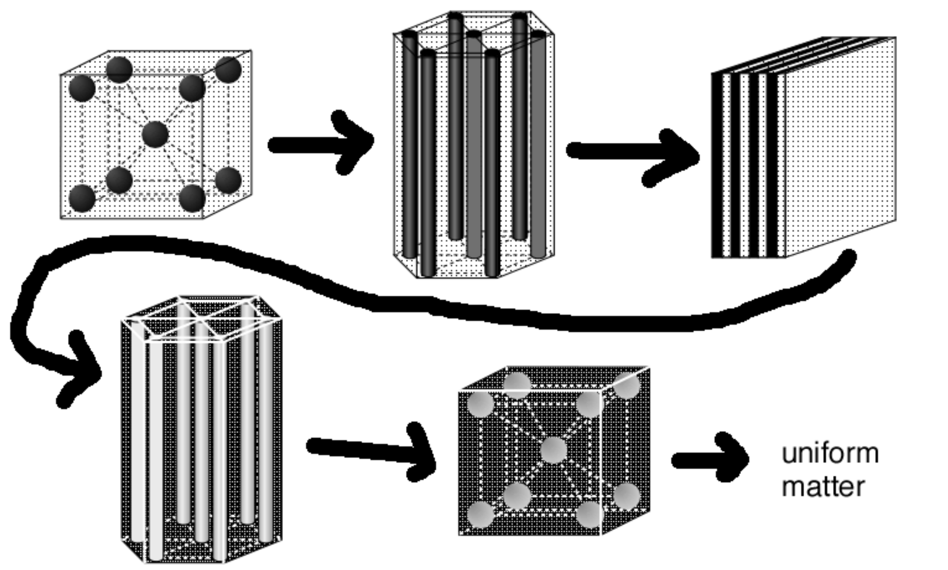


For large L, S(n) at $n<n_0$ is small
so that nuclei become more neutron-rich.

crust-core boundary density decrease with L.



spherical nuclei and pasta nuclei

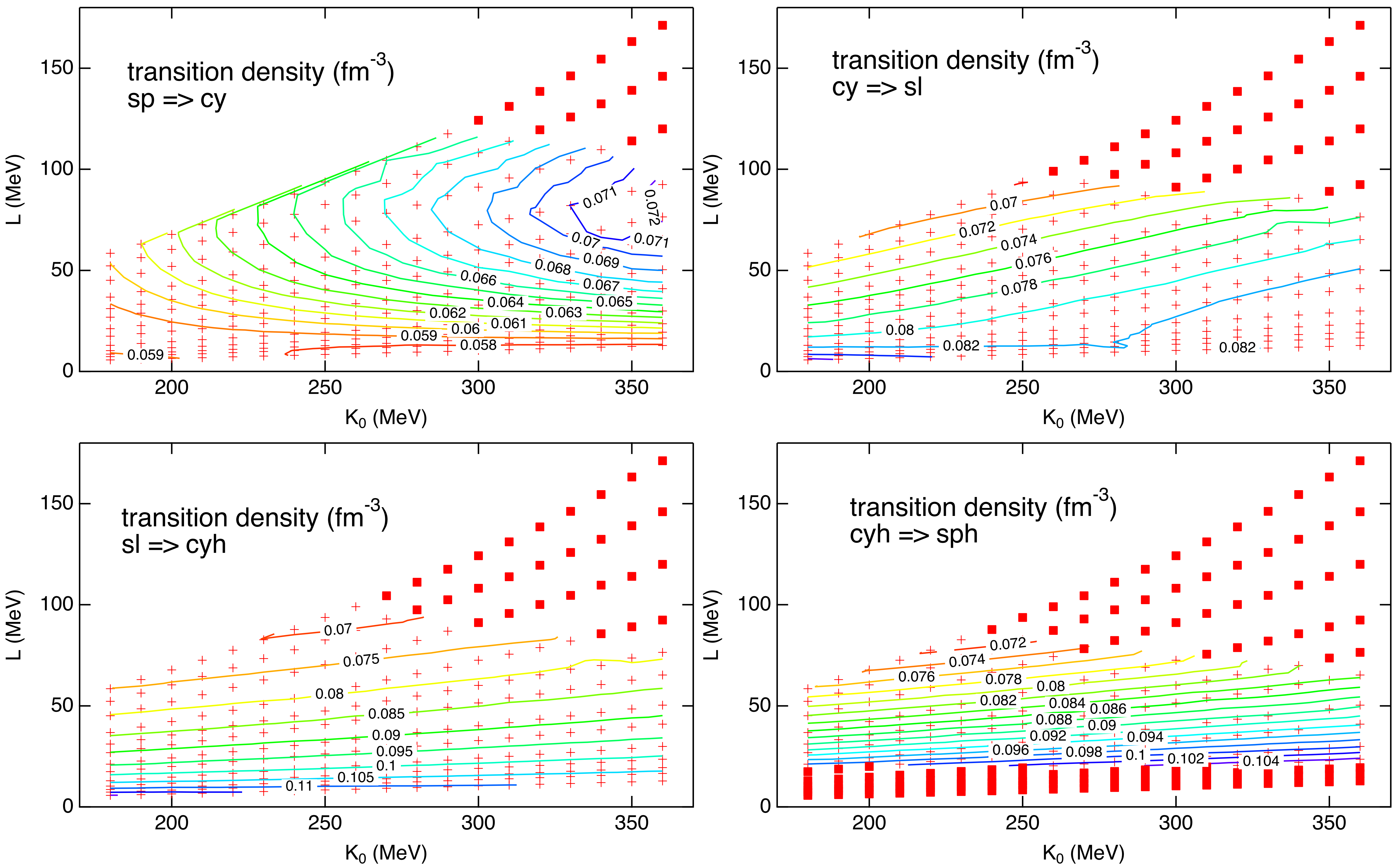


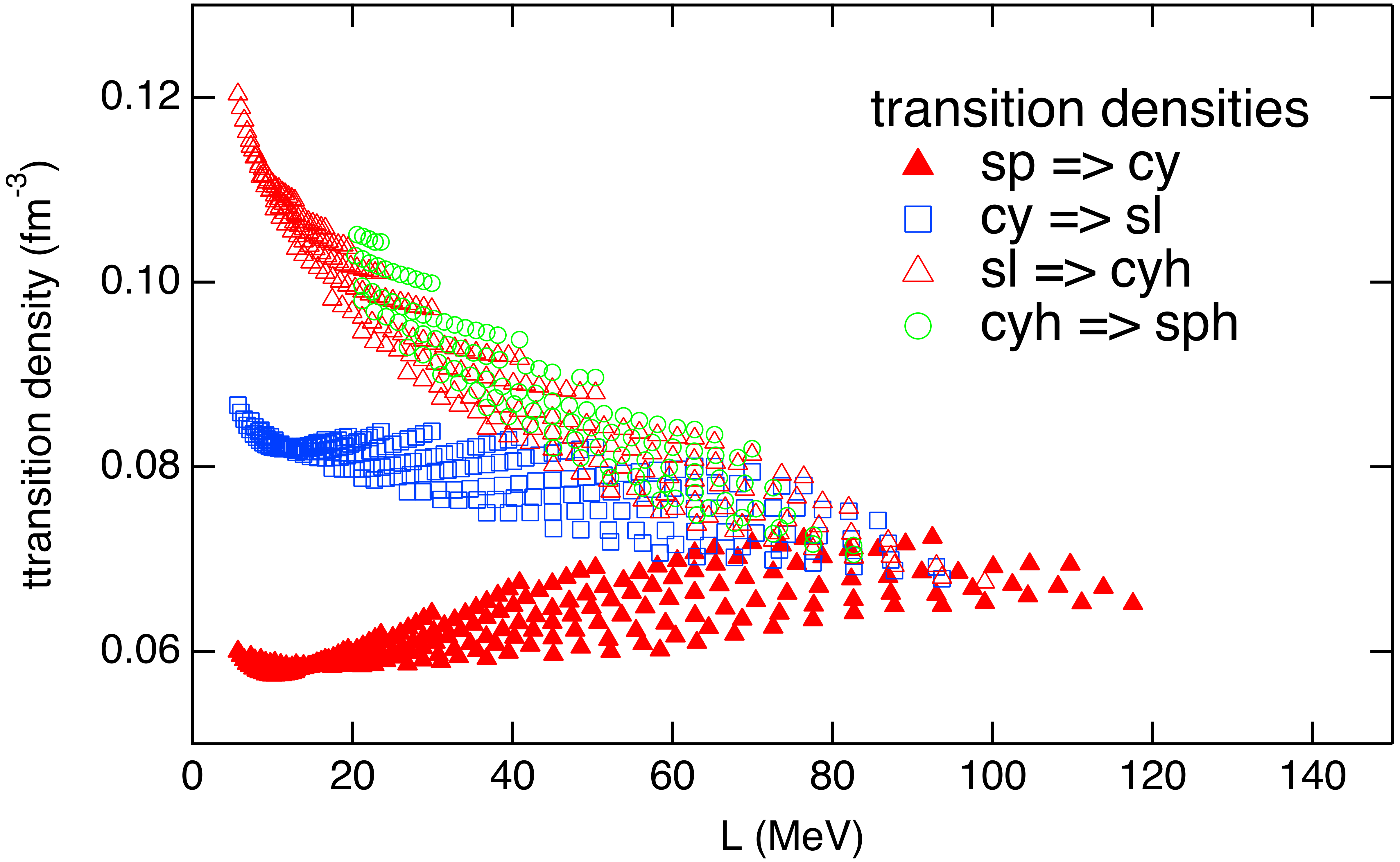
K.Oyamatsu, NPA561, 431 (1993)

Existence of pasta nuclei depends on the EOS.

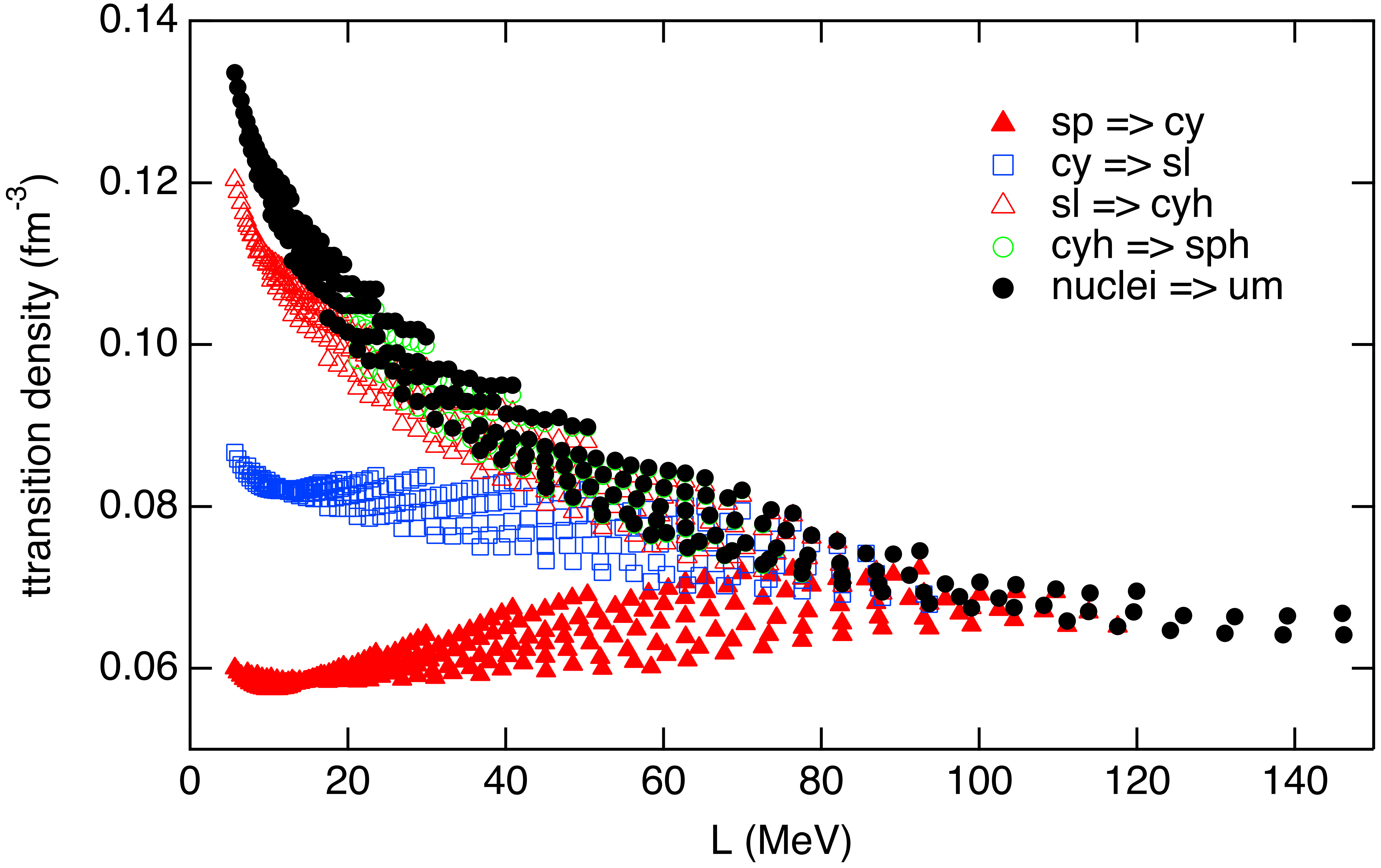
Pasta nuclei
in neutron star crusts

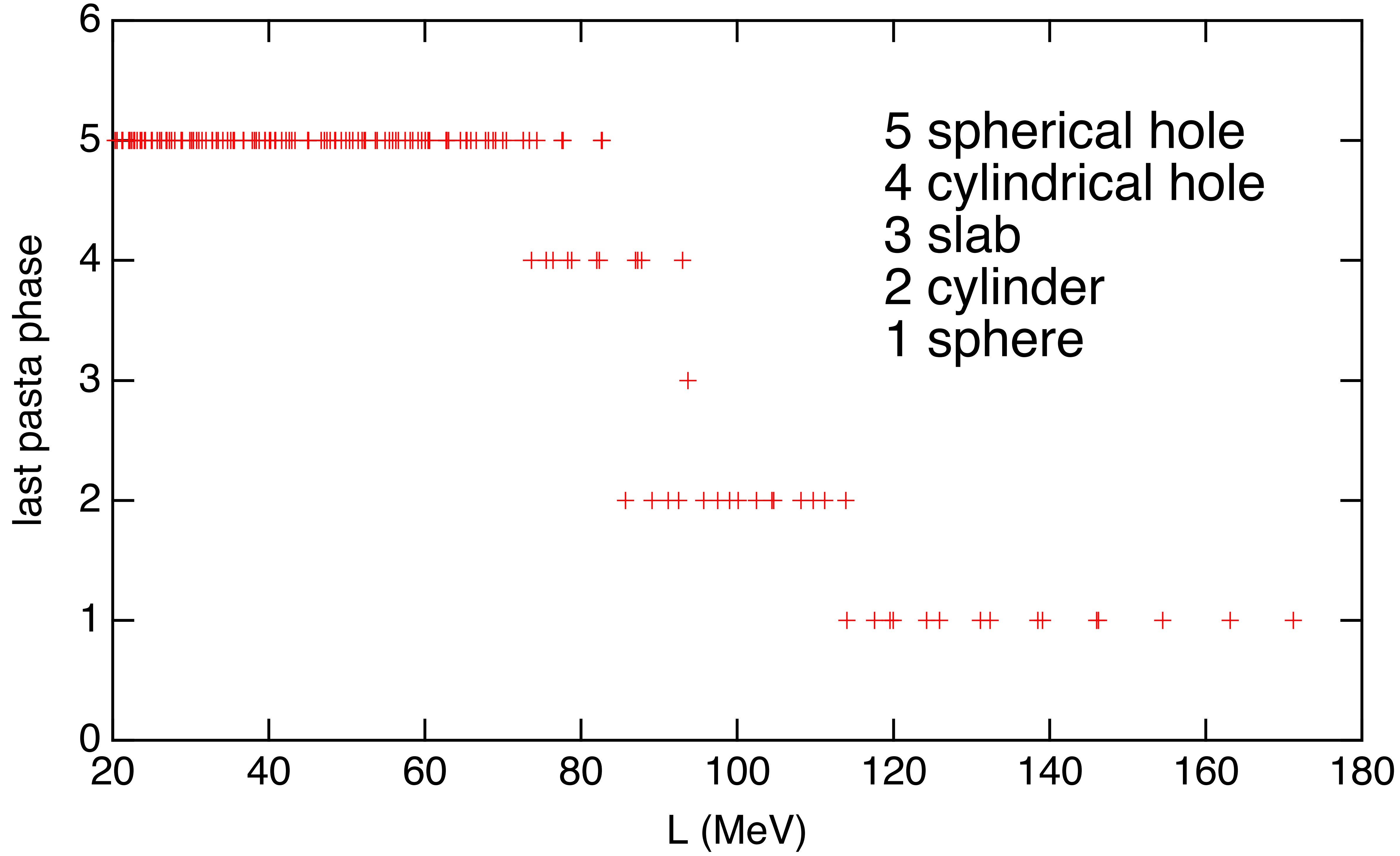
shape transition densities of pasta nuclei



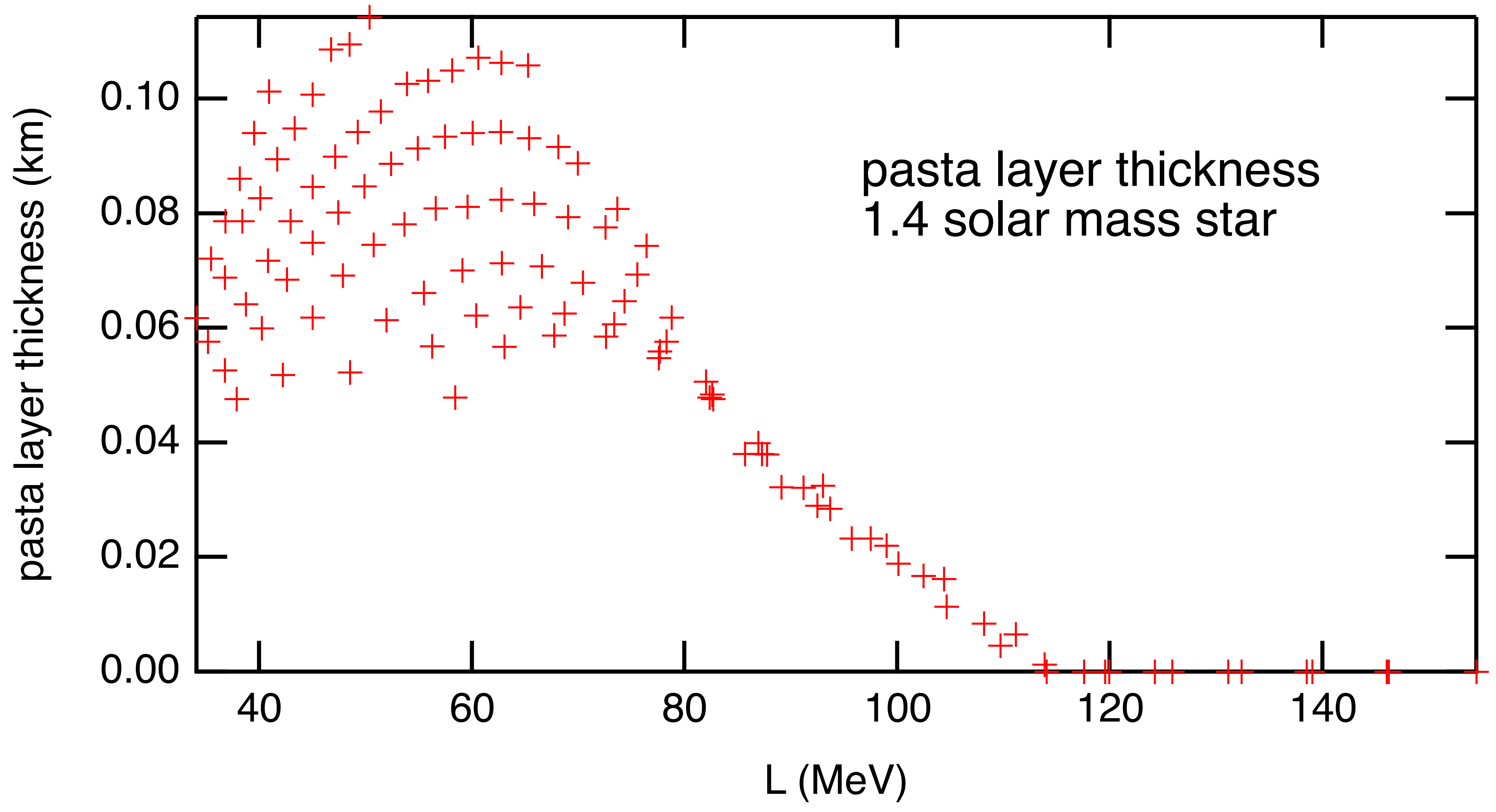


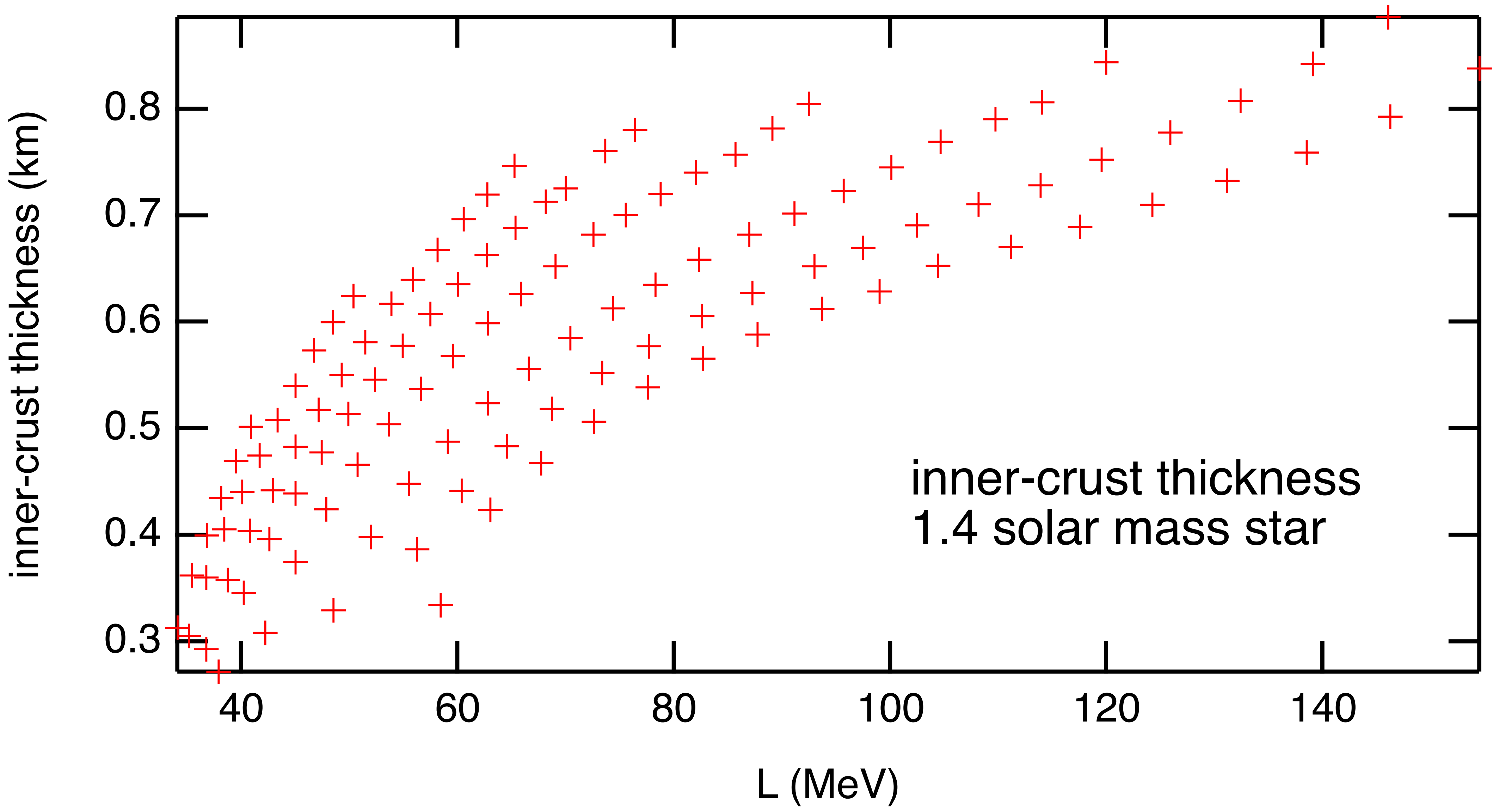
The last nuclear shape in the crust

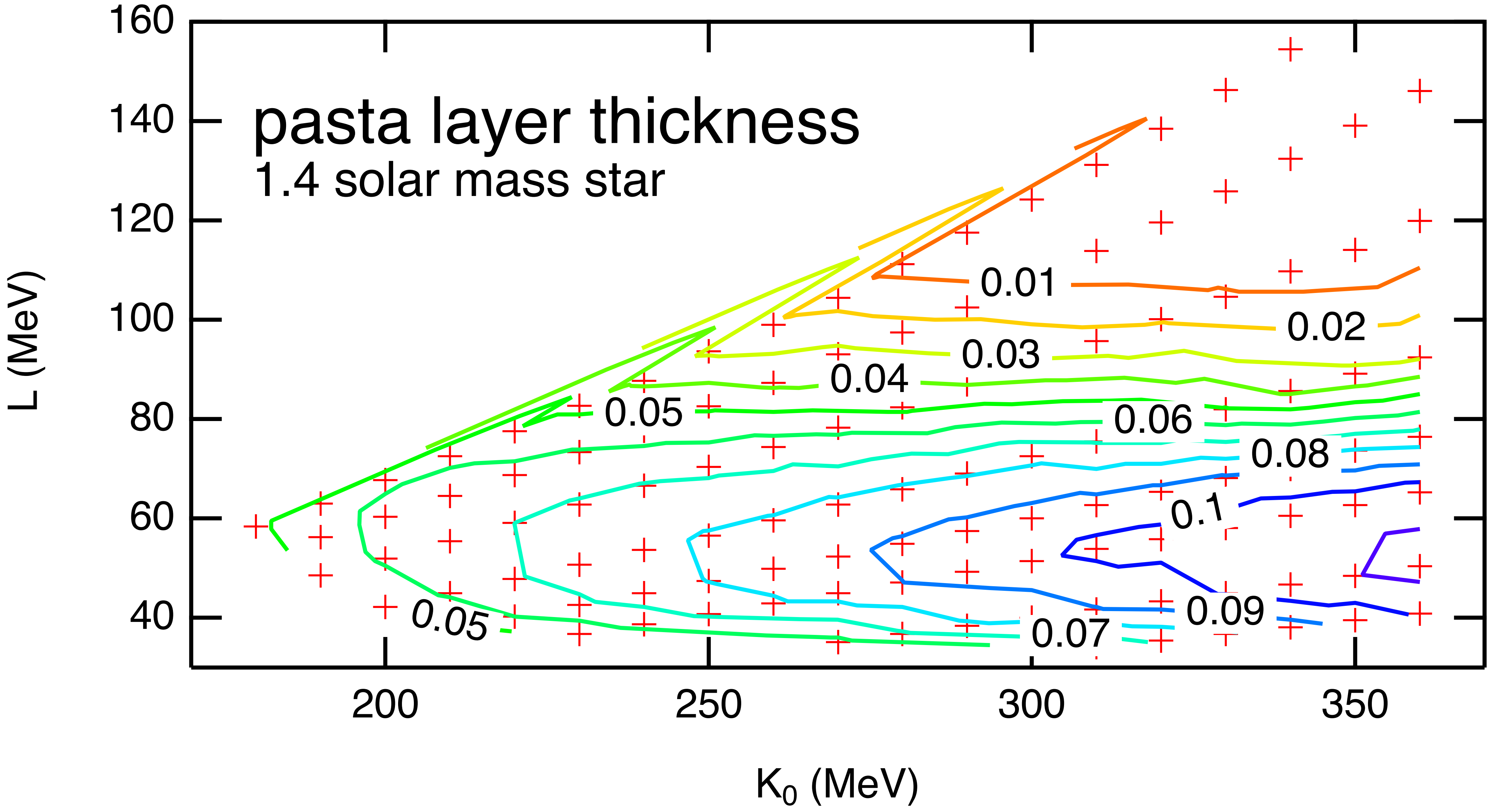


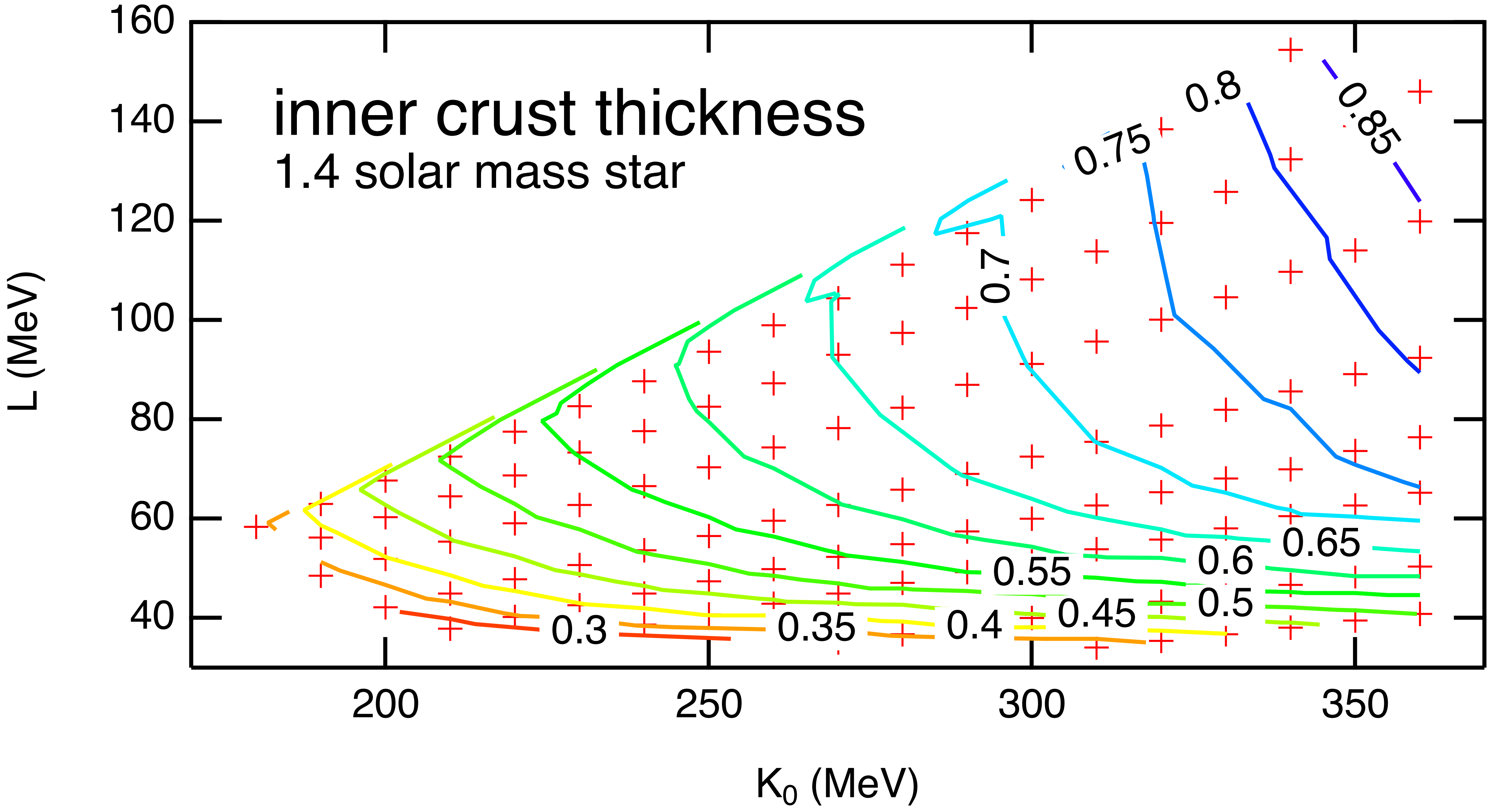


Pasta layer thickness (1.4 solar mass star)

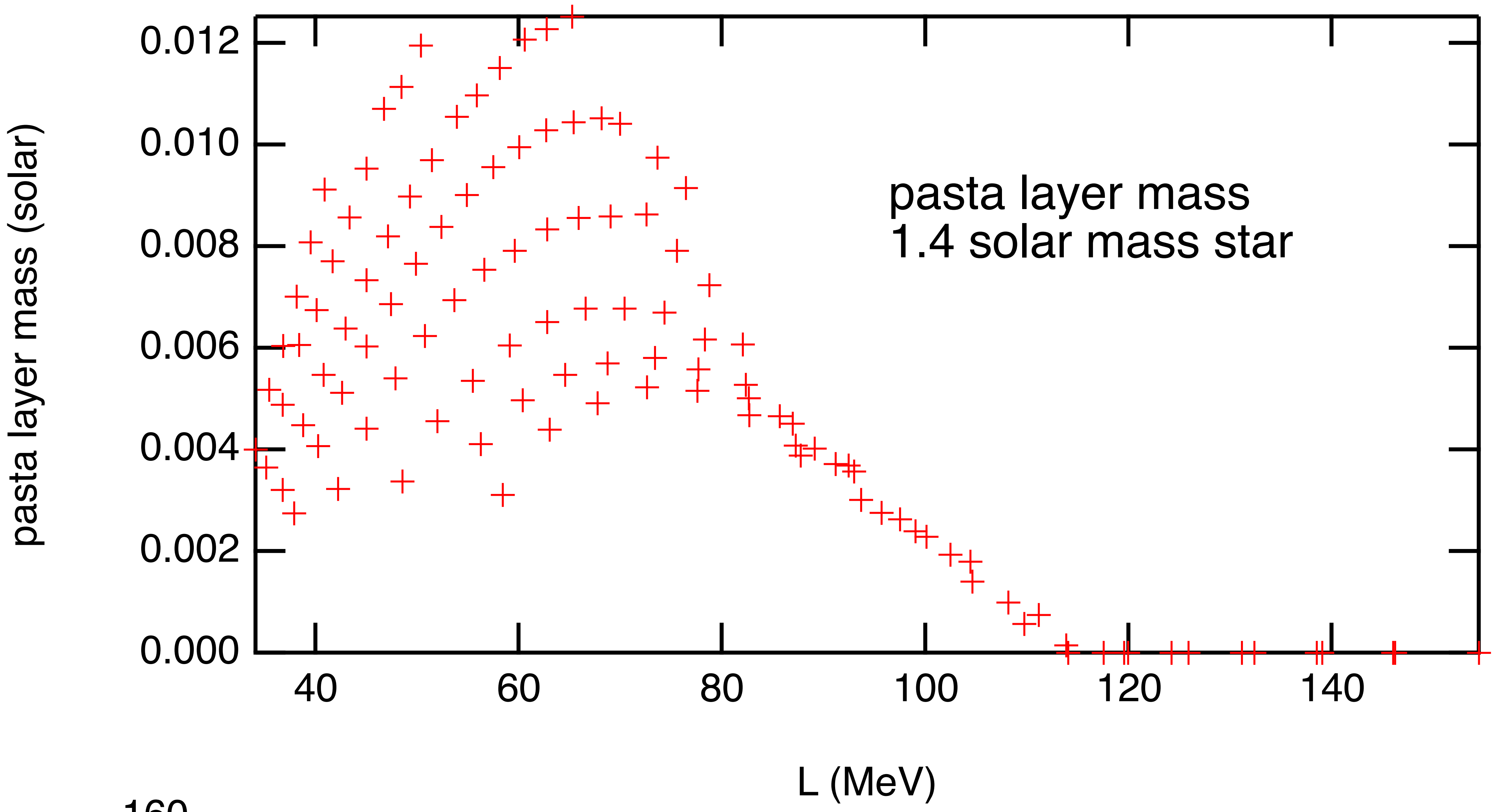


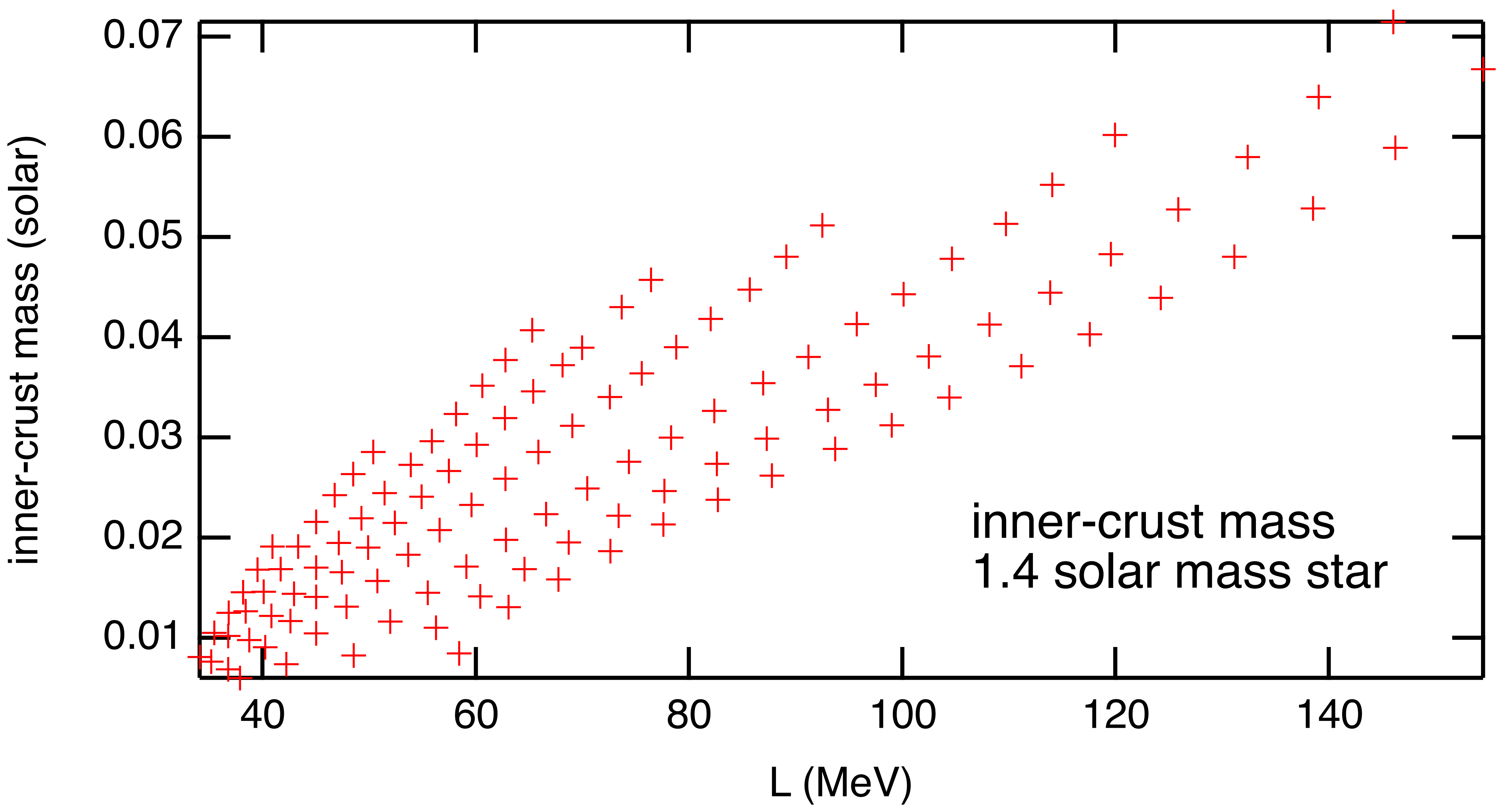


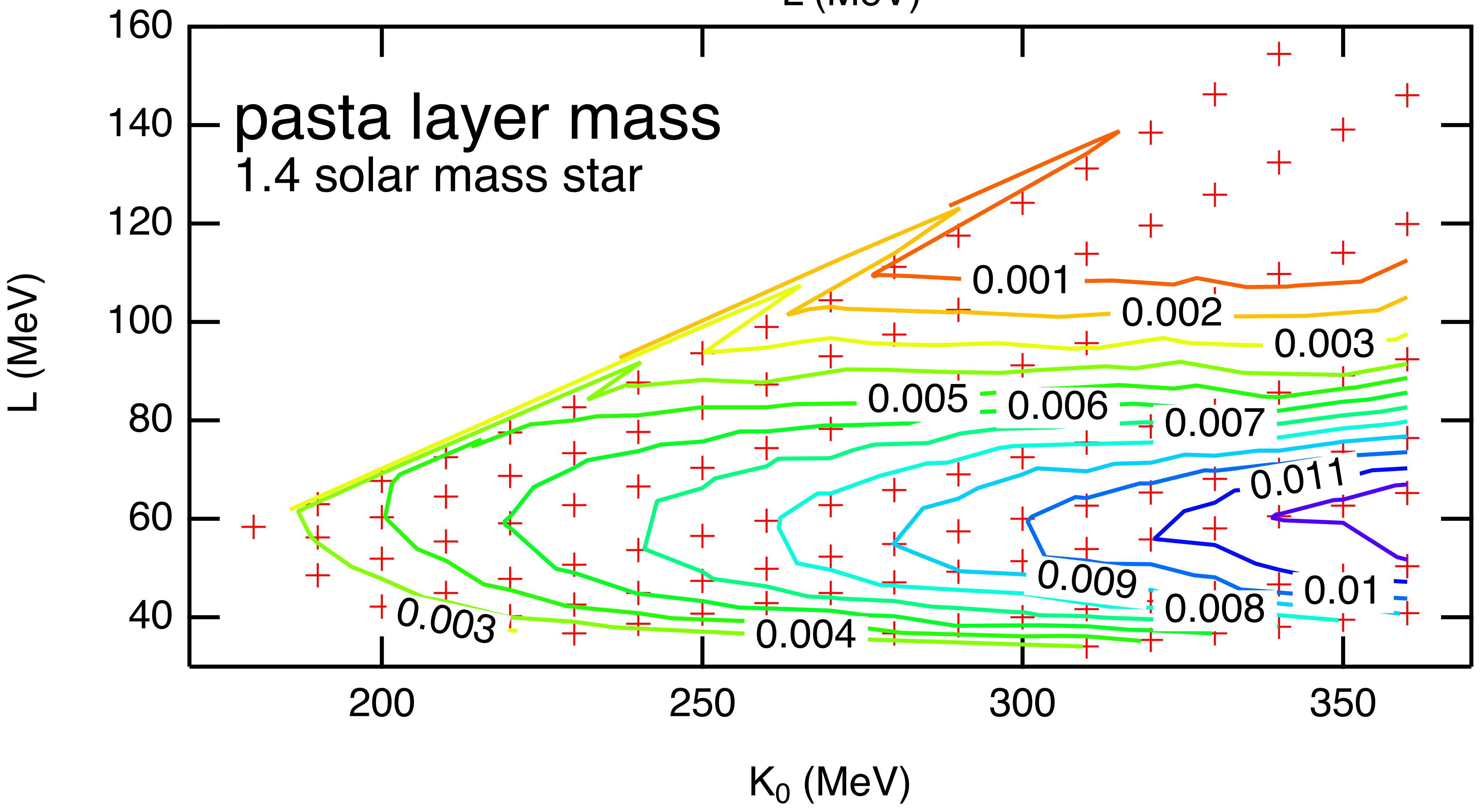


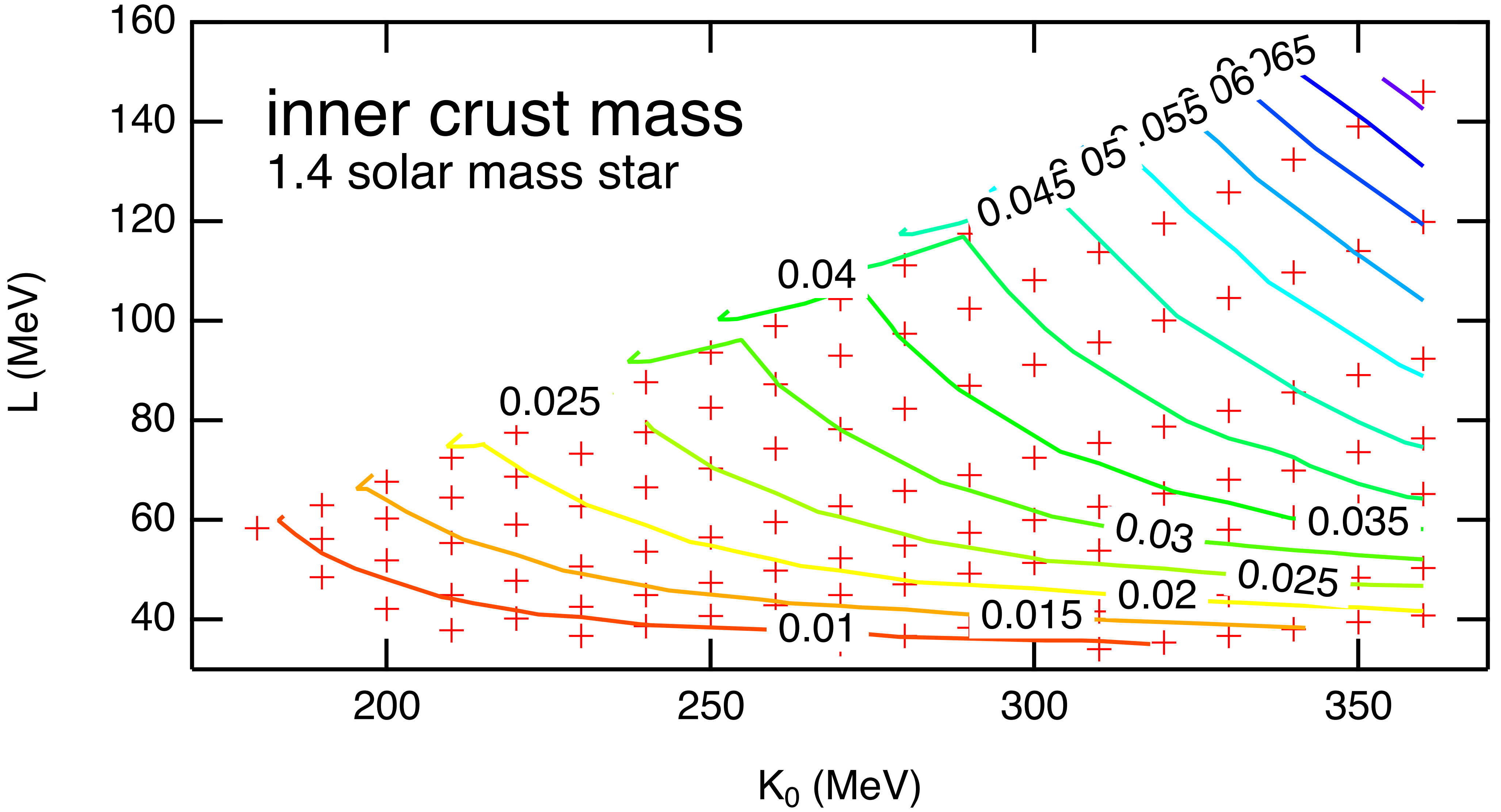


pasta layer mass (1.4 solar mass star)









Thickness of a pasta shape layer in NS with star mass M and radius R

- Sotani's useful fitting formula for thickness of a pasta layer
  - R(shape)/R=f(M, R, K_0 ,L)
  - This formula does not assume any specific EOS in the core and is a good perturbation formula with neutron star mass M and radius R.
  - This perturbation is reasonable because the layer thickness is thin compared with the star radius.

Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 16 June 2017 (MN LATEX style file v2.2)

Probing crustal structures from neutron star compactness

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arXiv:1706.04736v1 [astro-ph.HE] 15 Jun 2017

16 June 2017

ABSTRACT
With various sets of the parameters that characterize the equation of state (EOS) of nuclear matter, we systematically examine the thickness of a neutron star crust and of the pasta phases contained therein. Then, with respect to the thickness of the phase of spherical nuclei, the thickness of the cylindrical phase, and the crust thickness, we successfully derive fitting formulas that express the ratio of each thickness to the star's radius as a function of the star's compactness, the incompressibility of symmetric nuclear matter, and the density dependence of the symmetry energy. In particular, we find that the thickness of the phase of spherical nuclei has such a strong dependence on the stellar compactness as the crust thickness, but both of them show a much weaker dependence on the EOS parameters. Thus, via determination of the compactness, the thickness of the phase of spherical nuclei as well as the crust thickness can be constrained reasonably, even if the EOS parameters remain to be well-determined.

Key words: stars: neutron – equation of state

1 INTRODUCTION

Neutron stars help to probe the physics in extreme conditions mainly because the star is so compact that the density inside the star can be significantly beyond normal nuclear density (Haensel, Potekhin & Yakovlev 2007). Moreover, the surface magnetic field can be as high as $\sim 10^{15}$ G (Kouveliotou et al. 1998; Hurley et al. 1999), while the rotation period can be as short as ∼msec (Pulsar Group 2016). Thus, observations of neutron star phenomena associated with such compactness, high magnetic fields, and/or rapid rotation could leave an imprint of the properties of matter under such extreme conditions. However, the neutron star structure has yet to be fixed, because the equation of state (EOS) of matter in the star is still uncertain especially for a high density region. Even so, a conceptual picture of the neutron star structure is theoretically established.

Just below the star's surface lies an ocean composed of iron, under which matter forms a lattice structure due to the Coulomb interaction. This region is called a crust, where the matter behaves as a solid (or as a liquid crystal). The region below the crust corresponds to a core, where the matter becomes uniform and behaves as a fluid. The density at the base of the crust is expected to lie between ∼ (1/3–1) times normal nuclear density, depending on the EOS of nuclear matter (Oyamatsu & Iida 2007). This EOS is often characterized by several parameters that determine the Taylor expansion with respect to the nucleon density and neutron excess around the saturation point of symmetric nuclear matter (Lattimer 1981), which in turn can be constrained from terrestrial nuclear experiments (Oyamatsu & Iida 2003; Tsang et al. 2012). One of the key parameters that control the properties of matter in the crust thickness is known to be the slope parameter L of the symmetry energy (Oyamatsu & Iida 2007), which has yet to be fixed (Li 2017). This means that one may be able to extract the value of L from astronomical observations. In fact, after the discoveries of quasi-periodic oscillations in the soft-gamma repeaters (Watts & Strohmayer 2006), attempts to constrain L have been done by identifying the observed frequencies as the crustal torsional oscillations (Steiner & Watts 2009; Gearheart et al. 2011; Sotani et al. 2012, 2013a,b; Sotani 2014, 2016; Sotani, Iida & Oyamatsu 2016).

Additionally, the possible presence of non-spherical (pasta) nuclei in the deepest region of the crust of cold neutron stars has been theoretically considered since Lorenz, Ravenhall & Pethick (1993); Oyamatsu (1993) (see also Pethick & Ravenhall

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Summary

- OI EOS family gives the structure of nuclei in nuclear chart and in neutron stars as functions of (K_0, L) related to nuclear pressure.
- From the saturation properties of stable nuclei, we see correlations n_0-K_0, S_0-L and w_0-F_0.
- AME2016 excludes low and high L values, and favors $40 \leq L \leq 90$ (MeV).
- The structure of pasta layers in neutron star crusts depends mainly on L.
- For thickness of a pasta layer, we recommend useful Sotani's fitting formula.
- OI EOS family full tables will be available on line hopefully within a year.
  - We have been accumulating results and checking them now.
  - We hope the tables will be interesting and useful for you.